[In preparation]

Strong lensing of dark sirens and galaxies as a probe of the Hubble constant

Kyungmin Kim (KASI)

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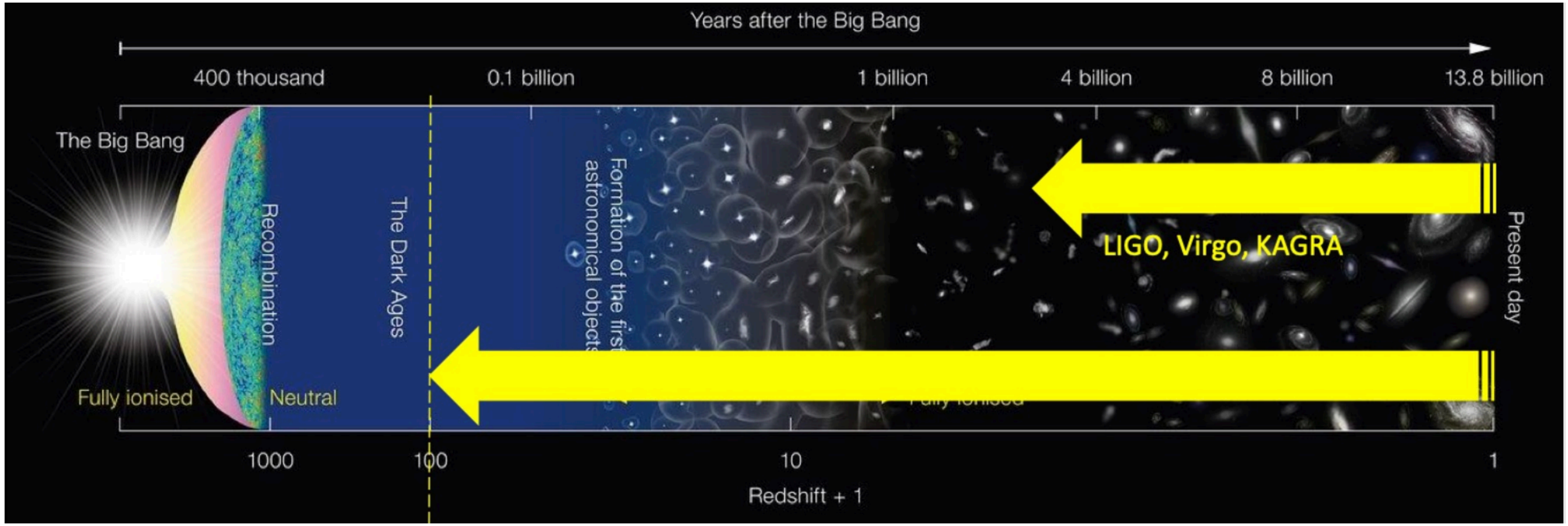
In collaboration with
Eungwang Seo, Zhoutao Li, Rachel Gray, and Martin Hendry (Glasgow, UK)

H_0 measurement using standard sirens

- We can utilize gravitational waves (GWs) as *cosmic probes*.



