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Deep Learning Spacetime from Quantum Data

Neural network-based approaches to holography and inverse problems

2026. 6. 11

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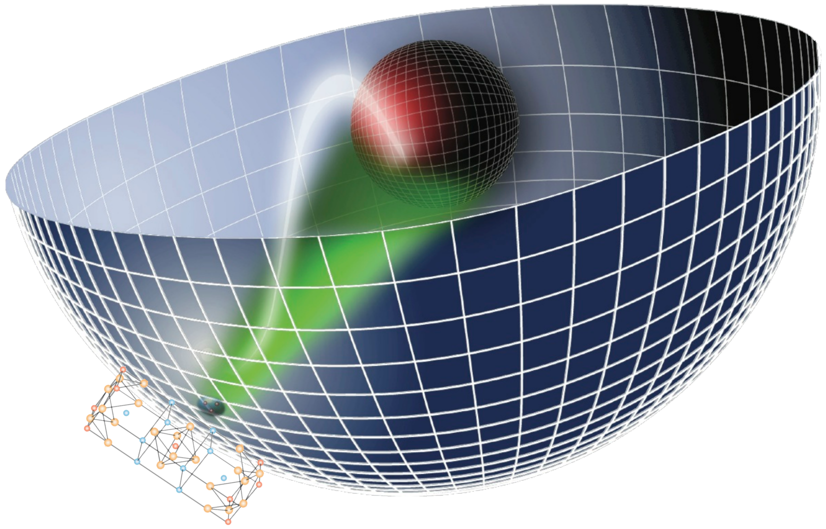
Deep Learning Spacetime from Quantum Data

Deep Learning $\boxed{\text{Bulk}}$ $\boxed{\text{Spacetime}}$ from $\boxed{\text{Boundary}}$ $\boxed{\text{Quantum Data}}$

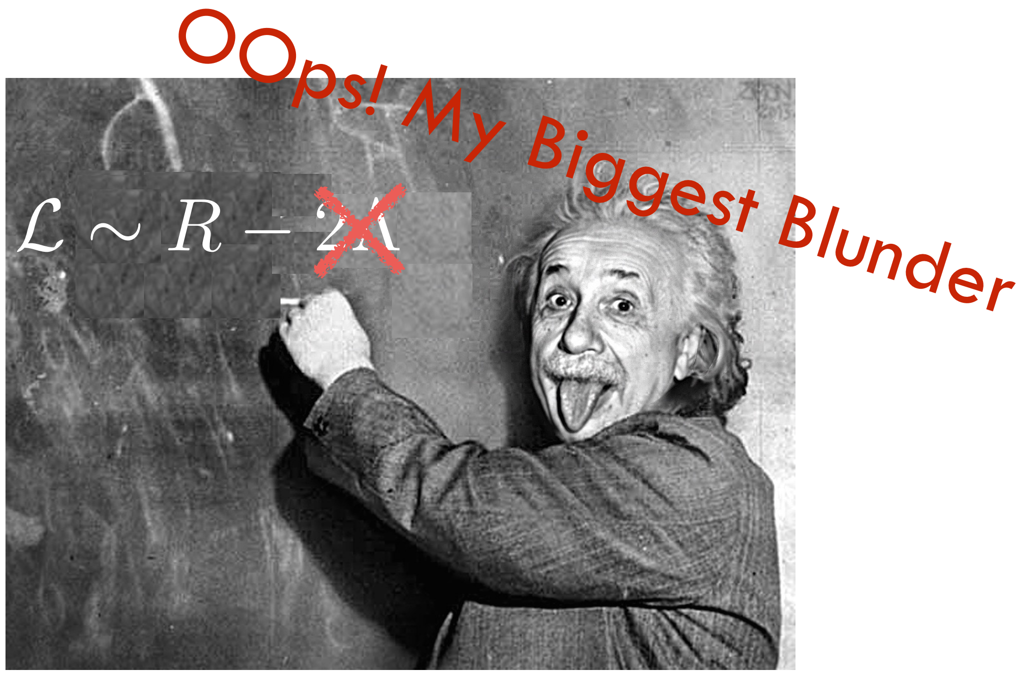
Holographic principle: Blackhole (spacetime)~Quantum Matter

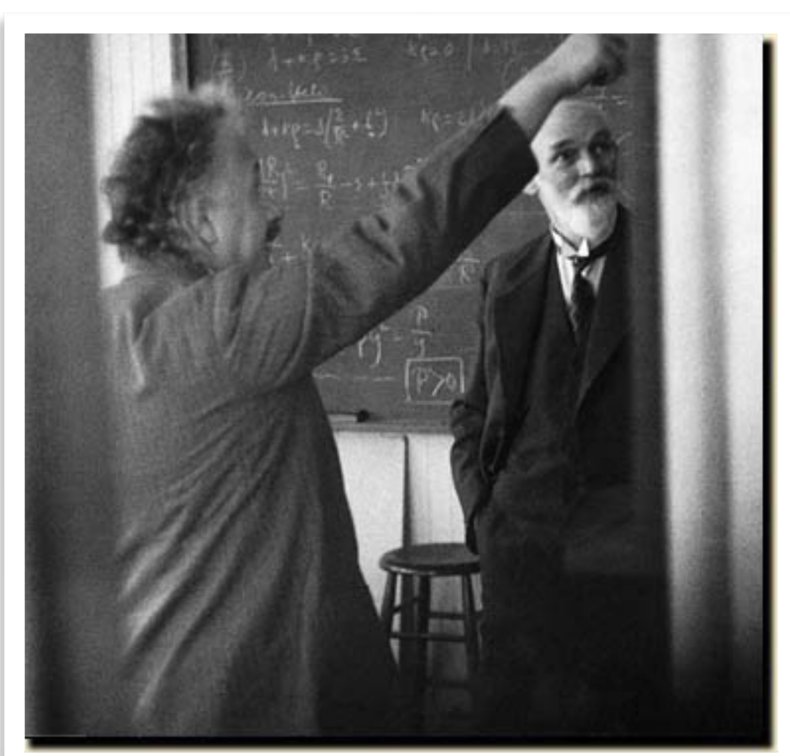
AdS/CMT
Anti de Sitter/Condensed Matter Theory

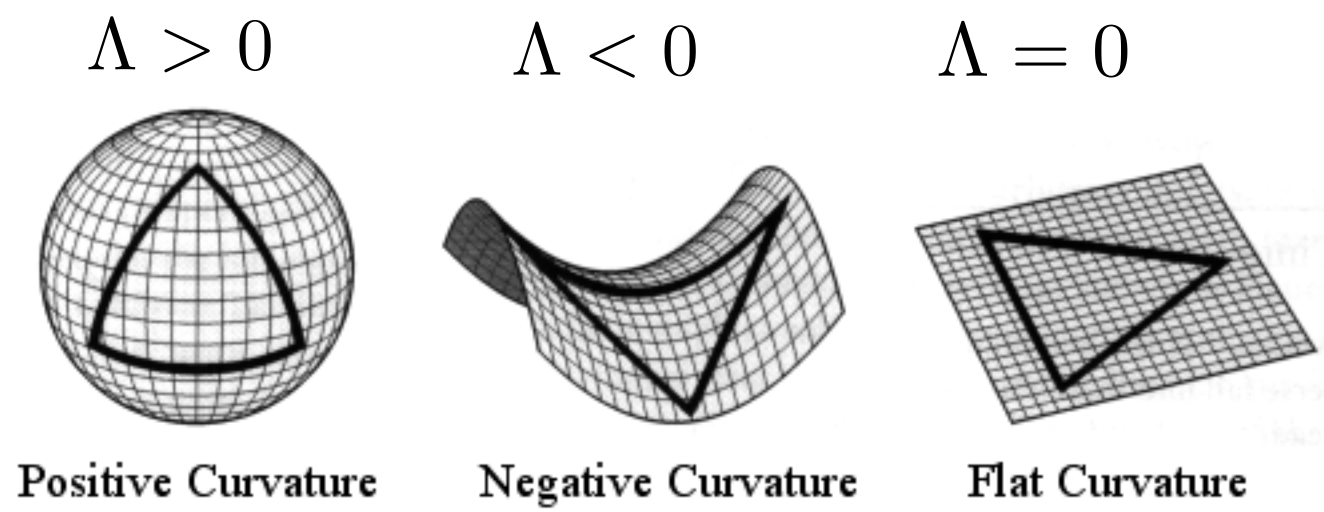


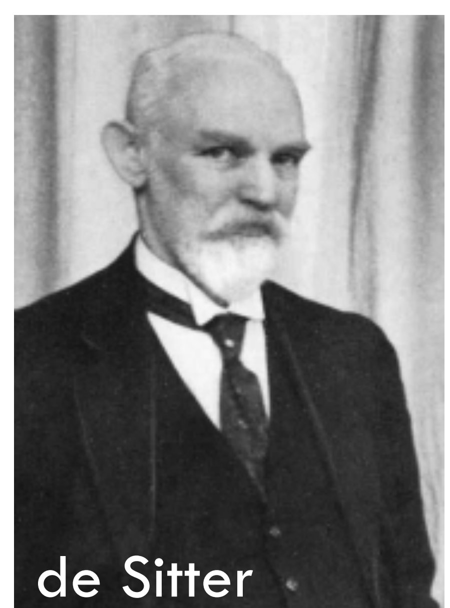
Anti de Sitter space

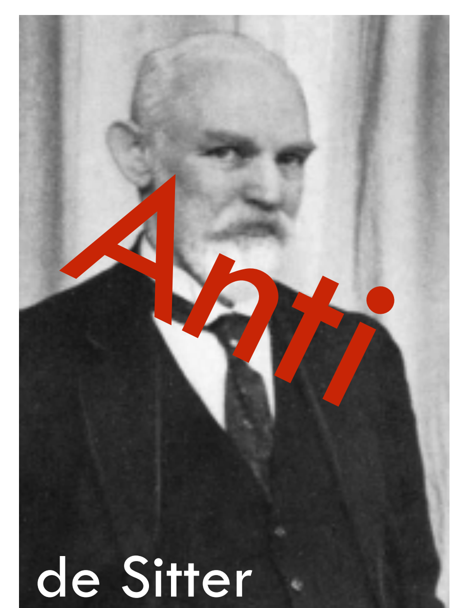
NOT the black-holes in the sky
BUT the black-holes in the box (so called AdS space)
In higher spacetime dimension











AdS
Anti de Sitter

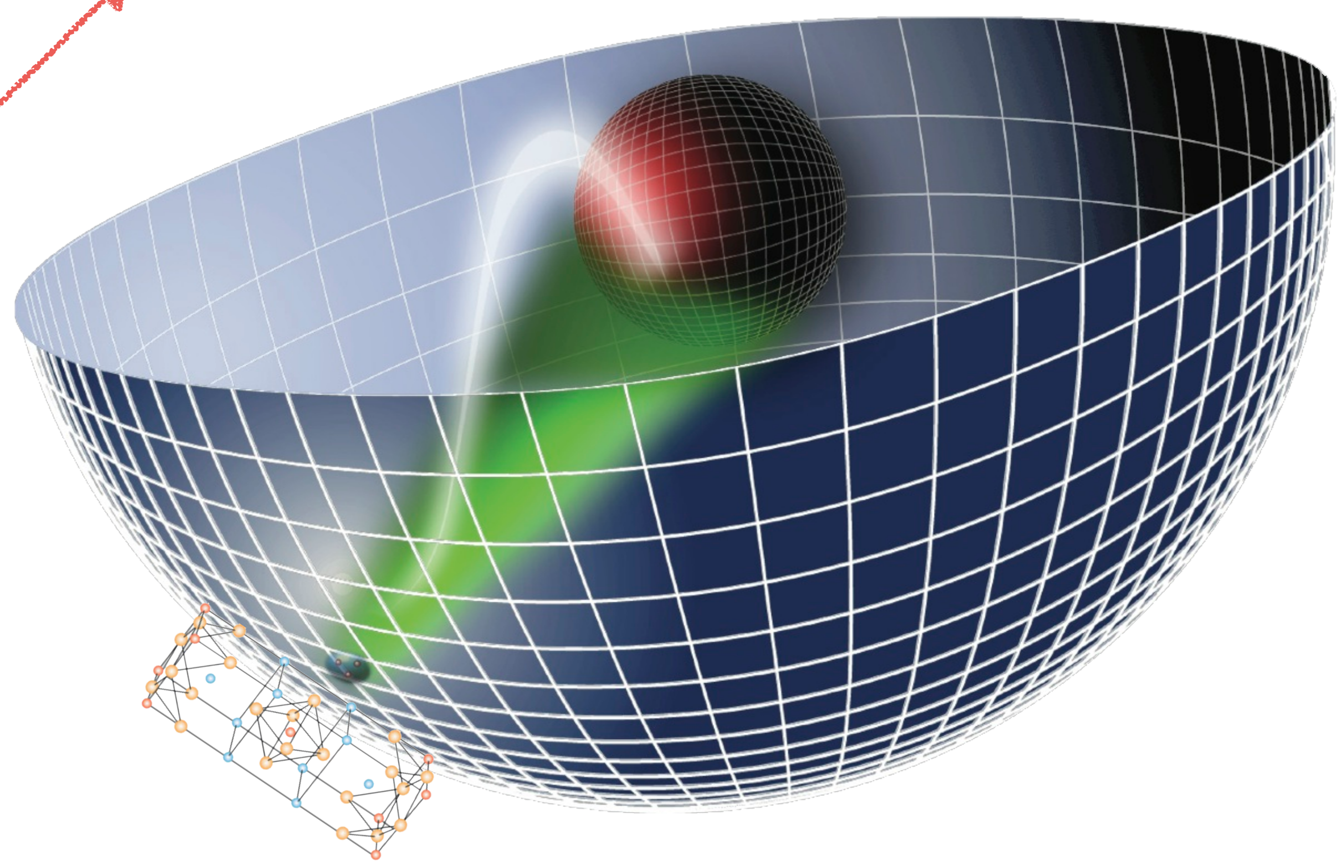
Holographic Principle

Why Holography?
Information is in 2D or 3D?