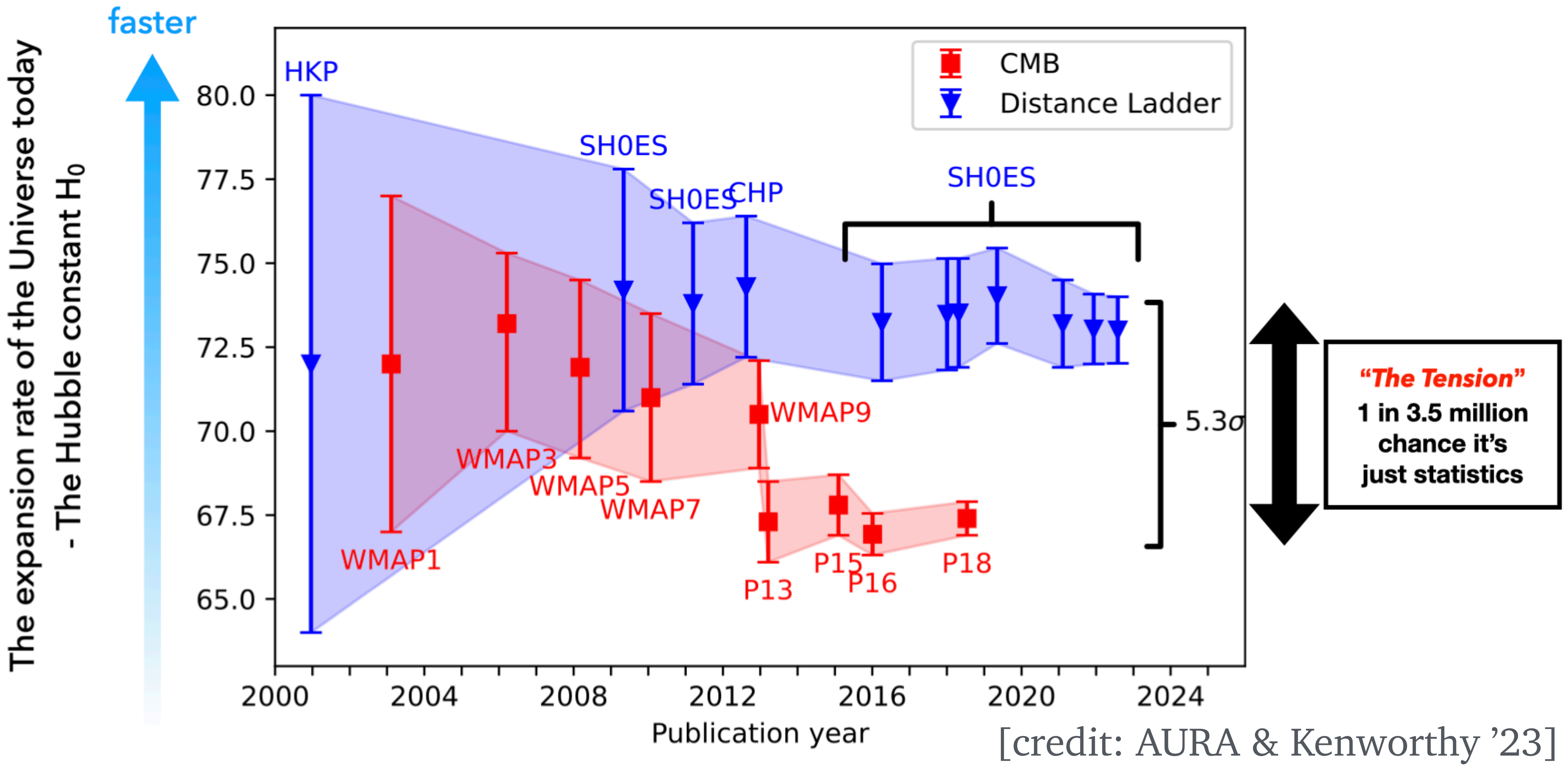
[credit: Bagnasco '22]

- GW observations enable us to measure the Hubble constant, H_0. ⇨ *standard sirens* [Schutz'86; Holz+'05]
  - w/ electromagnetic (EM) counterparts (e.g., GW170817 & GRB170817) ⇨ *bright sirens* [Abbott+'17; Smith+'25]
  - w/o EM counterparts ⇨ *dark sirens* [Soares-Santos+'19; Abbott+'21; Gray+'23; Ballard+'23; Bom+'24]