Holographic Principle

“Semi-classical” gravity system: 5D   Classical fields

Narrow sense



$$g_{\mu\nu} \sim T^{\mu\nu}$$

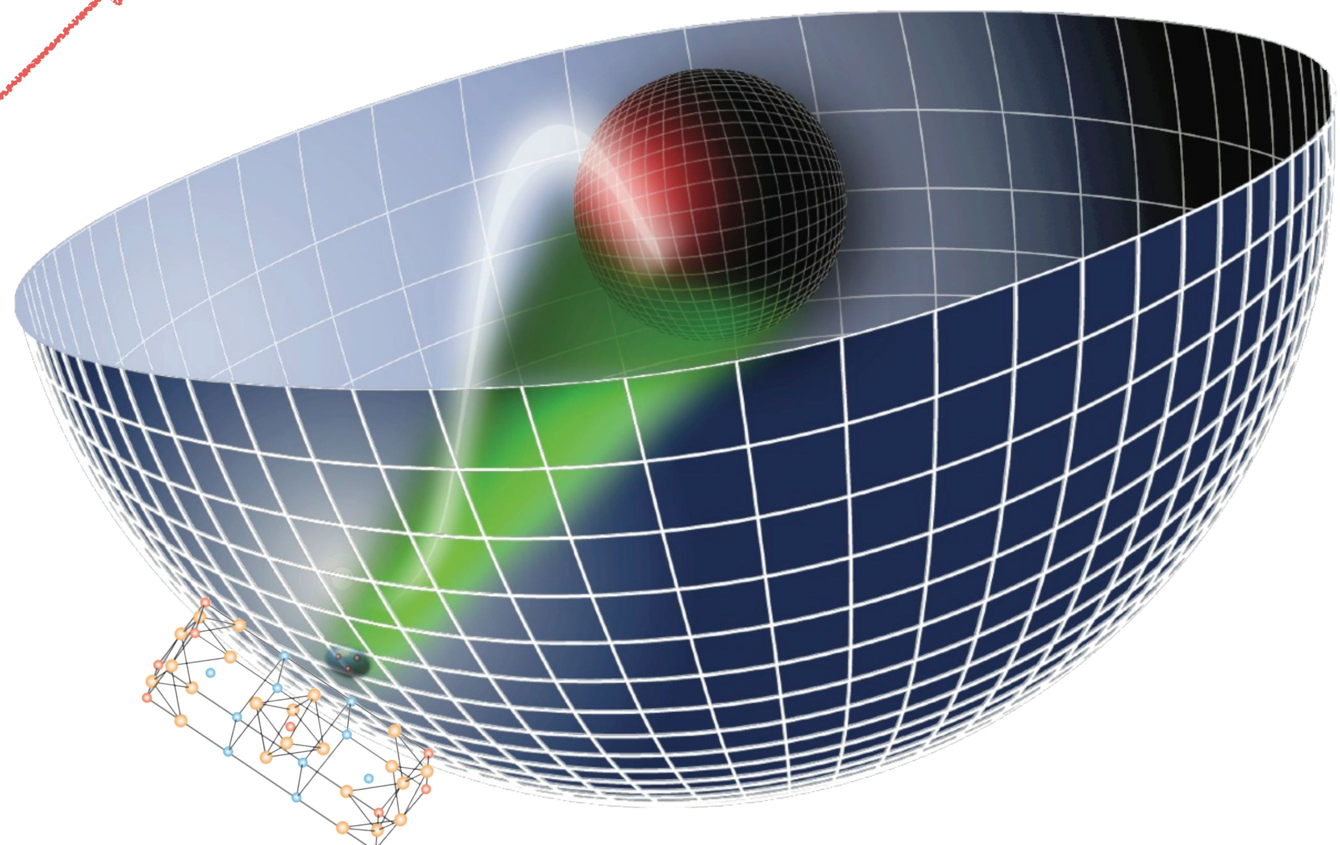
$$A_{\mu} \sim J^{\mu}$$

“Strongly” coupled field theory: 4D

Quantum operators

“Semi-classical” gravity system: 5D

Classical fields



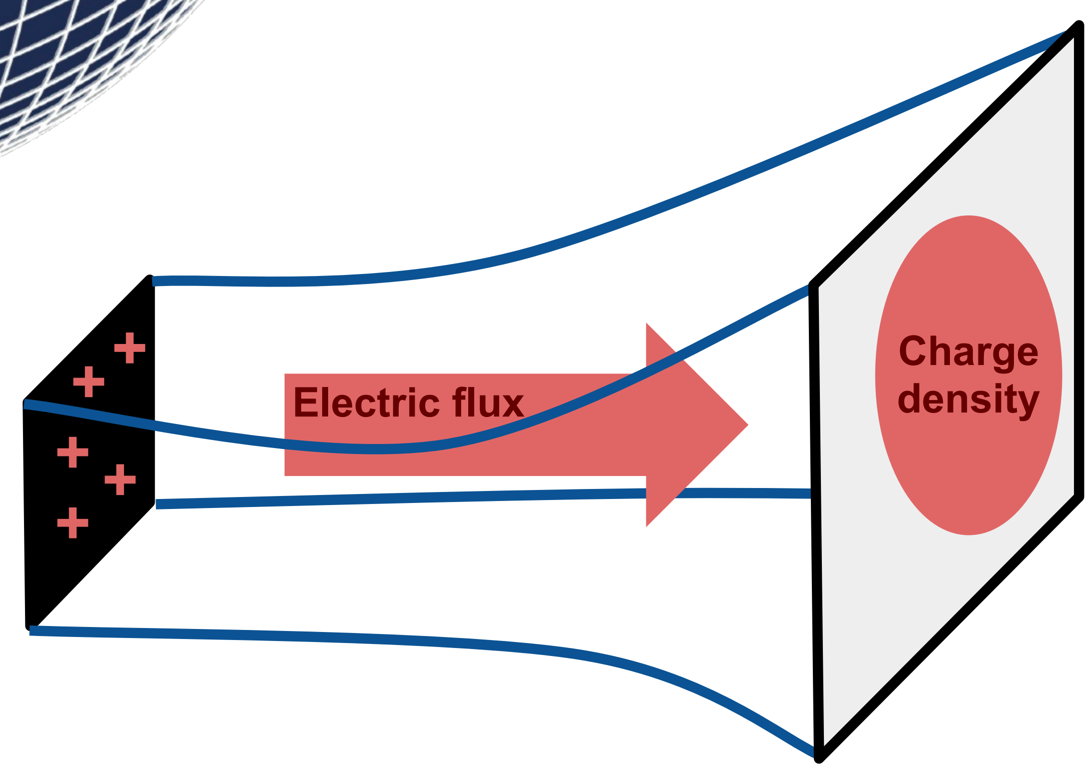
Narrow sense

$$g_{\mu\nu} \sim T^{\mu\nu}$$

$$A_{\mu} \sim J^{\mu}$$

“Strongly” coupled field theory: 4D

Quantum operators



Extra dimension

Gunnar Nordstrom (1913)

$$A_{\hat{\mu}}(x,z) = \{A_\mu(x), A_5(x)\}; \hat{\mu} = 0,1,2,3,5; \mu = 0,1,2,3$$

Extra dimension

Gunnar Nordstrom (1913)

$$A_{\hat{\mu}}(x,z) = \{A_\mu(x), A_5(x)\}; \hat{\mu} = 0,1,2,3,5; \mu = 0,1,2,3$$

His work was not very much appreciated at his home University;
when he applied for a travel grant, a colleague commented:
one can study the fourth dimension at home, without any trips abroad.

Extra dimension

Gunnar Nordstrom (1913)

$$A_{\hat{\mu}}(x,z) = \{A_\mu(x), A_5(x)\}; \hat{\mu} = 0,1,2,3,5; \mu = 0,1,2,3$$

Kaluza(1921)-Klein(1926) idea

$$g_{\hat{\mu}\hat{\nu}}(x,z) = \begin{pmatrix} g_{\mu\nu}(x) & A_\mu(x) \\ A_\nu(x) & \varphi(x) \end{pmatrix}$$

Extra dimension

Gunnar Nordstrom (1913)

$$A_{\hat{\mu}}(x,z) = \{A_\mu(x), A_5(x)\}; \hat{\mu} = 0,1,2,3,5; \mu = 0,1,2,3$$

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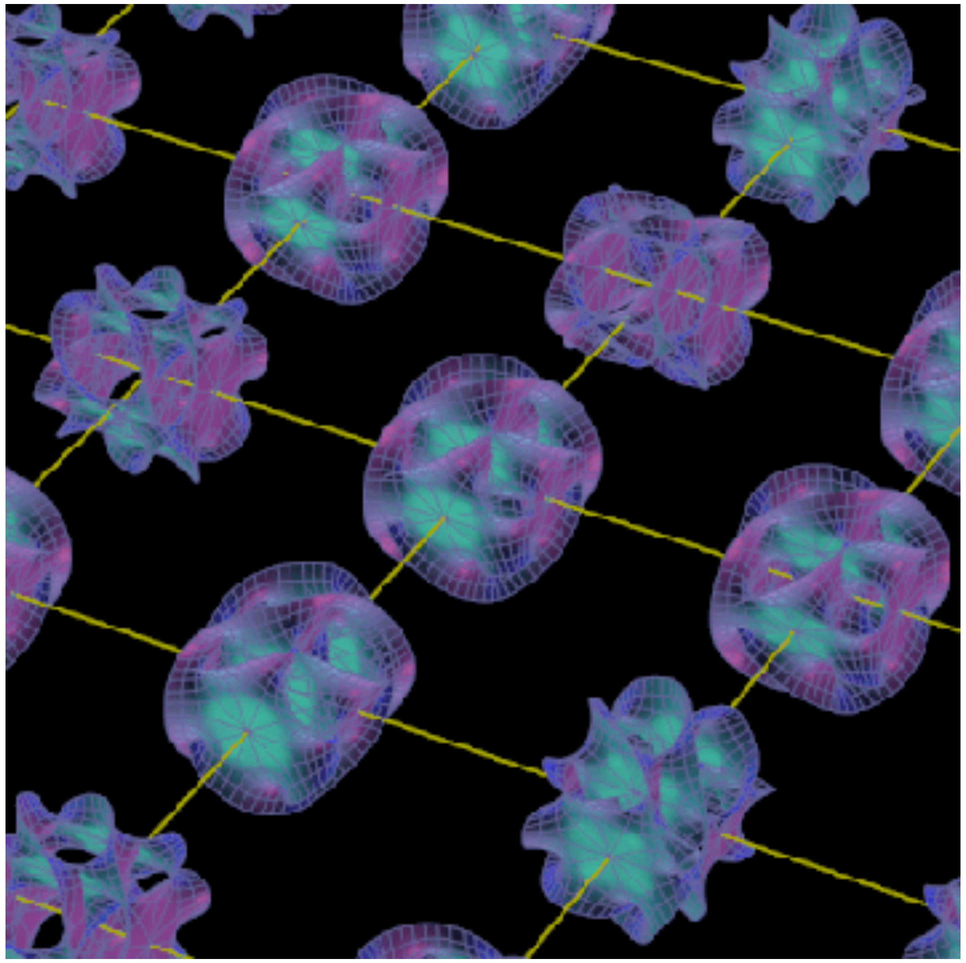
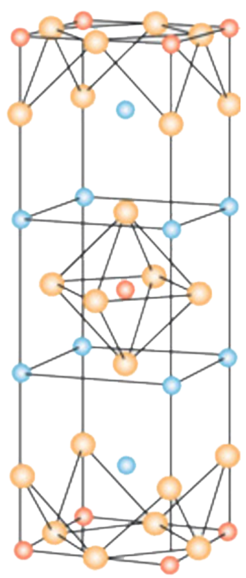
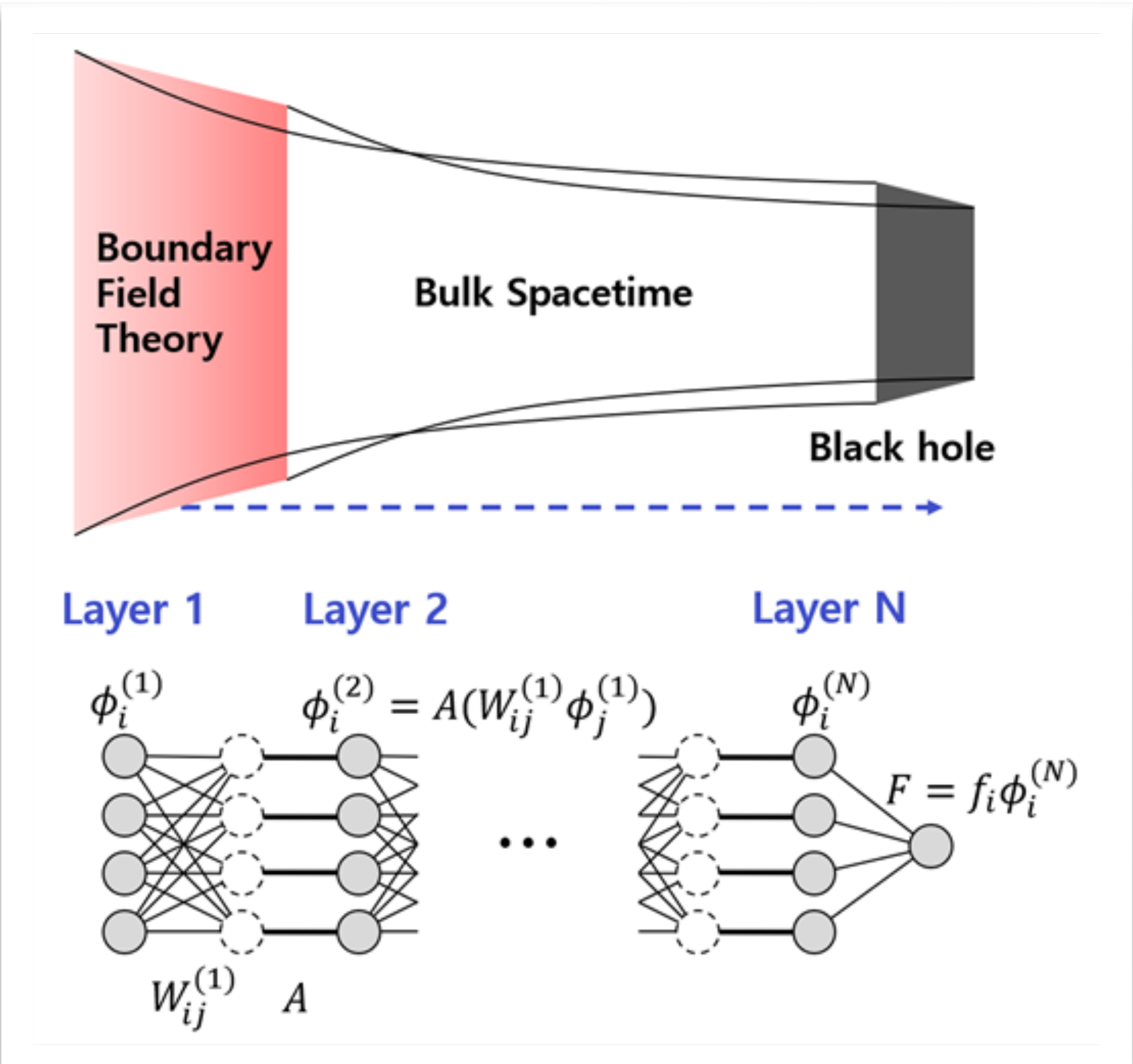


String theory in 10 dimension

Deep Learning Bulk Spacetime from Boundary data

- AdS(5D)/CMT(4D) Gravity in 5D = Quantum physics in 4D
- Holography, Holographic Principle
- Extra Dimensional Theory





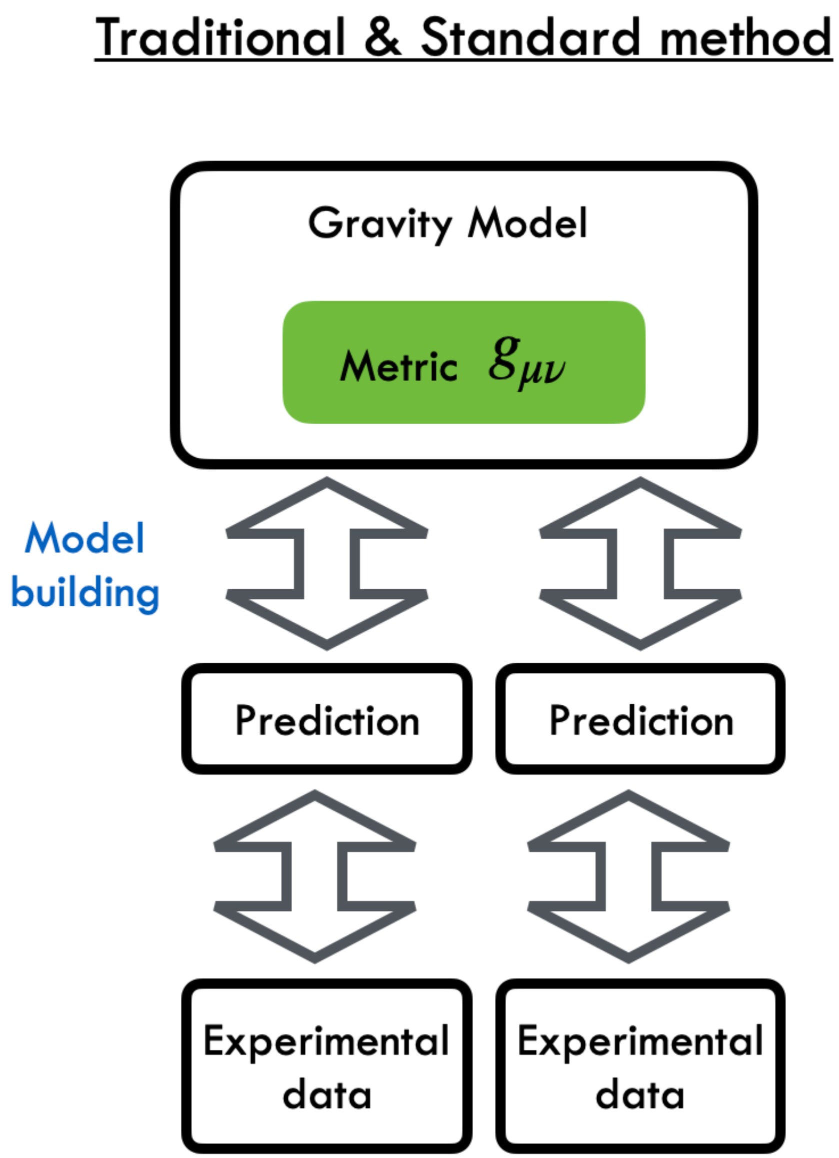
ML Questions:

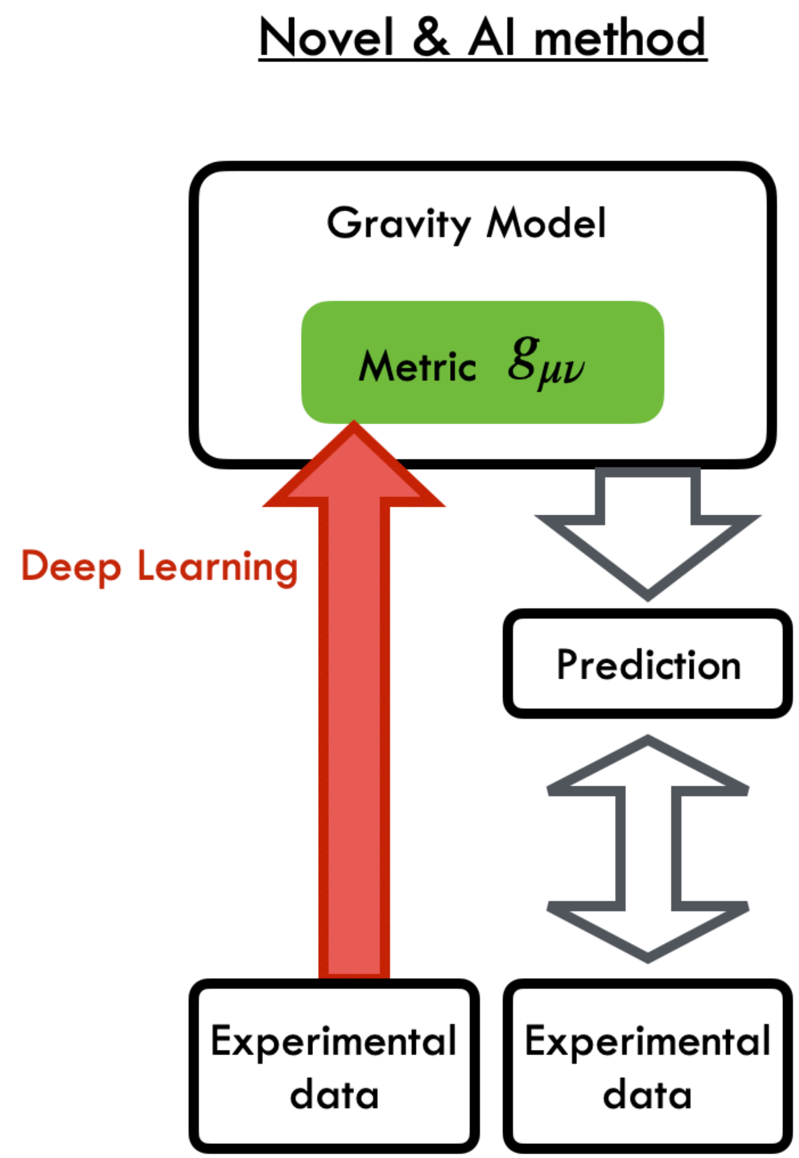
Can we understand
the extra (holographic) dimension
as a deep neural network?

Can we use a deep neural network
as a useful tool for holography?

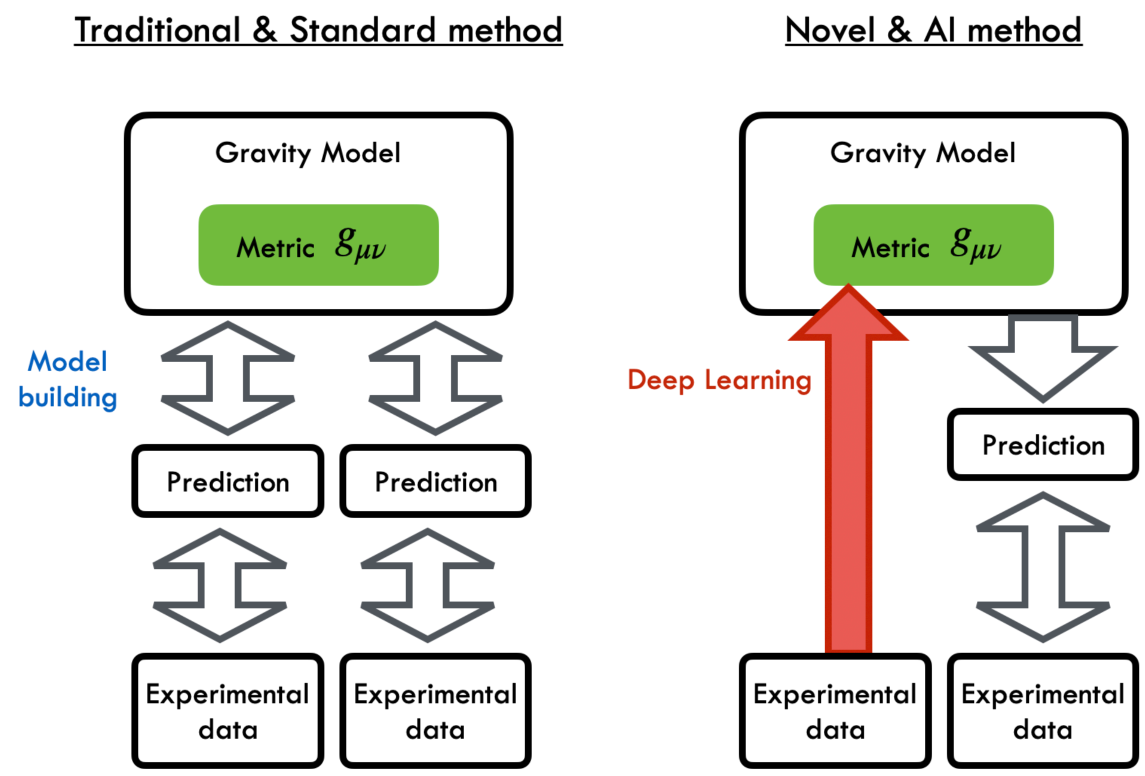
Answer:
Positive for both

Deep Learning as a methodology





Deep Learning as a methodology



Traditional method: from Bulk to Boundary

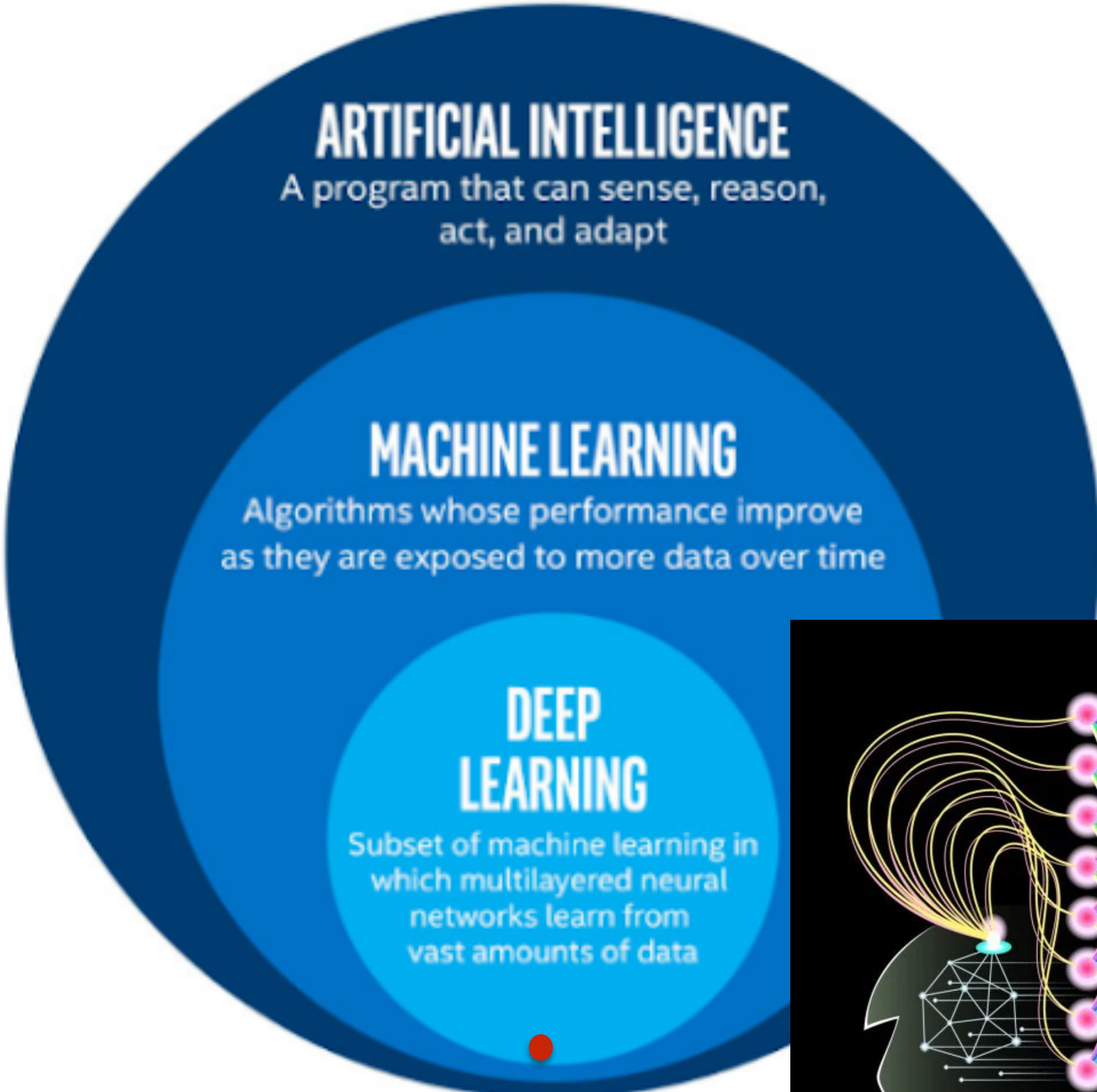
- Intuition, principle (ex: symmetry), "genius" etc required to make a model
- From a model, data are produced

AI method: from boundary to bulk

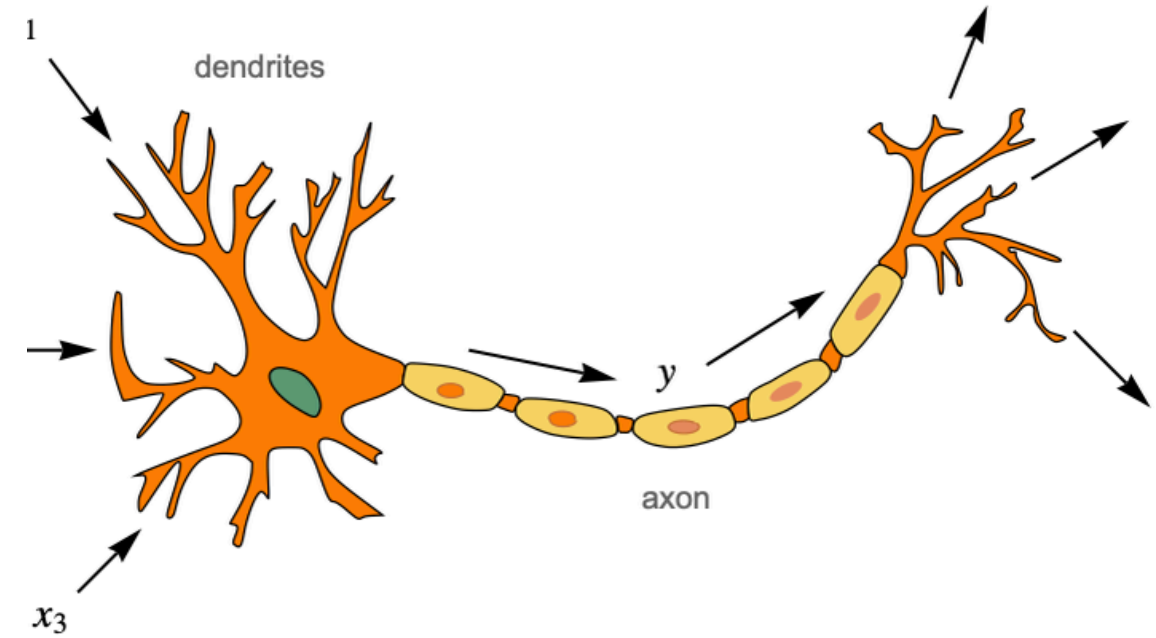
- Big data required
- Model yielding the answer is given by machine without any understanding
- Intuition, principle (ex: symmetry), etc implied by the model will be discovered by human

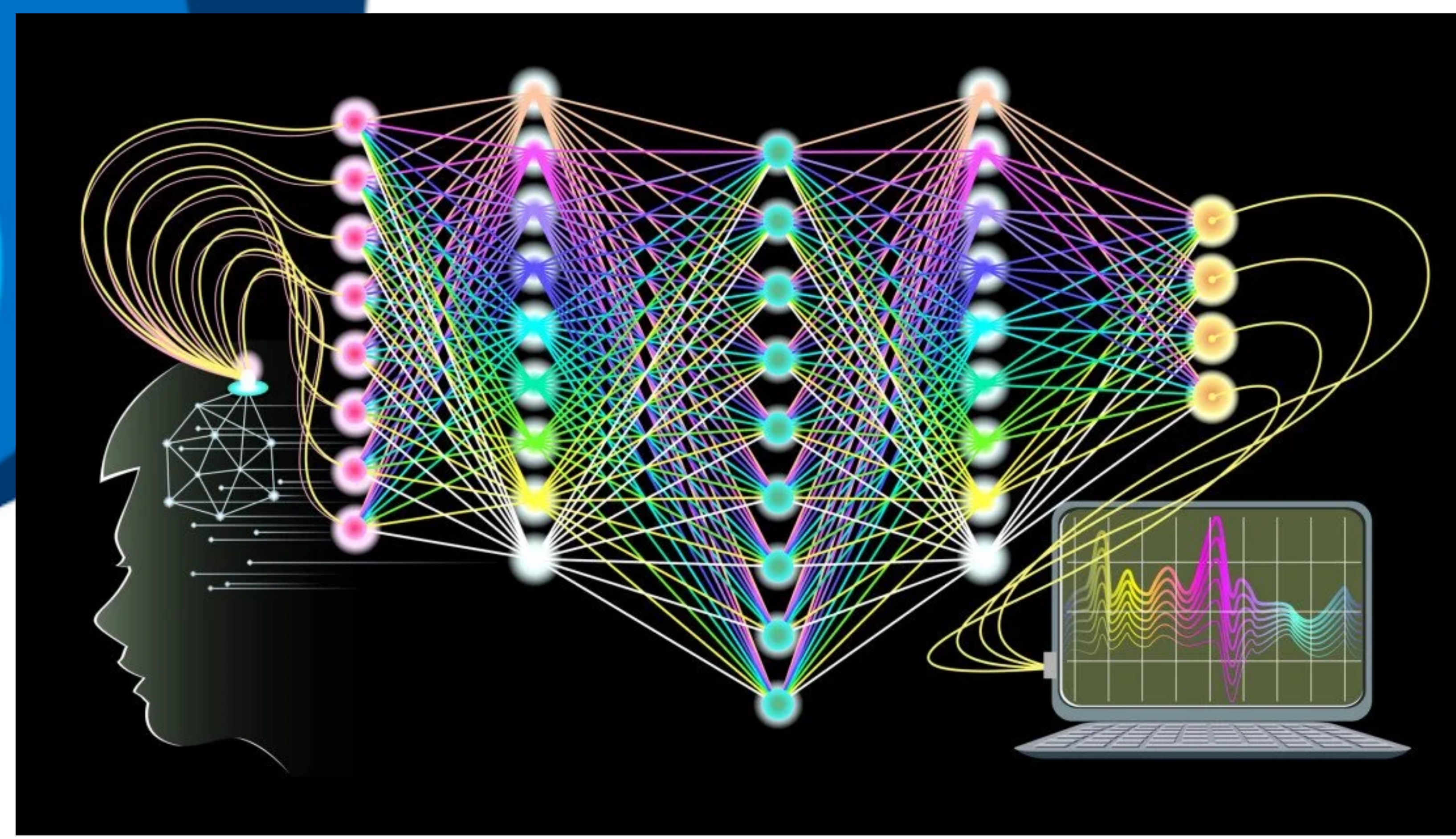
For a difficult problem,
once we are given a qualitative answer we can understand it more easily.
(for example, model of "T-linear resistivity + T^2 -Hall angle together")

Deep Learning 101



My talk

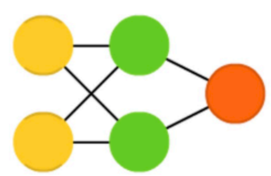




Machine learning = function approximator

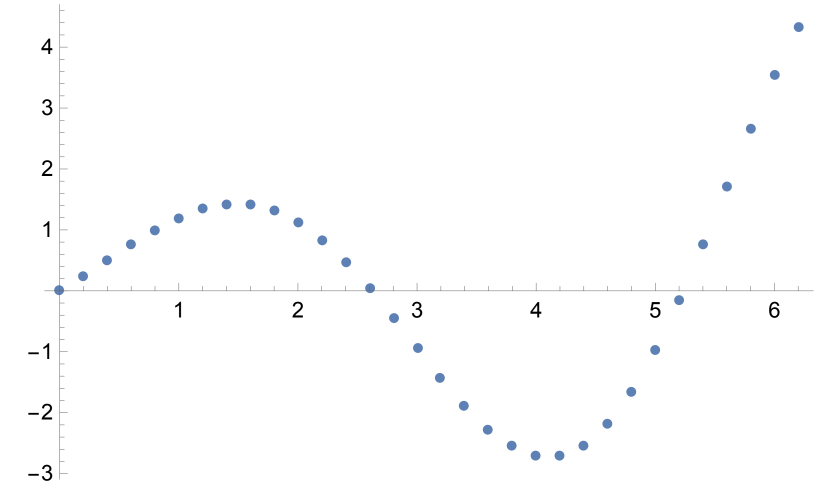
Input: a vector $(v_1, v_2, v_3, \ldots)$
Output: a value $f(v_1, v_2, v_3, \ldots)$

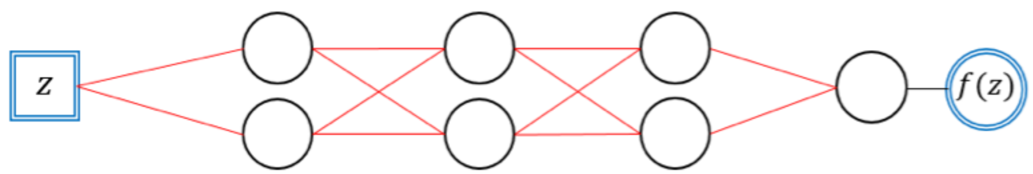
Network architecture = Function ansatz



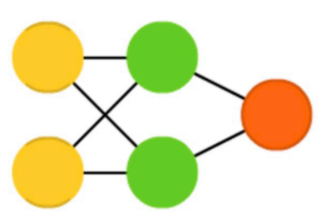
Perceptron model
[Rosenblatt 1958]
[Rumelhart, McClelland 1986]

Universal approximation theorem :
Any function can be approximated with more hidden units [Cybenko 1989] [Roux, Bengio 2008]





Perceptron model



$$f = W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right)$$

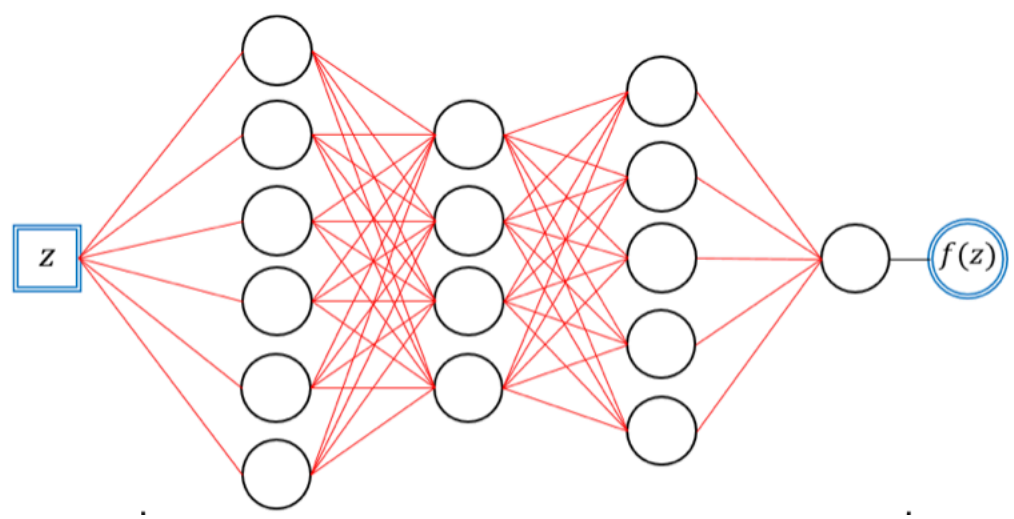
"Unit" (circles) : Vector components

"Weight" (lines) : Linear transformation to be optimized

"Activation function" (hidden line-end) : Nonlinear component-wise transf.

$$\varphi(x) \equiv \frac{1}{1 + e^{-x}}$$

- Training protocol :
  1) Prepare many sets $\{(x_j, f)\}$: input + output
  2) Train the network (adjust W) by lowering "Loss function" $E \equiv \sum_{\text{data}} \left| f - W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right) \right|$



Simple physic problem?