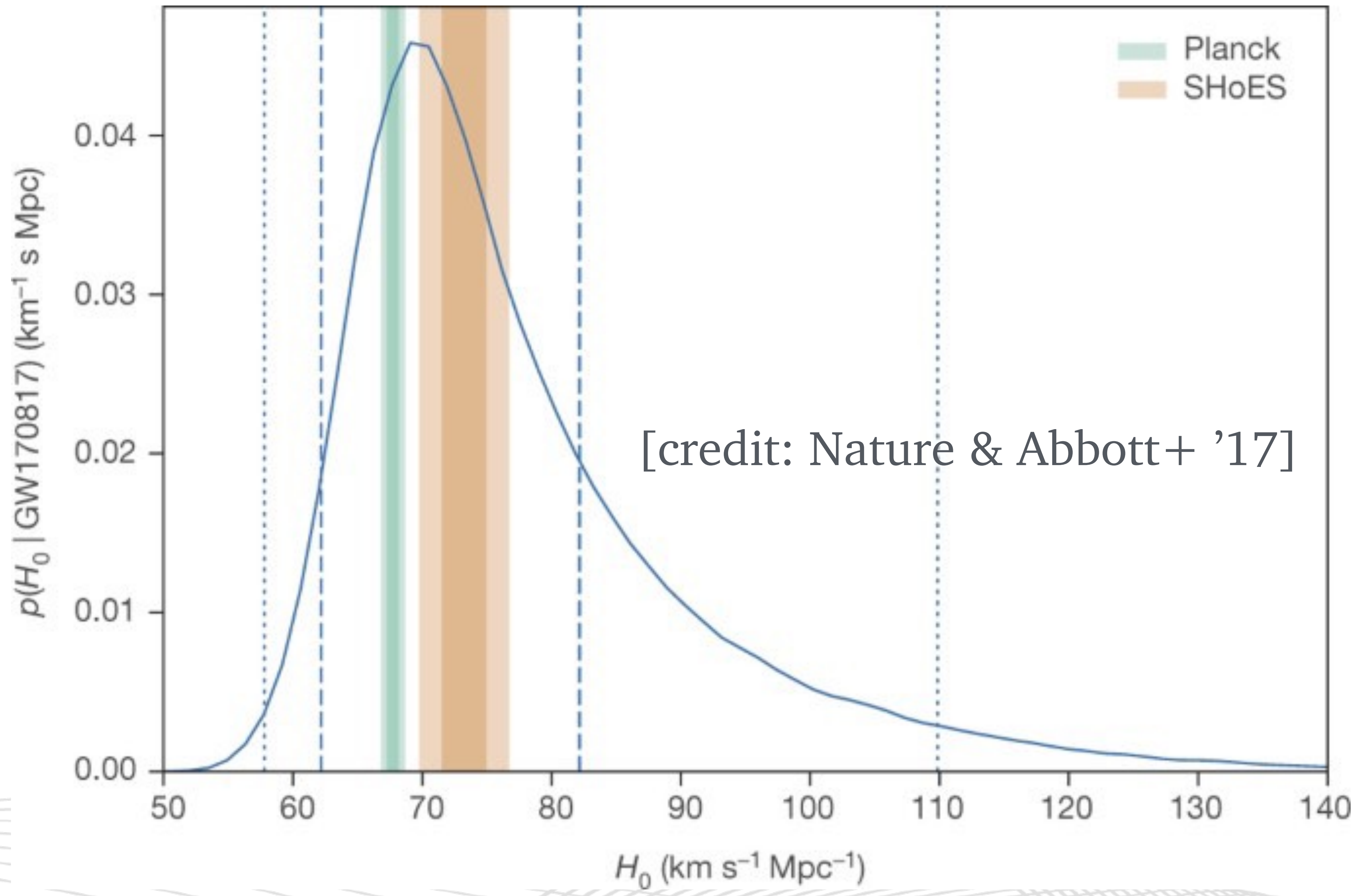
Hubble tension

- Hubble tension in EM-based H_0 measurements
  - early-Universe measurements (CMB) [Aghanim+'20]
    : $H_0 = (67.4 \pm 0.5)\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$
  - late-Universe measurements (Cepheid variable stars) [Riess+'22]
    : $H_0 = (72.3 \pm 1.4)\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$
- The tension sparks independent measurement methods like using standard sirens.
  - GW observations let us to infer the luminosity distance to GW sources directly w/o any cosmic distance ladders.

$$h_{\mathrm{GW}} \sim \frac{1}{d_L} \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$