Reference I

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AdS/Deep-Learning made easy: simple examples*

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[2]University of California–Merced, Department of Physics, Merced, CA, USA

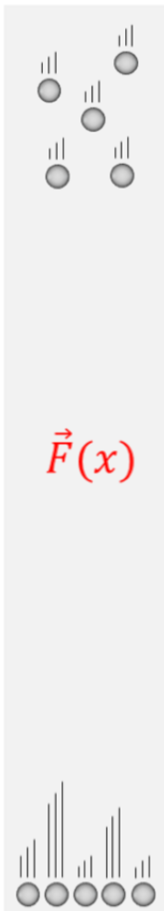
Abstract: Deep learning has been widely and actively used in various research areas. Recently, in gauge/gravity duality, a new deep learning technique called AdS/DL (Deep Learning) has been proposed. The goal of this paper is to explain the essence of AdS/DL in the simplest possible setups, without resorting to knowledge of gauge/gravity duality. This perspective will be useful for various physics problems: from the emergent spacetime as a neural network to classical mechanics problems. For prototypical examples, we choose simple classical mechanics problems. This method is slightly different from standard deep learning techniques in the sense that we not only have the right final answers but also obtain physical understanding of learning parameters.

Keywords: gauge/gravity duality, holographic principle, machine learning

DOI: 10.1088/1674-1137/abfc36

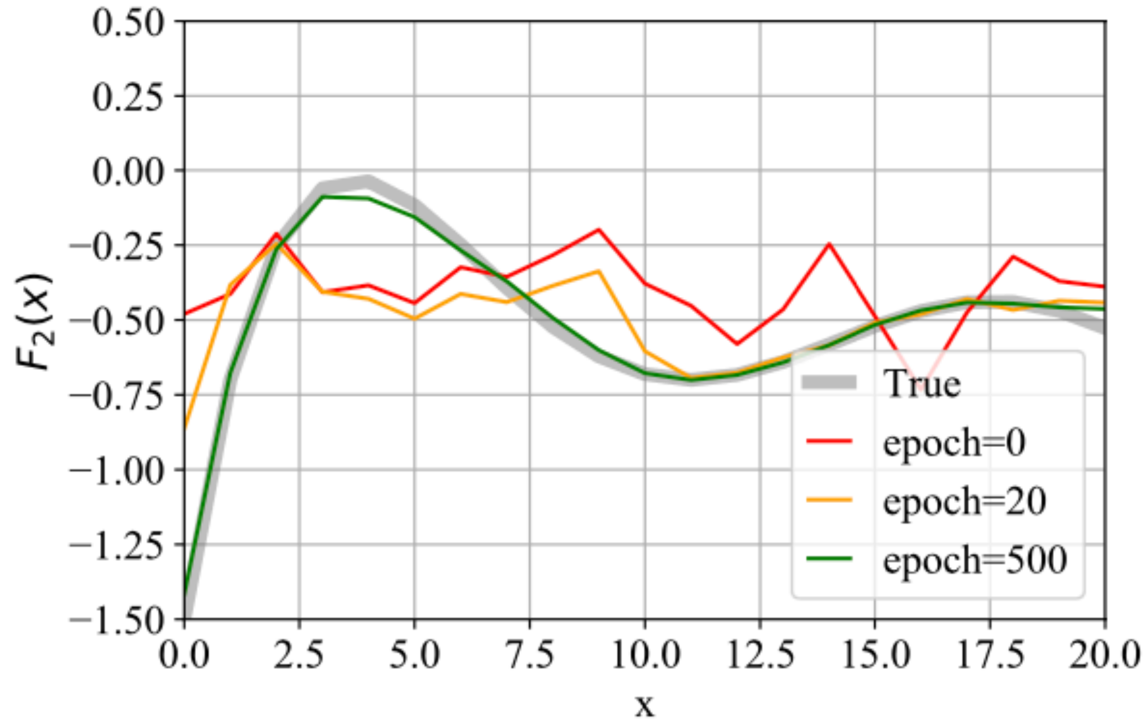
Deep Learning for ODE: classical mechanics

x_i, v_i



v_f

$$m\dot{v} = mg - F(x)$$
$$\dot{x} = v$$



1. (Data)

$$(v_i, x_i) \longrightarrow$$

3. (Numerical ODE solving)

$$m\dot{v} = mg - F(x) \longrightarrow$$
$$\dot{x} = v$$

5. (Loss)

$$L = \mathrm{avg}(\,|v_f - V_f|\,) \longleftarrow$$

4. (Solution)

$$V_f$$

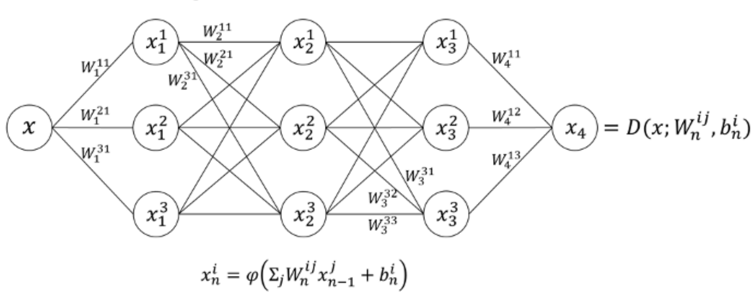
$$\frac{\partial L}{\partial W_n}, \frac{\partial L}{\partial b_n}$$

6. (Parameters optimizing)

Machine learning = function approximator

$$F(x) = D(x; W_n, b_n)$$

2. (Deep neural network)



Generalization

AdS/Deep learning: optical conductivity

$$m\ddot{x} = F$$

$$A(z)f''(z)+B(z)f'(z)+C(z)f(z) = F(z)$$

$$\boxed{(fA_x')' + \frac{w^2}{f}A_x = \frac{4\mu^2}{\gamma^2 r_h^2} r^2 A_x\,.}$$

$$\sigma(\omega) = \frac{1}{e^2}\frac{A_x'}{i\omega A_x}\bigg|_{r=0}\,.$$

$$A(z)f''(z)+B(z)f'(z)+C(z)f(z) = D(z)g(z)$$

$$E(z)g''(z)+F(z)g'(z)+G(z)g(z) = H(z)f(z)$$

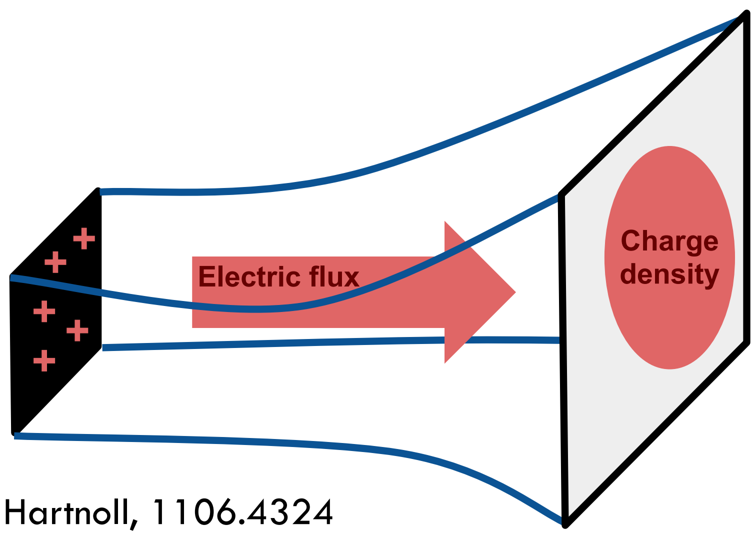
$$\boxed{\begin{aligned}
\partial_z^2 A_x &= \zeta\,\partial_z A_x + \left(\frac{z^2\mu^2}{f} - \xi\right) A_x + \frac{iz\mu}{f}\Phi\,,\\
\partial_z^2 \Phi &= \zeta\,\partial_z \Phi + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right)\Phi - \frac{iz\alpha^2\mu}{f}A_x\,,\\
\zeta &:= \frac{2i\omega}{(1-z)f'(1)} - \frac{f'(z)}{f(z)}, \qquad \xi := \frac{\omega^2}{f(z)^2} + \frac{i\omega}{(1-z)f'(1)}\left(\frac{i\omega}{(1-z)f'(1)} - \frac{1}{1-z} - \frac{f'(z)}{f(z)}\right)
\end{aligned}}$$

Holographic conductivity

- Einstein-Maxwell system

$$S_{\mathrm{EM}} = \int_M \mathrm{d}^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2\right]$$

- Reissner-Nordstrom-AdS black hole
  ~ Boundary field theory at finite temperature and density



Hartnoll, 1106.4324

$$+ \quad (f\delta A_x')' + \frac{\omega^2}{f}\delta A_x - \frac{4\mu^2 r^2}{\gamma^2 r_+^2}\delta A_x = 0$$

$$\delta A_x(r,\omega) = \frac{E}{i\omega} + J_x(\omega) r + \cdots$$

Source Expectation value

$$\sigma(\omega) = \frac{1}{e^2}\frac{A_x'}{i\omega A_x}\bigg|_{r=0}.$$

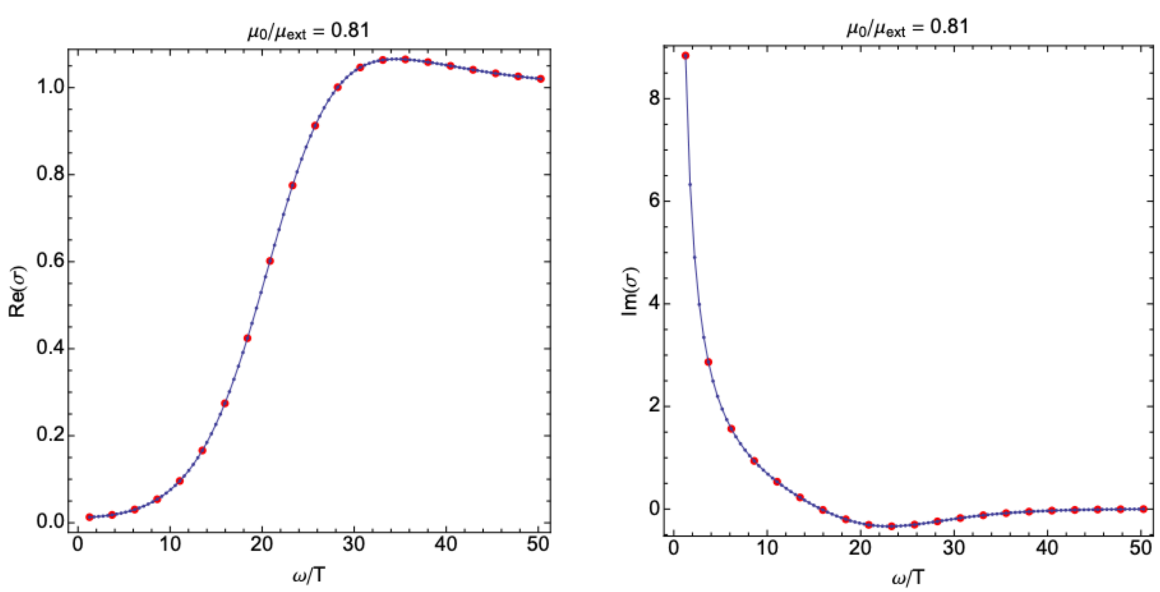


Fig. 5. Holographic optical conductivity.

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Deep learning bulk spacetime from boundary optical conductivity

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AdS/Deep learning: optical conductivity

Action

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

EOM

$$R_{ab} - \frac{1}{2}g_{ab}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right) - F_{ac}F_b^{\ c} - \frac{1}{2}\sum_{I=1}^{2}\partial_a X_I \partial_b X_I = 0\,,$$

$$\nabla^a F_{ab} = 0\,, \qquad \nabla_a\nabla^a X_I = 0\,,$$

Background

$$\mathrm{d}s^2 = \frac{1}{z^2}\left[-f(z)\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + \mathrm{d}x^2 + \mathrm{d}y^2\right]\,, \qquad f(z) = 1 - \frac{\alpha^2}{2}z^2 - \left(1 - \frac{\alpha^2}{2} + \frac{\mu^2}{4}\right)z^3 + \frac{\mu^2}{4}z^4$$

$$A = \mu\,(1-z)\,\mathrm{d}t\,, \qquad X_1 = \alpha\, x\,, \quad X_2 = \alpha\, y$$

Flucutation EOM I

$$\delta g_{tx} = e^{-i\omega t}\frac{h_{tx}(z)}{z^2}\,, \qquad \delta A_x = e^{-i\omega t}a_x(z)\,, \qquad \delta X_1 = e^{-i\omega t}\frac{\psi_x(z)}{\alpha}$$

$$a_x''(z) + \frac{f'(z)}{f(z)}a_x'(z) + \left(\frac{\omega^2}{f(z)^2} - \frac{\mu^2 z^2}{f(z)}\right)a_x(z) - \frac{i\mu z}{f(z)}\phi(z) = 0\,, \qquad \phi(z) := -\frac{f(z)\psi_x'(z)}{\omega\, z}$$

$$\phi''(z) + \frac{f'(z)}{f(z)}\phi'(z) + \left(\frac{\omega^2}{f(z)^2} - \frac{\alpha^2}{f(z)} - \frac{f'(z)}{z f(z)}\right)\phi(z) + \frac{i\alpha^2\mu z}{f(z)}a_x(z) = 0\,,$$

$$\boxed{\begin{aligned}\sigma(\omega) &= \frac{1}{i\omega}G^R_{j^x j^x}(\omega) = \frac{1}{i\omega}\frac{a_x^{(R)}}{a_x^{(S)}} \\ &= \frac{A_x'(z_{\mathrm{fin}})}{i\omega A_x(z_{\mathrm{fin}})} - \frac{1}{f'(1)}\end{aligned}}$$

Flucutation EOM II

$$A_x(z) := (1-z)^{-\frac{i\omega}{f'(1)}}a_x(z)\,, \qquad \Phi(z) := (1-z)^{-\frac{i\omega}{f'(1)}}\phi(z)\,,$$

$$\boxed{\begin{aligned}\partial_z^2 A_x &= \zeta\,\partial_z A_x + \left(\frac{z^2\mu^2}{f} - \xi\right)A_x + \frac{iz\mu}{f}\Phi\,, \\ \partial_z^2 \Phi &= \zeta\,\partial_z \Phi + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right)\Phi - \frac{iz\alpha^2\mu}{f}A_x\,, \\ \zeta &:= \frac{2i\omega}{(1-z)f'(1)} - \frac{f'(z)}{f(z)}\,, \quad \xi := \frac{\omega^2}{f(z)^2} + \frac{i\omega}{(1-z)f'(1)}\left(\frac{i\omega}{(1-z)f'(1)} - \frac{1}{1-z} - \frac{f'(z)}{f(z)}\right)\end{aligned}}$$

AdS/Deep learning: optical conductivity

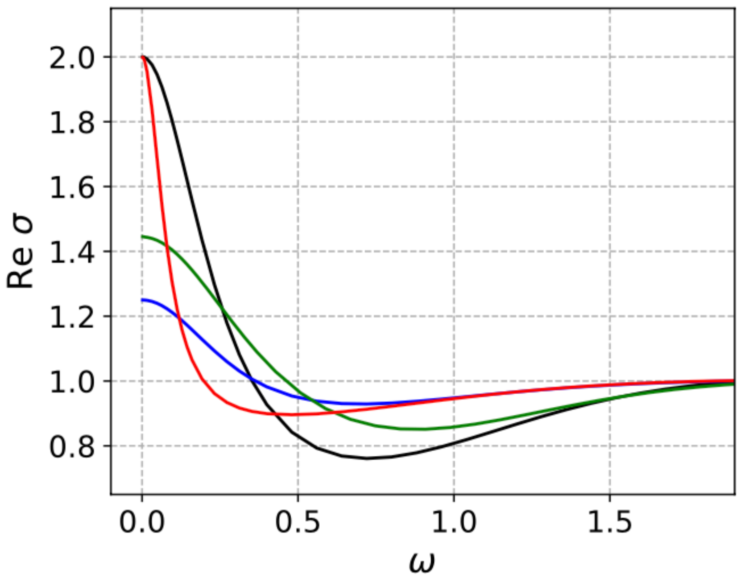
Setup

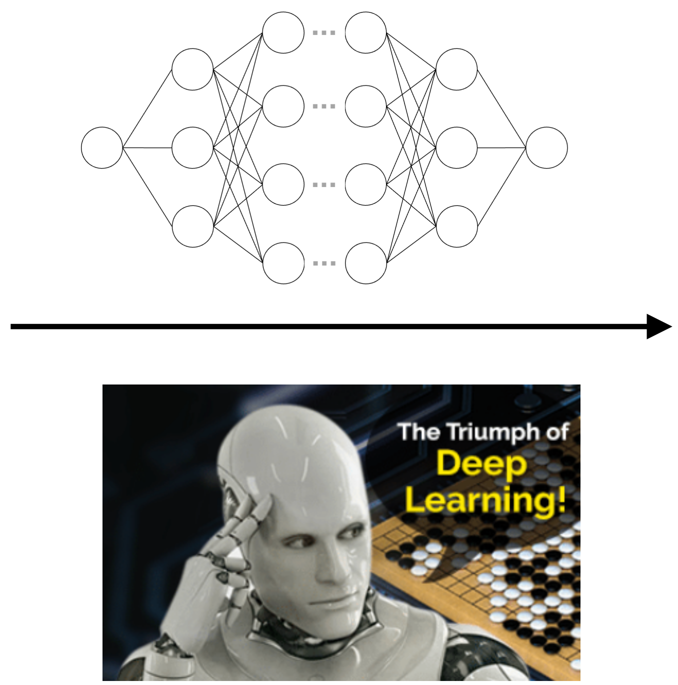
$$\mathcal{S} = \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

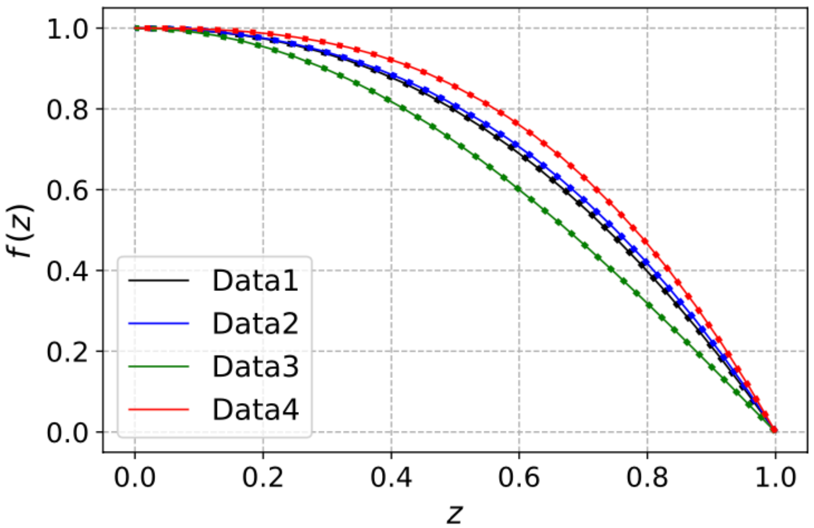
$$ds^2 = \frac{1}{z^2}\left[-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^2 + dy^2\right],$$

$$A = \mu\,(1-z)\,dt\,, \qquad X_1 = \alpha\,x\,, \quad X_2 = \alpha\,y$$

What is the bulk metric giving the conductivity at boundary







Harder problem

arXiv > hep-th > arXiv:2502.10245

High Energy Physics – Theory

[Submitted on 14 Feb 2025]

Deep learning–based holography for T–linear resistivity

Byoungjoon Ahn, Hyun–Sik Jeong, Chang–Woo Ji, Keun–Young Kim, Kwan Yun

Science

Current Issue

HOME > SCIENCE > VOL. 377, NO. 6602 > STRANGER THAN METALS

REVIEW | PHYSICS

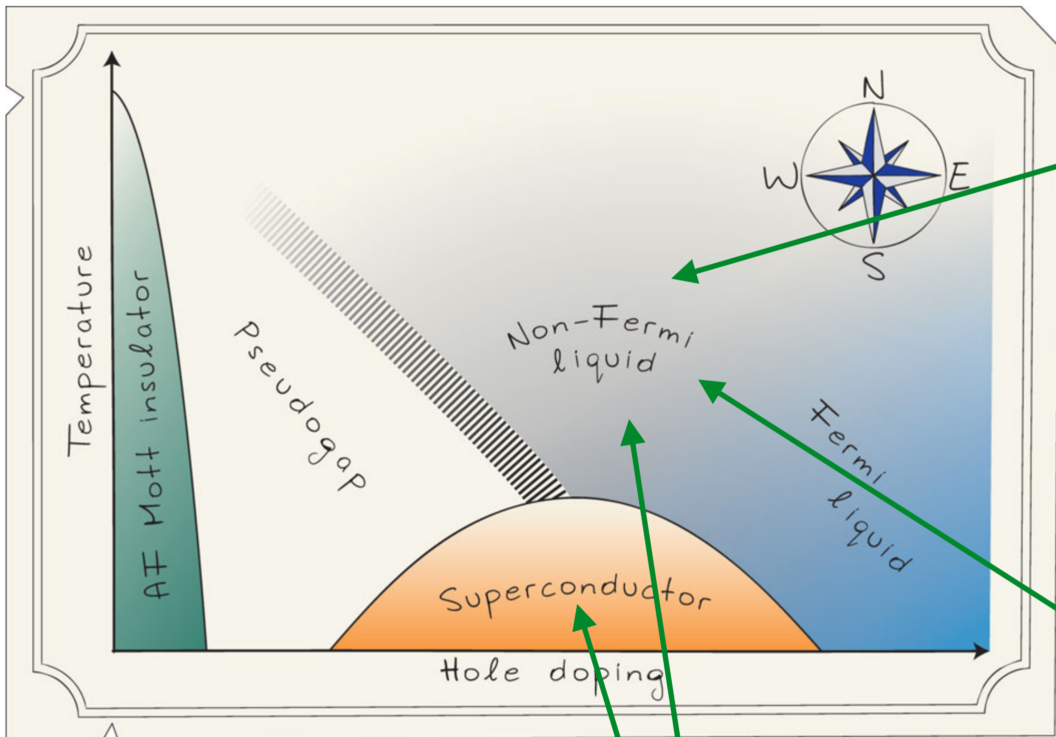
Stranger than metals

PHILIP W. PHILLIPS , NIGEL E. HUSSEY , AND PETER ABBAMONTE Authors Info & Affiliations

SCIENCE • 8 Jul 2022 • Vol 377, Issue 6602 • DOI: 10.1126/science.abh4273

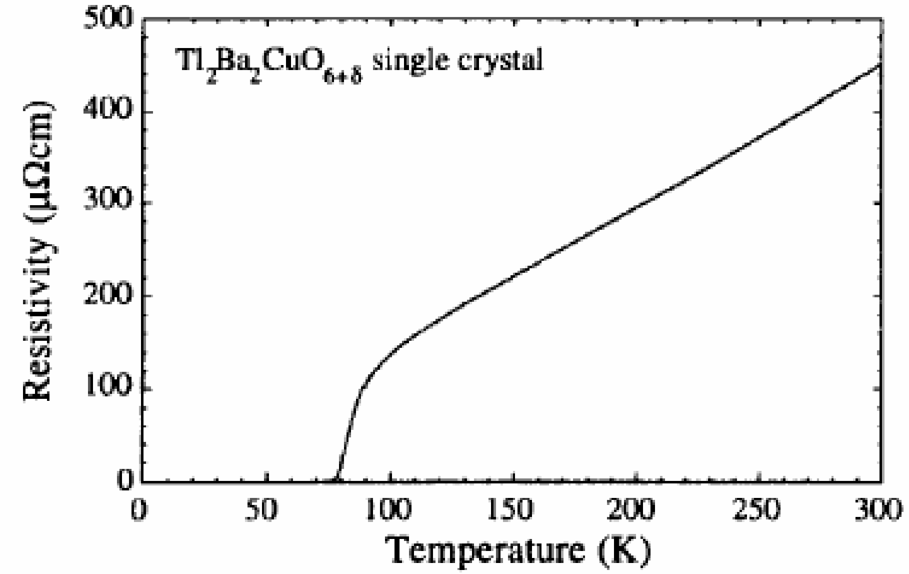
Some universal properties in CMT

Cuprate phase diagram

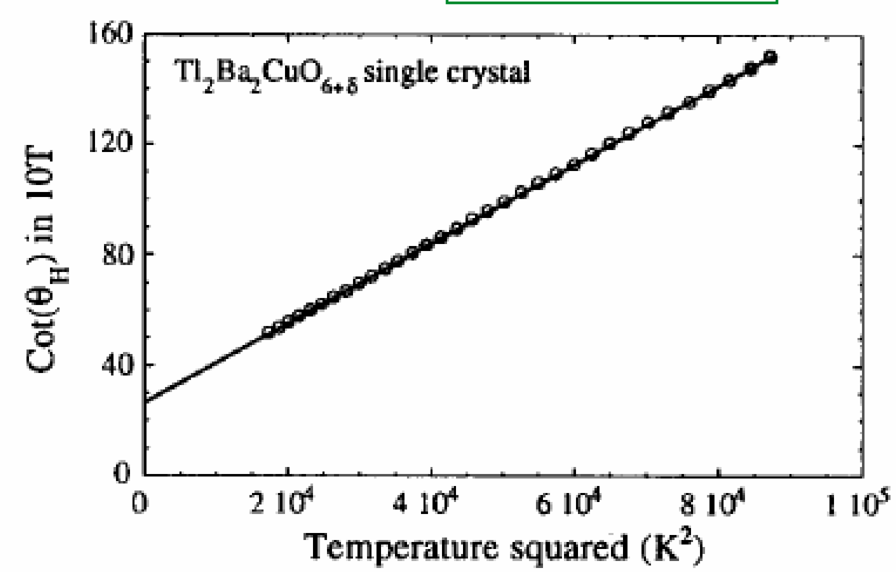


[Peter Wahl: 2012, Nature Physics]

- DC resistivity $\boxed{\rho \sim T}$

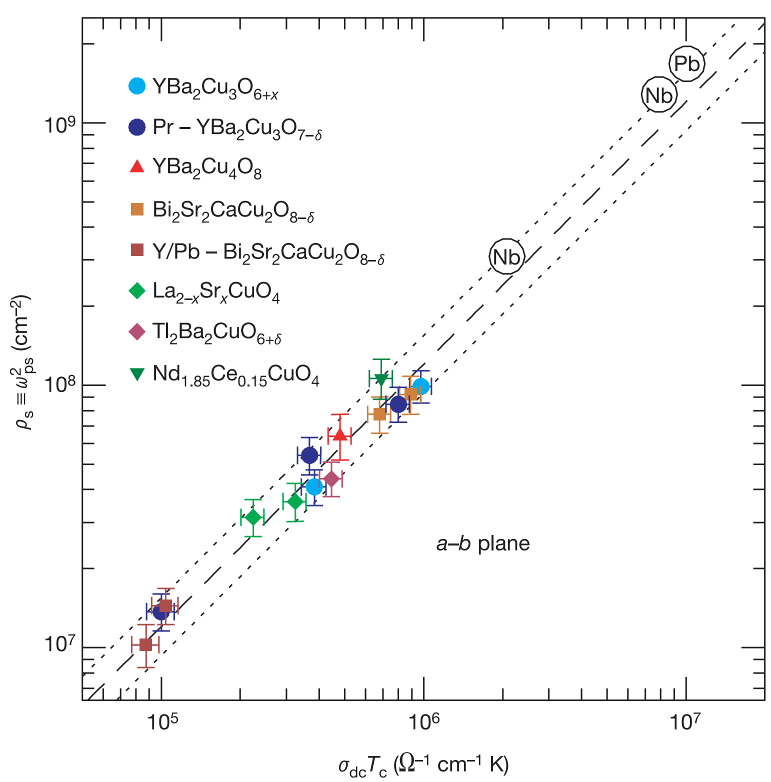


- Hall angle $\boxed{\sigma_{xx}/\sigma_{xy} \sim T^2}$



Mackenzie, 1997

- Homes law $\boxed{\rho_s(T=0) = C\sigma_{DC}(T_c)T_c}$



Some universal properties in CMT

	$\rho \propto T$ as $T \to \infty$	$\rho \propto T$ as $T \to 0$	Extended criticality	$\cot\Theta_H \propto T^2$ (at low H)	Modified Kohler's (at low H)	H-linear MR (at high H)	Quadrature MR
UD p-cuprates	✓ [6]	× [20]	× [21]	✓ [22]	✓ [23]	-	-
OP p-cuprates	✓ [4]	-	-	✓ [24]	✓ [25]	✓ [26]	× [27]
OD p-cuprates	✓ [6]	✓ [8]	✓ [8]	✓ [28]	× [29]	✓ [29]	✓ [29]
$La_{2-x}Ce_xCuO_4$	× [30]	✓ [31]	✓ [31]	× [32]	× [33]	✓ [34]	× [34]
Sr_2RuO_4	✓ [35]	× [36]	× [37]	× [38]	× [36]	× [36]	× [36]
$Sr_3Ru_2O_7$	✓ [10]	✓ [10]	× [10]	×	-	-	-
$FeSe_{1-x}S_x$	× [39]	✓ [40]	× [40]	✓ [41]	✓ [41]	✓* [42]	✓* [42]
$BaFe_2(As_{1-x}P_x)_2$	× [43]	✓ [44]	× [44]	-	✓ [45]	✓ [46]	✓ [46]
$Ba(Fe_{1/3}Co_{1/3}Ni_{1/3})_2As_2$	-	✓ [47]	× [47]	-	-	✓ [47]	✓ [47]
$YbRh_2Si_2$	× [48]	✓ [49]	✓ [50]	✓ [51]	-	-	-
$YbBAl_4$	× [52]	✓** [52]	✓** [52]	-	-	-	-
$CeCoIn_5$	× [53]	✓ [54]	× [54]	✓ [53]	✓ [53]	-	-
$CeRh_6Ge_4$	× [55]	✓ [55]	× [55]	-	-	-	-
$(TMTSF)_2PF_6$	-	✓ [56]	✓ [56]	-	-	-	-
MATBG	✓ [57]	✓ [58]	✓ [58]	✓ [59]	-	-	-

a

	$\rho \propto T$ as $T \to 0$	$\rho \propto T$ as $T \to \infty$	$\sigma \propto \omega^{-2/3}$	Quadrature MR	Extended criticality	Experimental Prediction
Phenomenological						
MFL	✓ [65]	× [65]	×	×	×	loop currents [104]
EFL	- [b]	-	-	×	×	loop currents [105]
Numerical						
ECFL	×	✓ [106]	-	-	×	×
HM (QMC/ED/CA)	- [107]	✓ [107–111]	×	-	-	-
DMFT/EDMFT	✓ [112]	✓ [113, 114]	×	-	✓ [114]	-
QCP	✓ [115]	-	-	-	×	-
Gravity-based						
SYK	✓ [116, 117]	✓[c] [117]	×	✓[d] [118]	-	×
AdS/CFT	✓ [119]	✓ [119]	✓[e] [88, 123]	×	×	×
AD/EMD	✓ [124–126]	✓ [88, 123, 124, 126, 127]	✓ [88, 123, 127]	×	✓ [123]	Fractional A-B [126]

No concrete holography model of "T-linear resistivity + T^2 -Hall angle together" yet, even though there are many interesting holography models partly successful?

ML Example: Holographic model

EMD(Einstein Maxwell Dilaton) model [ArXiv:1005.4690][hep-th], [ArXiv:1401.5436][hep-th]

$$S = \int \mathrm{d}^{p+1}x \sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 + V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\partial\psi_i^2\right].$$ $\longrightarrow$ Many variations

$$Z(\phi) \sim e^{\gamma\phi}, \qquad V(\phi) \sim V_0 e^{-\delta\phi}, \qquad Y(\phi) \sim e^{\lambda\phi}$$

$$\mathrm{d}s^2 = r^{\frac{2\theta}{p-1}}\left[-f(r)\frac{\mathrm{d}t^2}{r^{2z}} + \frac{L^2\mathrm{d}r^2}{r^2 f(r)} + \frac{\mathrm{d}\vec{x}^2}{r^2}\right], \quad A = Qr^{\zeta - z}\mathrm{d}t\,, \quad \phi = \kappa \ln r$$

ML Example: Holographic model

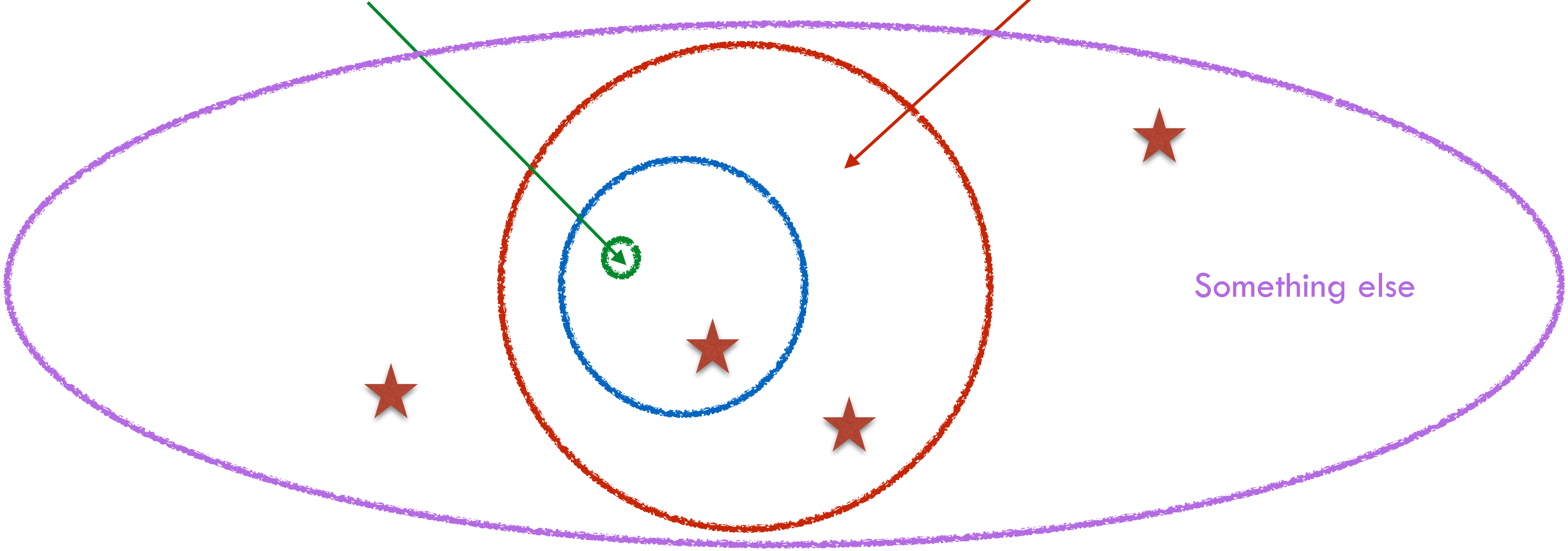
EMD(Einstein Maxwell Dilaton) model

$$S = \int \mathrm{d}^{p+1}x \sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 + V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\partial\psi_i^2\right]. \longrightarrow$$

Many variations

$$Z(\phi) \sim e^{\gamma\phi}, \qquad V(\phi) \sim V_0 e^{-\delta\phi}, \qquad Y(\phi) \sim e^{\lambda\phi}$$

$$\mathrm{d}s^2 = r^{\frac{2\theta}{p-1}}\left[-f(r)\frac{\mathrm{d}t^2}{r^{2z}} + \frac{L^2\mathrm{d}r^2}{r^2 f(r)} + \frac{\mathrm{d}\vec{x}^2}{r^2}\right], \quad A = Qr^{\zeta - z}\mathrm{d}t\,, \quad \phi = \kappa\ln r$$



Towards holographic strange model

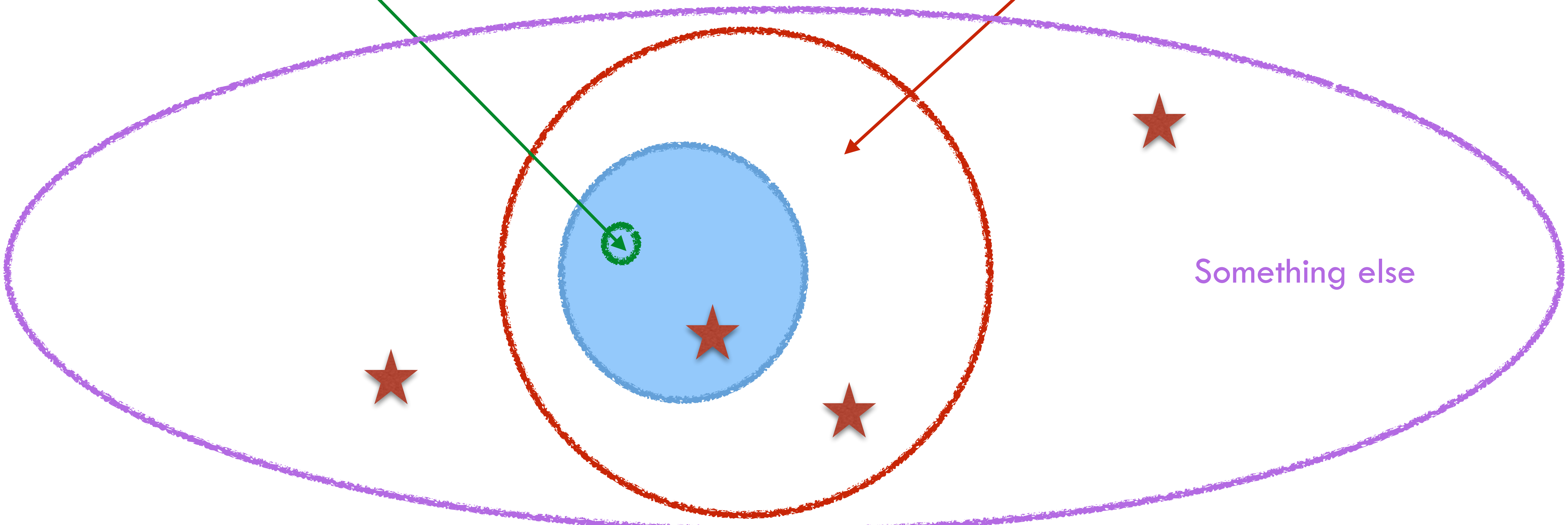
EMD(Einstein Maxwell Dilaton) model

$$S=\int \mathrm{d}^{p+1}x\,\sqrt{-g}\left[R-\frac{1}{2}\partial\phi^2-\frac{1}{4}Z(\phi)F^2+V(\phi)-\frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\partial\psi_i^2\right].$$

$\longrightarrow$ Many variations

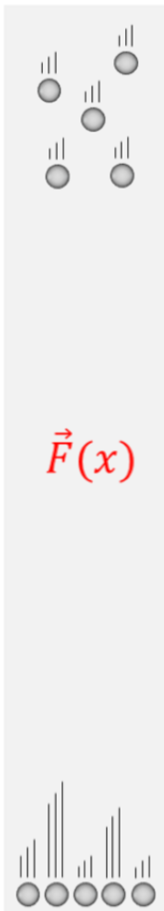
$$Z(\phi)\sim e^{\gamma\phi},\qquad V(\phi)\sim V_0e^{-\delta\phi},\qquad Y(\phi)\sim e^{\lambda\phi}$$

$$\mathrm{d}s^2=r^{\frac{2\theta}{p-1}}\left[-f(r)\frac{\mathrm{d}t^2}{r^{2z}}+\frac{L^2\mathrm{d}r^2}{r^2f(r)}+\frac{\mathrm{d}\vec{x}^2}{r^2}\right],\quad A=Qr^{\zeta-z}\mathrm{d}t\,,\quad \phi=\kappa\ln r$$



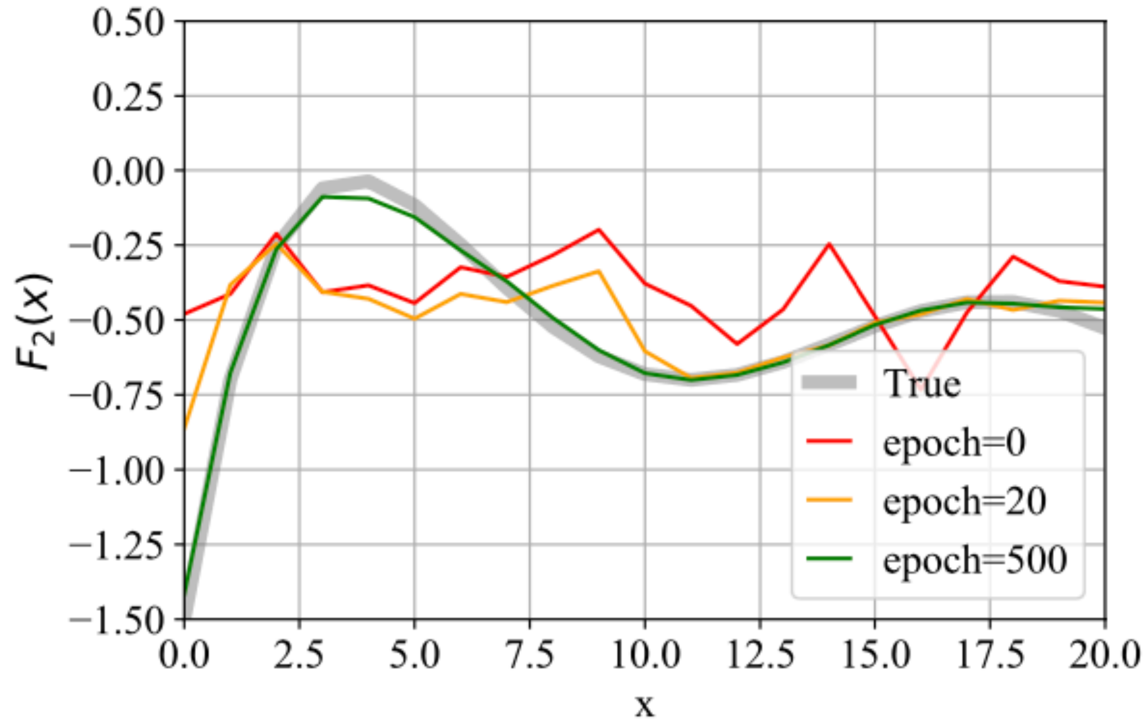
Deep Learning for ODE: classical mechanics

x_i, v_i



v_f

$$m\dot{v} = mg - F(x)$$
$$\dot{x} = v$$



1. (Data)　3. (Numerical ODE solving)　5. (Loss)　4. (Solution)

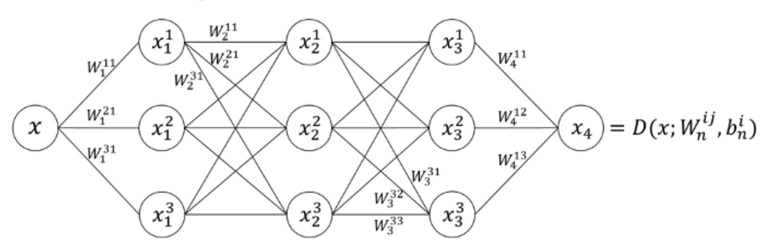
$$(v_i, x_i) \longrightarrow \begin{array}{c} m\dot{v} = mg - F(x) \\ \dot{x} = v \end{array} \longrightarrow L = \mathrm{avg}(\,|v_f - V_f|\,) \longleftarrow V_f$$

$$\frac{\partial L}{\partial W_n}, \frac{\partial L}{\partial b_n}$$

Machine learning = function approximator

$$F(x) = D(x; W_n, b_n)$$

2. (Deep neural network)　6. (Parameters optimizing)



$$x_n^i = \varphi\left(\Sigma_j W_n^{ij} x_{n-1}^j + b_n^i\right)$$

Deep Learning for ODE: classical mechanics

physics informed neural networks

1. (Data)

$$\left(v_i,\ V_f\right) \longrightarrow$$

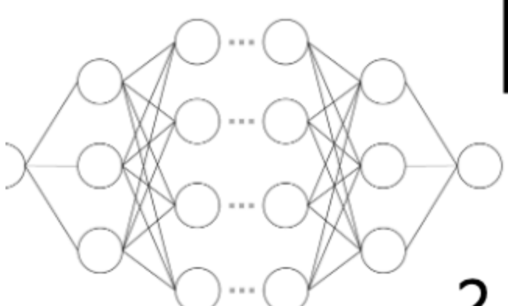
3. (Loss)

$$L = \mathrm{avg_t}(\ |eom(t)|\ ) + |v(t_i) - v_i| + \left|v\left(t_f\right) - v_f\right|$$
$$+ |x'(t) - v(t)| + |x(t_i) - x_i| + \left|x_f\right|$$

$$eom(t) = mv'(t) - \left(mg - F\left(x(t)\right)\right) = 0$$

$$\frac{\partial L}{\partial W_n}, \frac{\partial L}{\partial b_n}$$

$$F(x) = D_3(x; W_n, b_n)$$



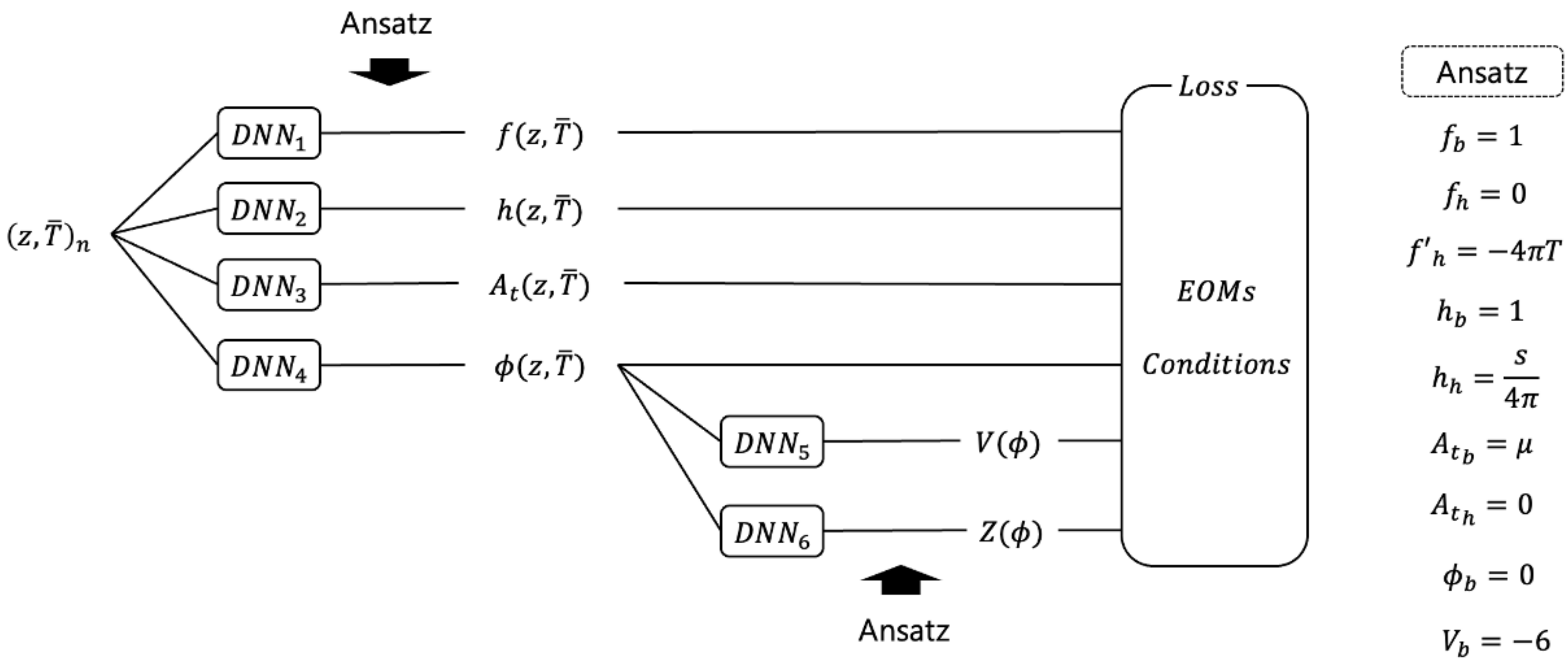
$$x(t) = D_1(t; W_n, b_n)$$
$$v(t) = D_2(t; W_n, b_n)$$

2. (Deep neural networks)

4. (Parameters optimizing)

$$\frac{\partial L}{\partial W_n}, \frac{\partial L}{\partial b_n}, \frac{\partial L}{\partial W_n}, \frac{\partial L}{\partial b_n}$$

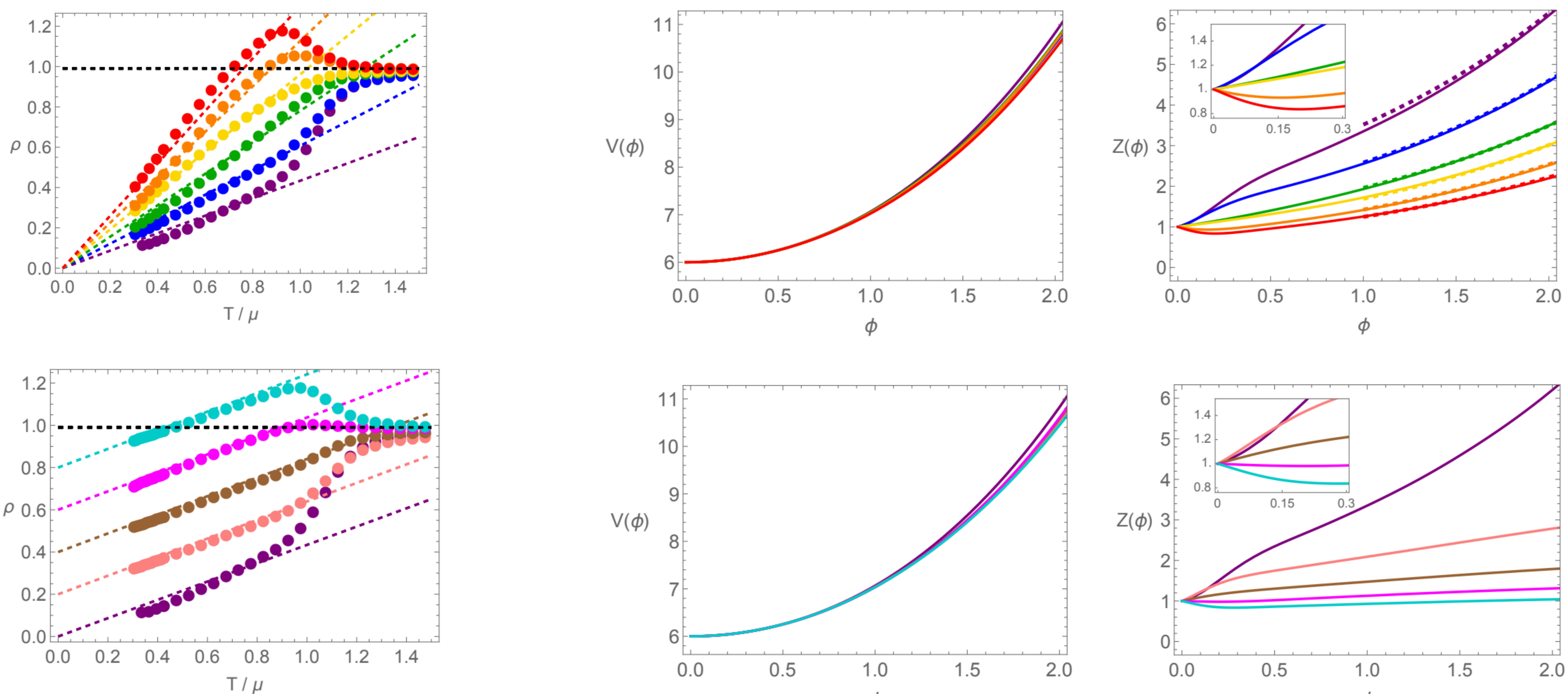
Towards holographic strange model



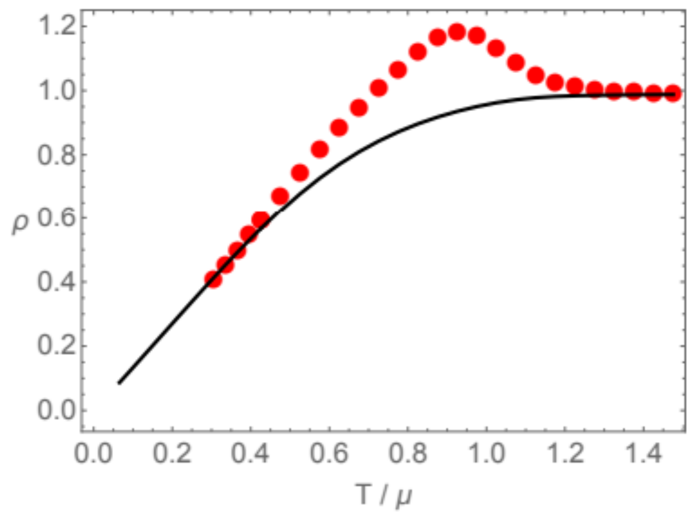
$$Loss = \frac{1}{N}\sum_{n=1}^{N}\left(\sum_{i=1}^{4} |EOM_i((z,\bar{T})_n)| + \sum_j |Condition_j((z,\bar{T})_n)| \right) + L_{etc}$$

$$S=\int \mathrm{d}^{p+1}x\,\sqrt{-g}\left[R-\frac{1}{2}\partial\phi^2-\frac{1}{4}Z(\phi)F^2+V(\phi)-\frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\partial\psi_i^2\right].$$

Gubser Rocha model $\quad V(\phi)=6\cosh\frac{\phi}{\sqrt{3}},\quad Z(\phi)=e^{\frac{\phi}{\sqrt{3}}}\quad Y(\phi)=1$

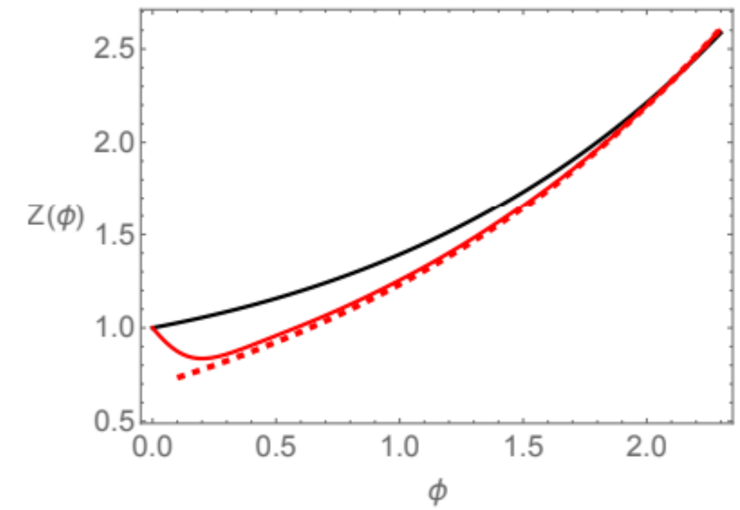


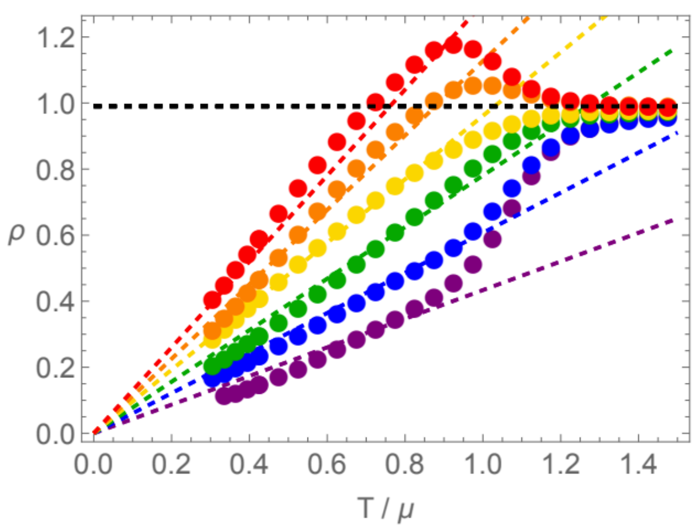
Towards holographic strange model

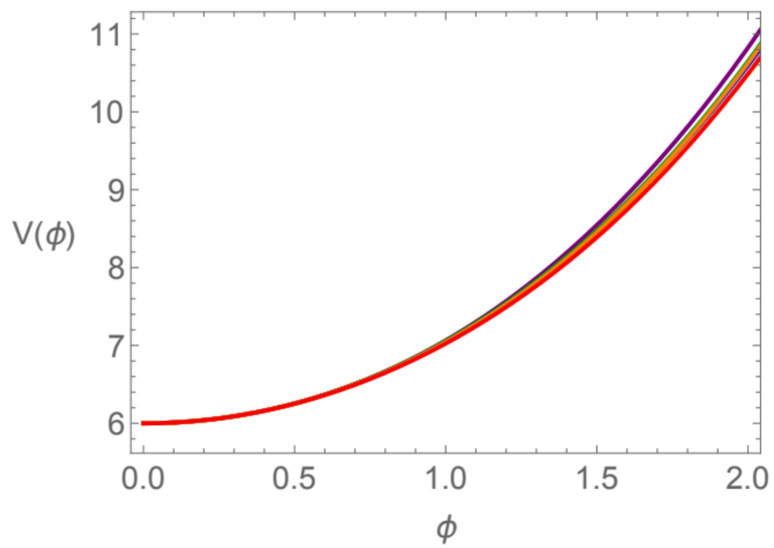


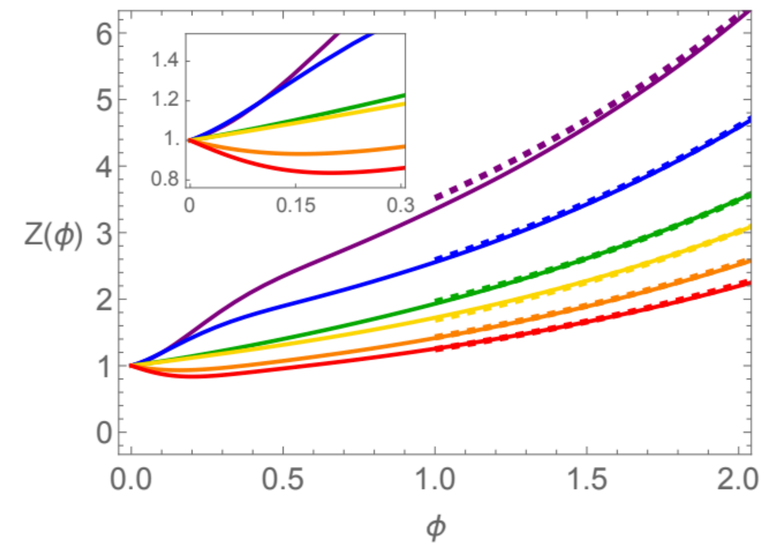
$$= \int \mathrm{d}^{p+1}x\,\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 + V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\right.$$

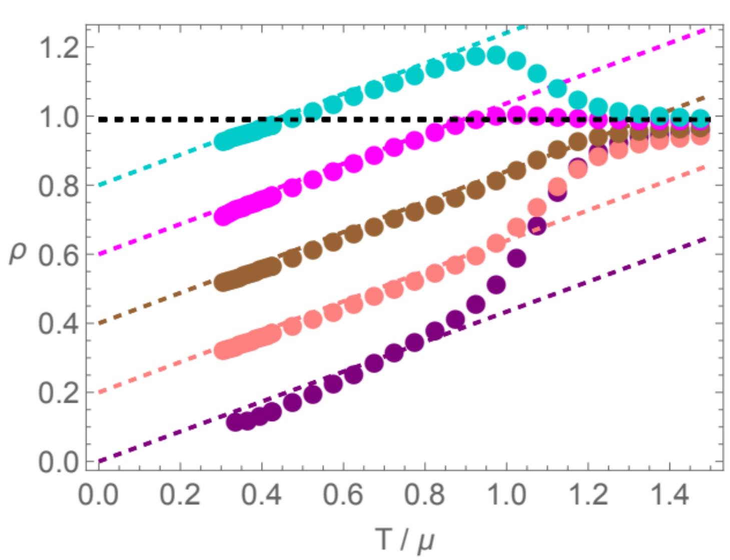
$$V(\phi) = 6\cosh\frac{\phi}{\sqrt{3}}, \qquad Z(\phi) = e^{\frac{\phi}{\sqrt{3}}}$$

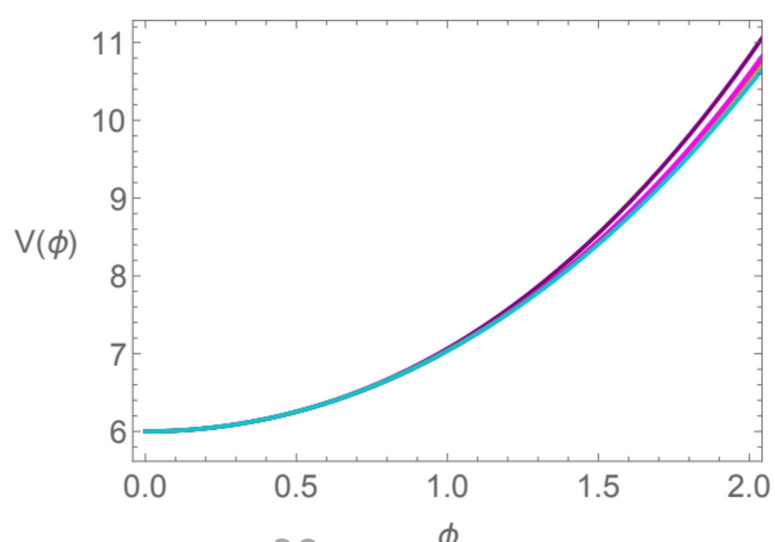


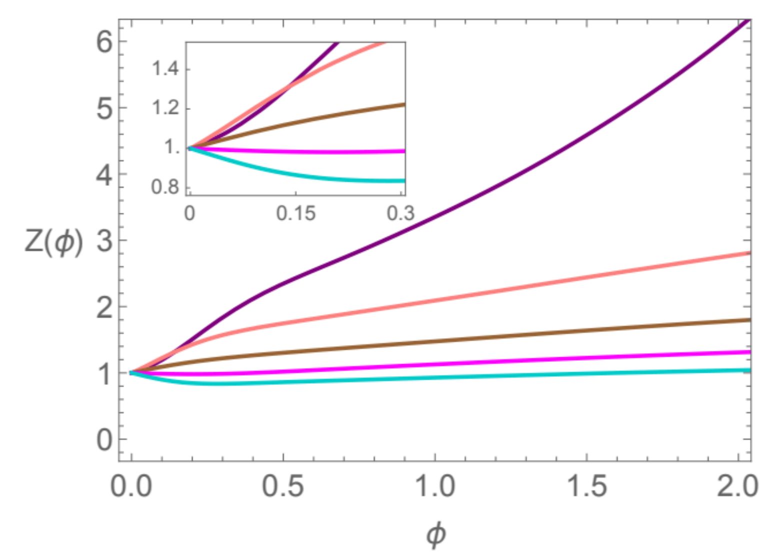












Towards holographic strange model

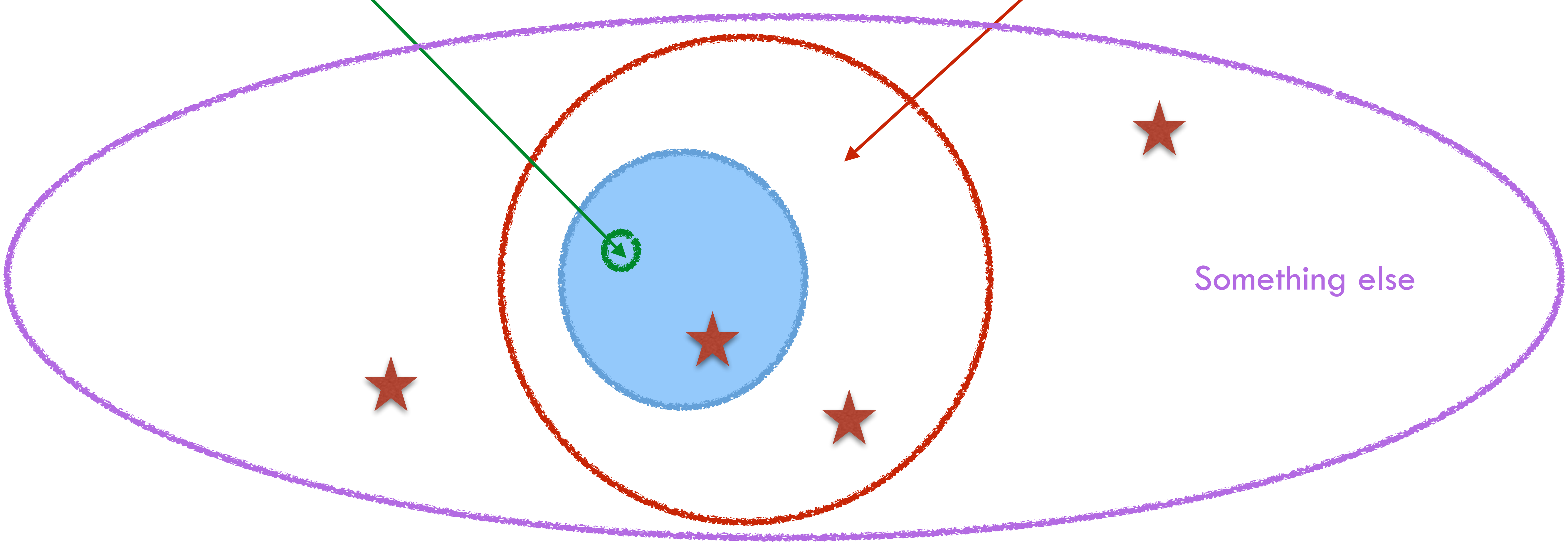
EMD(Einstein Maxwell Dilaton) model

$$S=\int \mathrm{d}^{p+1}x\,\sqrt{-g}\left[R-\frac{1}{2}\partial\phi^2-\frac{1}{4}Z(\phi)F^2+V(\phi)-\frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\partial\psi_i^2\right].\longrightarrow$$

Many variations

$$Z(\phi)\sim e^{\gamma\phi},\qquad V(\phi)\sim V_0e^{-\delta\phi},\qquad Y(\phi)\sim e^{\lambda\phi}$$

$$\mathrm{d}s^2=r^{\frac{2\theta}{p-1}}\left[-f(r)\frac{\mathrm{d}t^2}{r^{2z}}+\frac{L^2\mathrm{d}r^2}{r^2f(r)}+\frac{\mathrm{d}\vec{x}^2}{r^2}\right],\quad A=Qr^{\zeta-z}\mathrm{d}t\,,\quad \phi=\kappa\ln r$$



Ongoing and future work

- Methodology development

  - Neural ODE, Neural integral

  - PINN (Physics Informed Neural Network)

  - PDE

- Other physical quantities

  - ARPES: Fermionic spectral function

  - Quantum info: complexity, entanglement entropy, etc

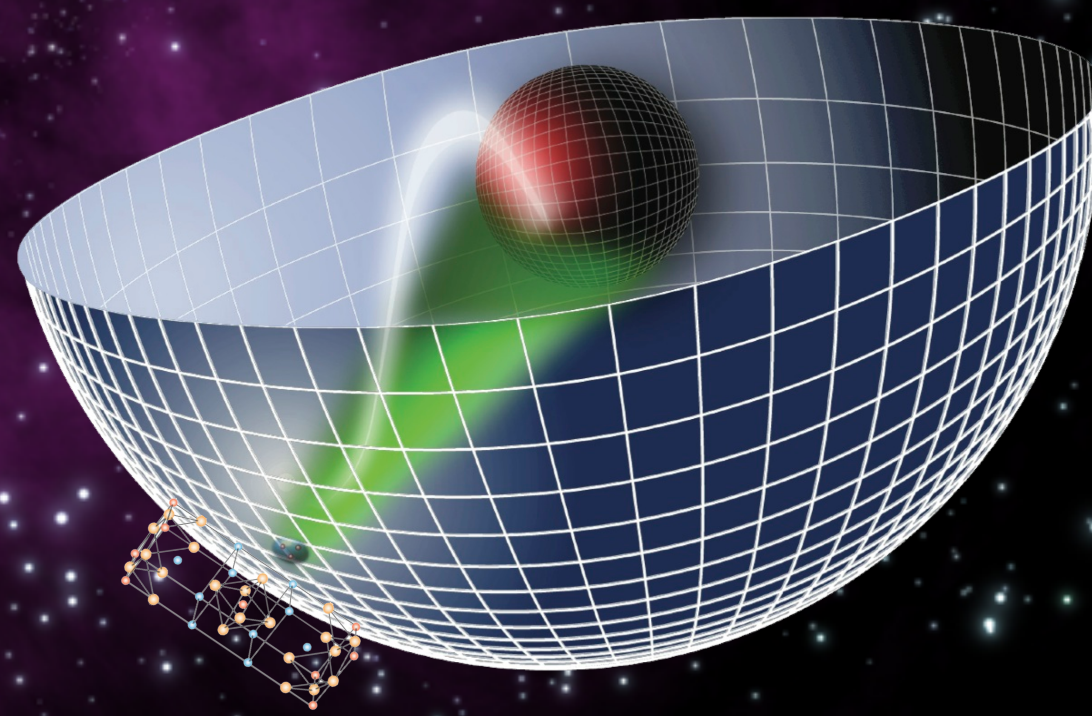
  - Applications to other physics problems (including ODE, PDE, Integral)

- Figuring out action itself for a specific problem

  - so far, the form of the action is fixed

  - Linear T resistivity + T^2 Hall angle together

Thank you

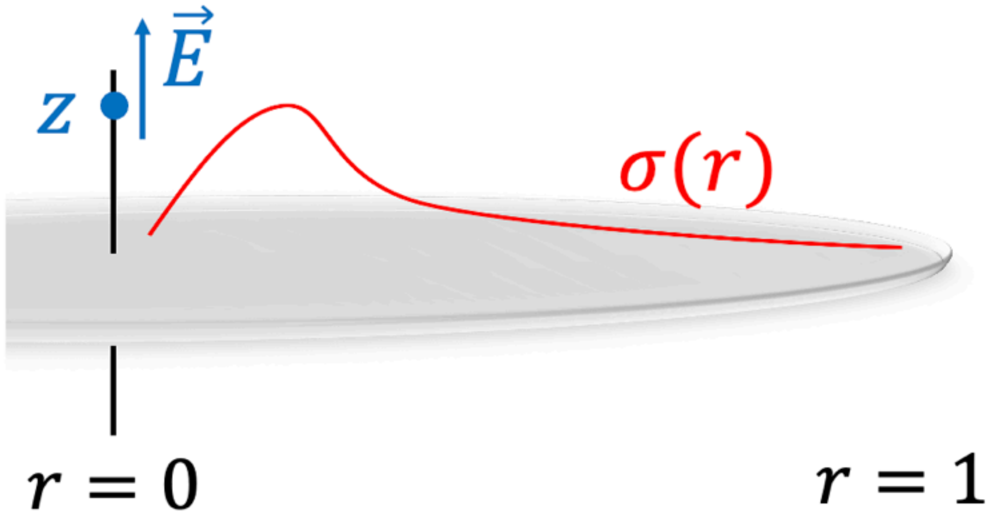


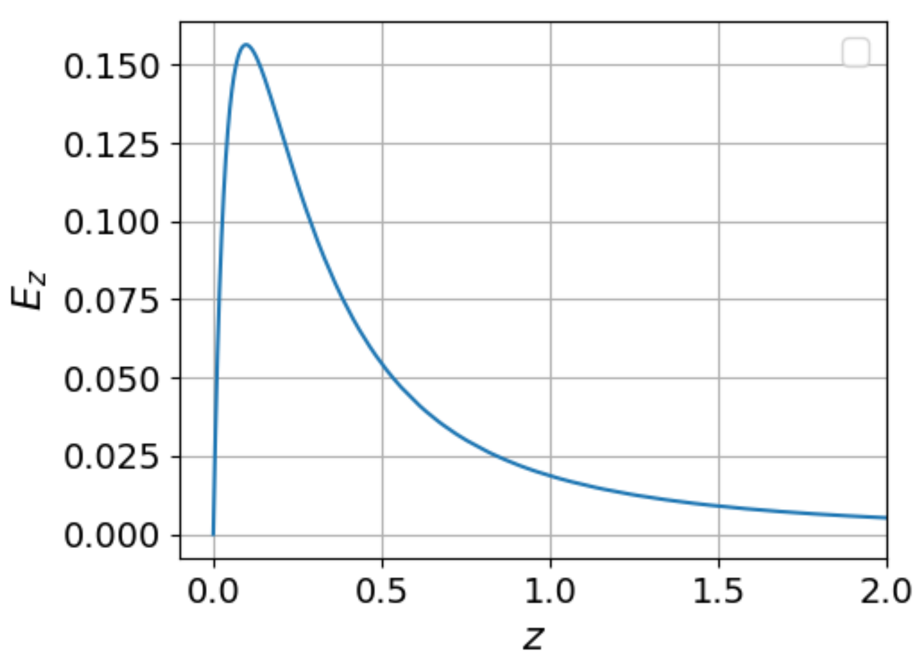
Deep Learning for integral: electrostatics

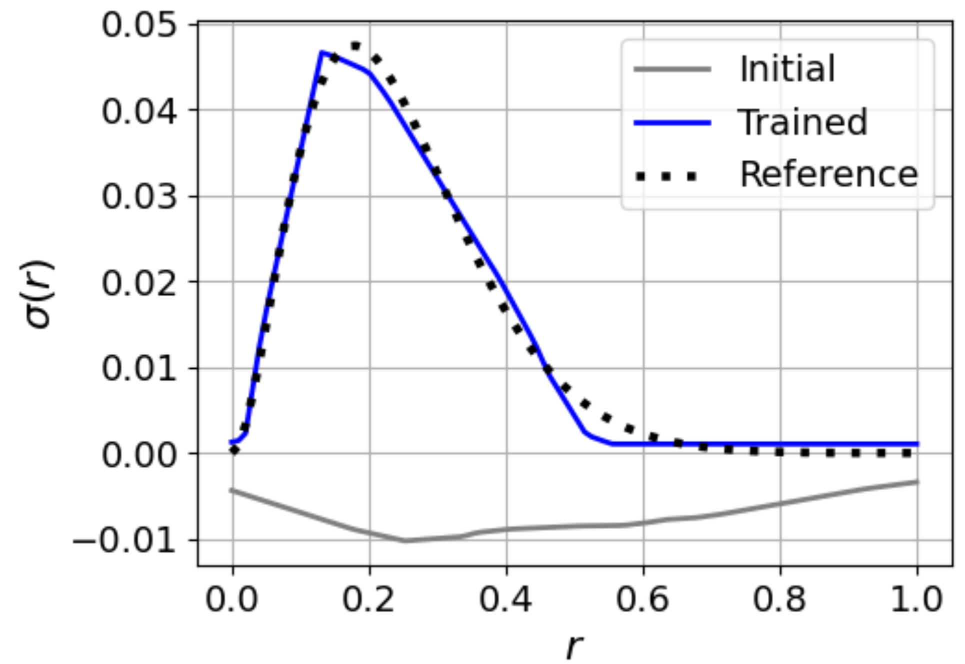
$$E(z) = \int_0^1 dr\, 2\pi r \frac{z\,\sigma(r)}{\sqrt{r^2+z^2}^3}$$

Loss

$|E^{(N)}(z) - E(z)|$







AdS/Deep learning: entanglement entropy

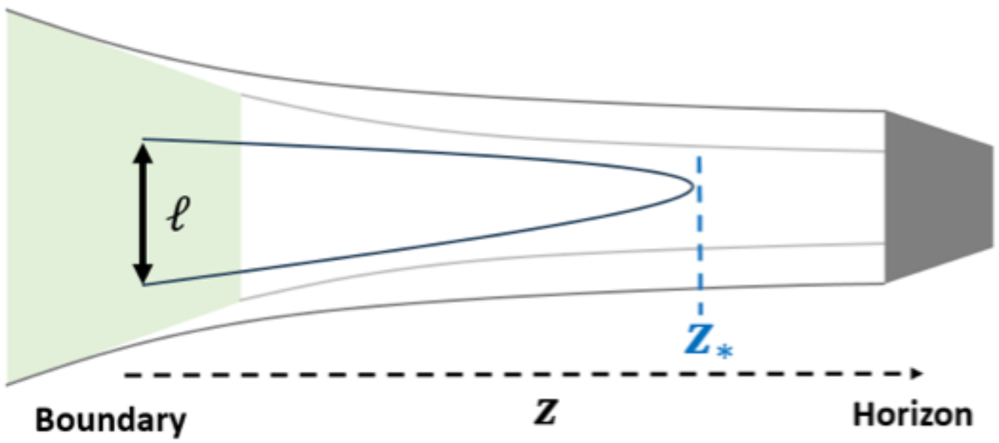
$$E(z) = \int_0^1 dr\; 2\pi r \frac{z\,\sigma(r)}{\sqrt{r^2+z^2}^3}$$

$$A(\alpha) = \int F[\, f_1(r;\alpha), f_2(r;\alpha), \cdots]\, dr$$

$$B(\alpha) = \int G[\, f_1(r;\alpha), f_2(r;\alpha), \cdots]\, dr$$

$$\ell(z_*) = \int_0^{z_*} dz \frac{2z^2}{\sqrt{z_*^4 - z^4}} \frac{1}{\sqrt{f(z)}}$$

$$C(z_*) := -1 + \int_0^{z_*} dz \cdot \frac{z_*}{z^2} \left(\sqrt{1 - \frac{z^2}{z_*^2}} \sqrt{\frac{1}{f(z)}} - 1 \right)$$



$$\bar{\sigma} := \frac{\sigma(\ell(z_*))}{s} = \frac{1}{z_*^2} + \frac{C(z_*)}{z_*} \frac{2}{\ell(z_*)} + \frac{4\pi}{\ell(z_*)^2} \left(\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})} \right)^2$$

Reference II

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Holographic reconstruction of black hole spacetime: machine learning and entanglement entropy

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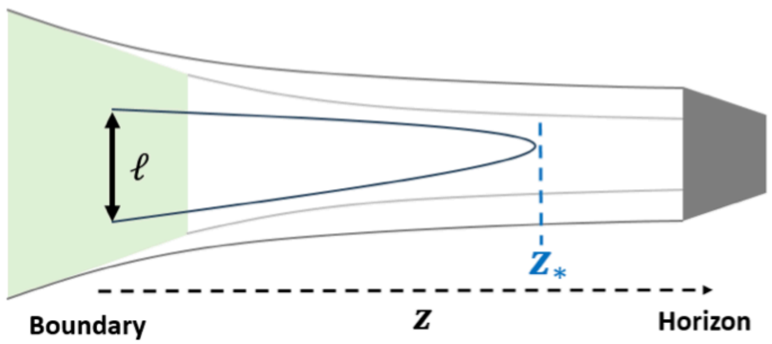
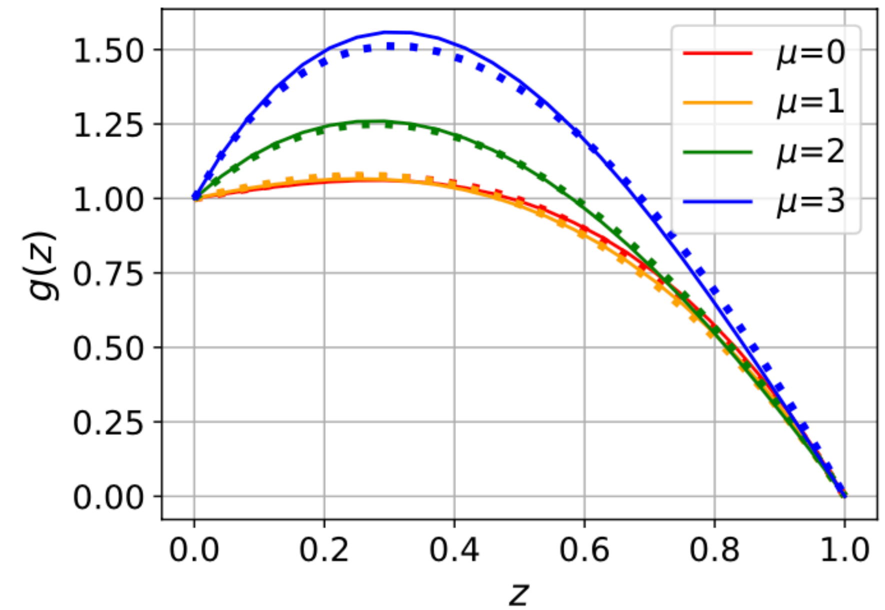
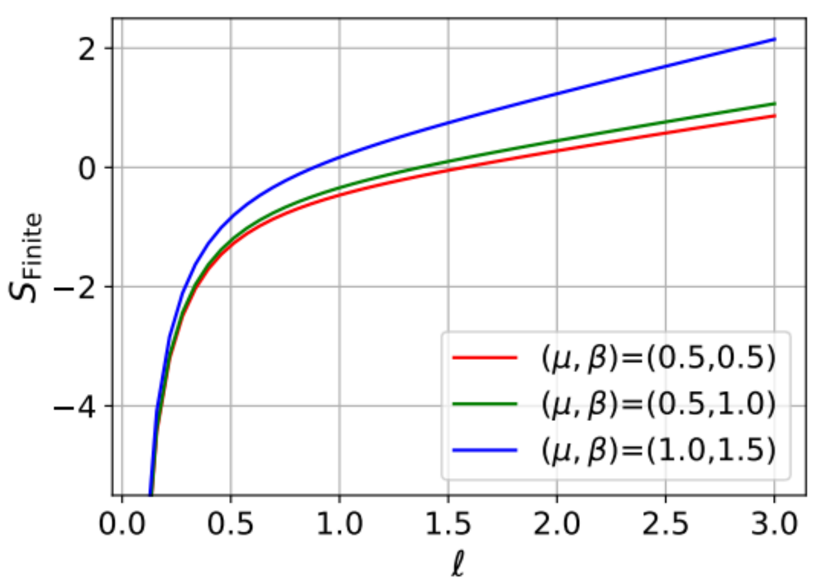
AdS/Deep learning: entanglement entropy

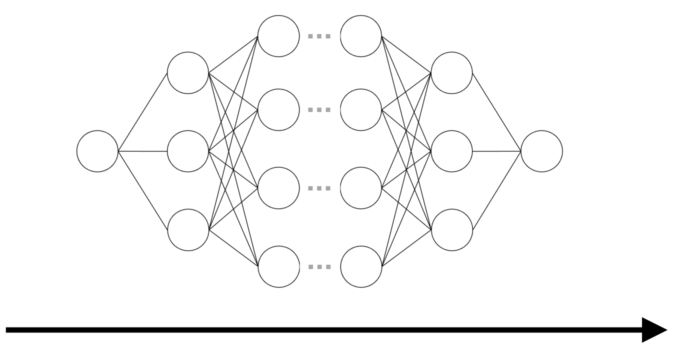
Setup Gubser-Rocca case

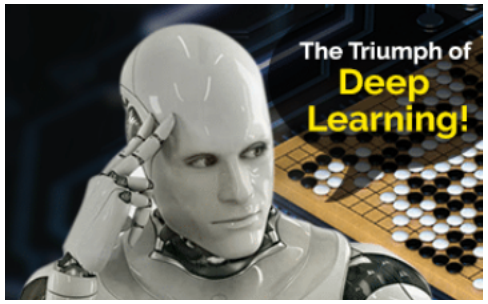
$$S = \int \mathrm{d}^4x\sqrt{-g}\left(R + 6\cosh\phi - \frac{1}{4}e^{\phi}F^2 - \frac{3}{2}(\partial\phi)^2 - \frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right) \qquad ds^2 = \frac{L^2}{z^2}\left[-g(z)dt^2 + \frac{dz^2}{g(z)} + h(z)(dx^2+dy^2)\right]$$

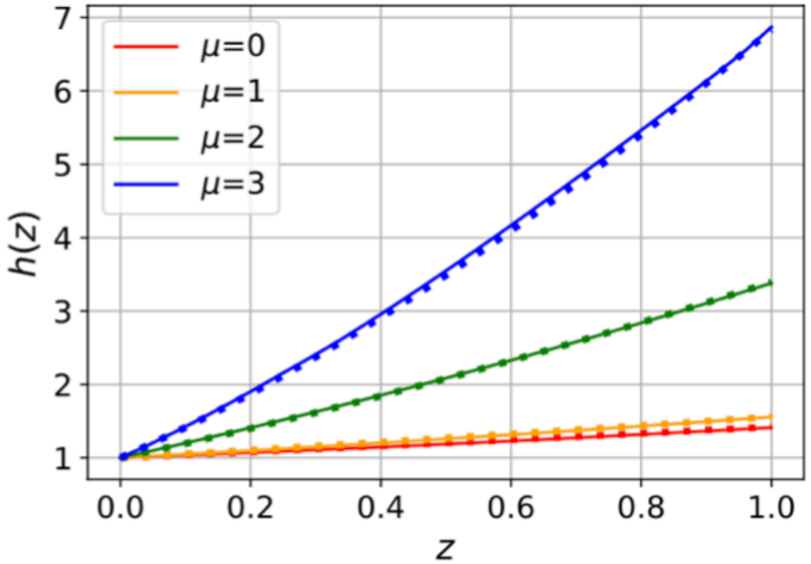




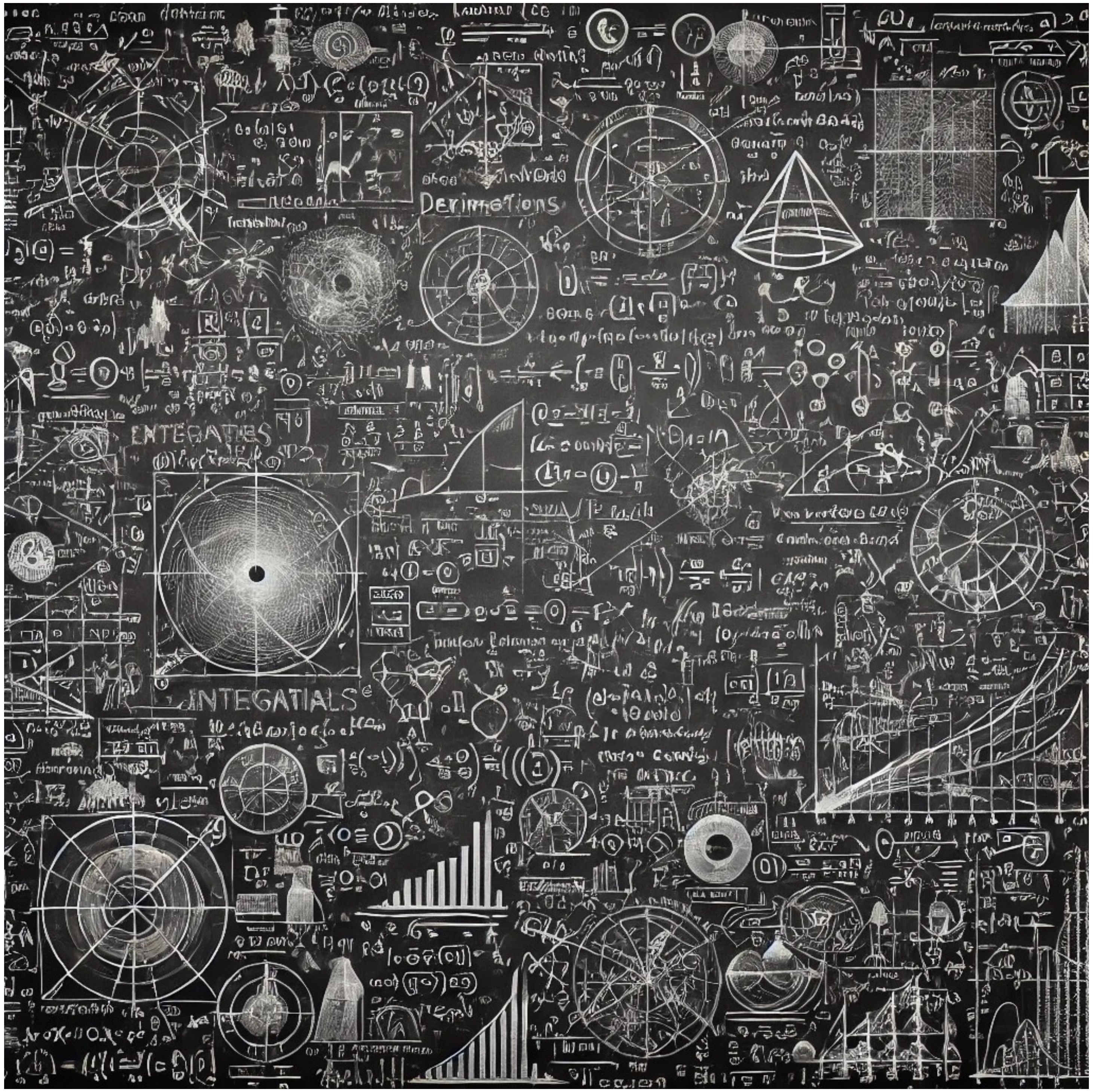








Problem classification



Complicated equations by “ChatGPT”

Differential equation

$$A(z)f''(z)+B(z)f'(z)+C(z)f(z) = D(z)g(z)$$

$$E(z)g''(z)+F(z)g'(z)+G(z)g(z) = H(z)f(z)$$

Ex) boundary transport
by bulk fluctuation equations

Integral equation

$$A(\alpha) = \int F[\, f_1(r;\alpha), f_2(r;\alpha), \cdots]\, dr$$

$$B(\alpha) = \int G[\, f_1(r;\alpha), f_2(r;\alpha), \cdots]\, dr$$

Ex) boundary entanglement entropy
by bulk metric

Inverse Problem
Optimization problem