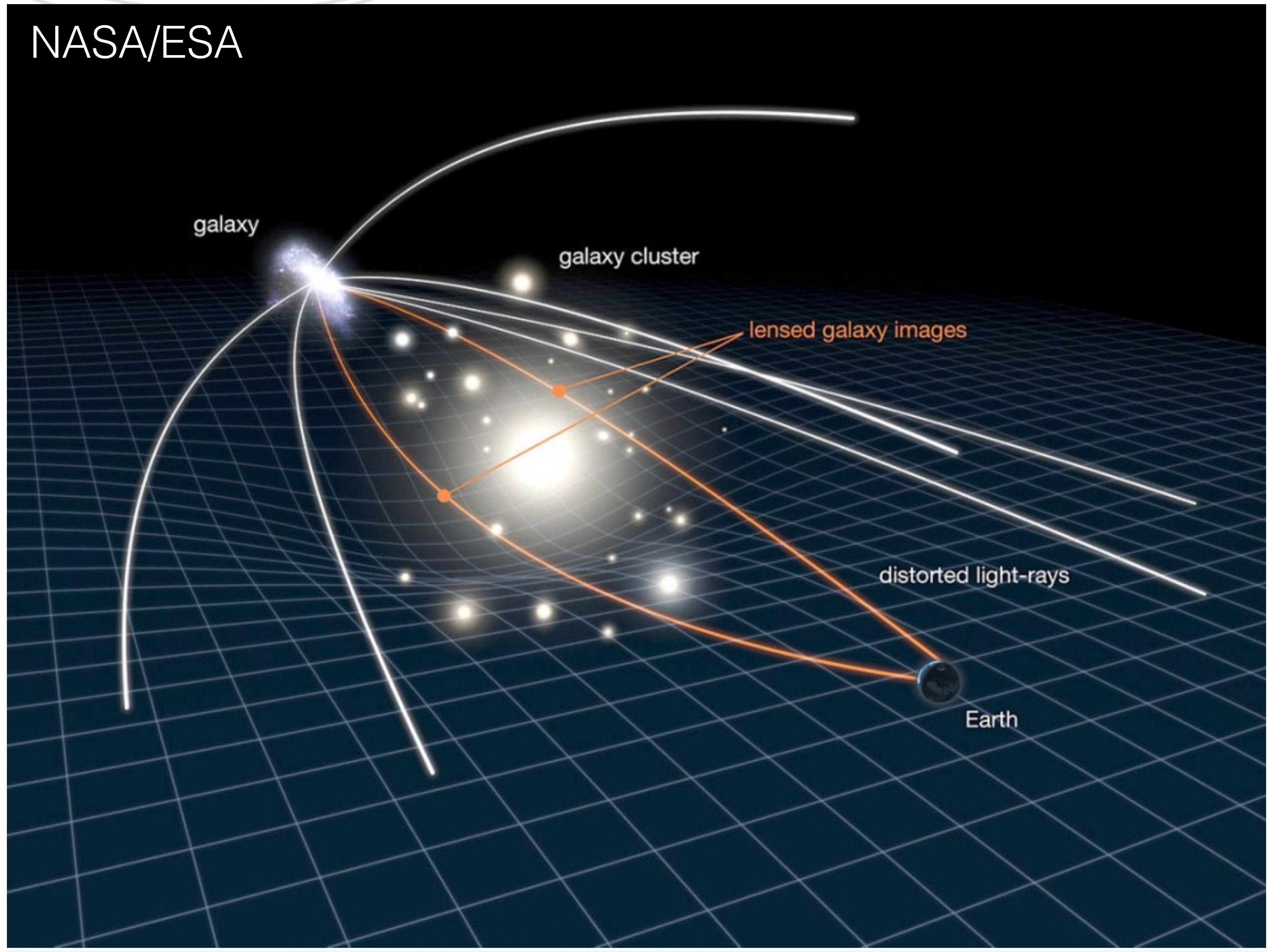
[credit: AURA & Kenworthy '23]

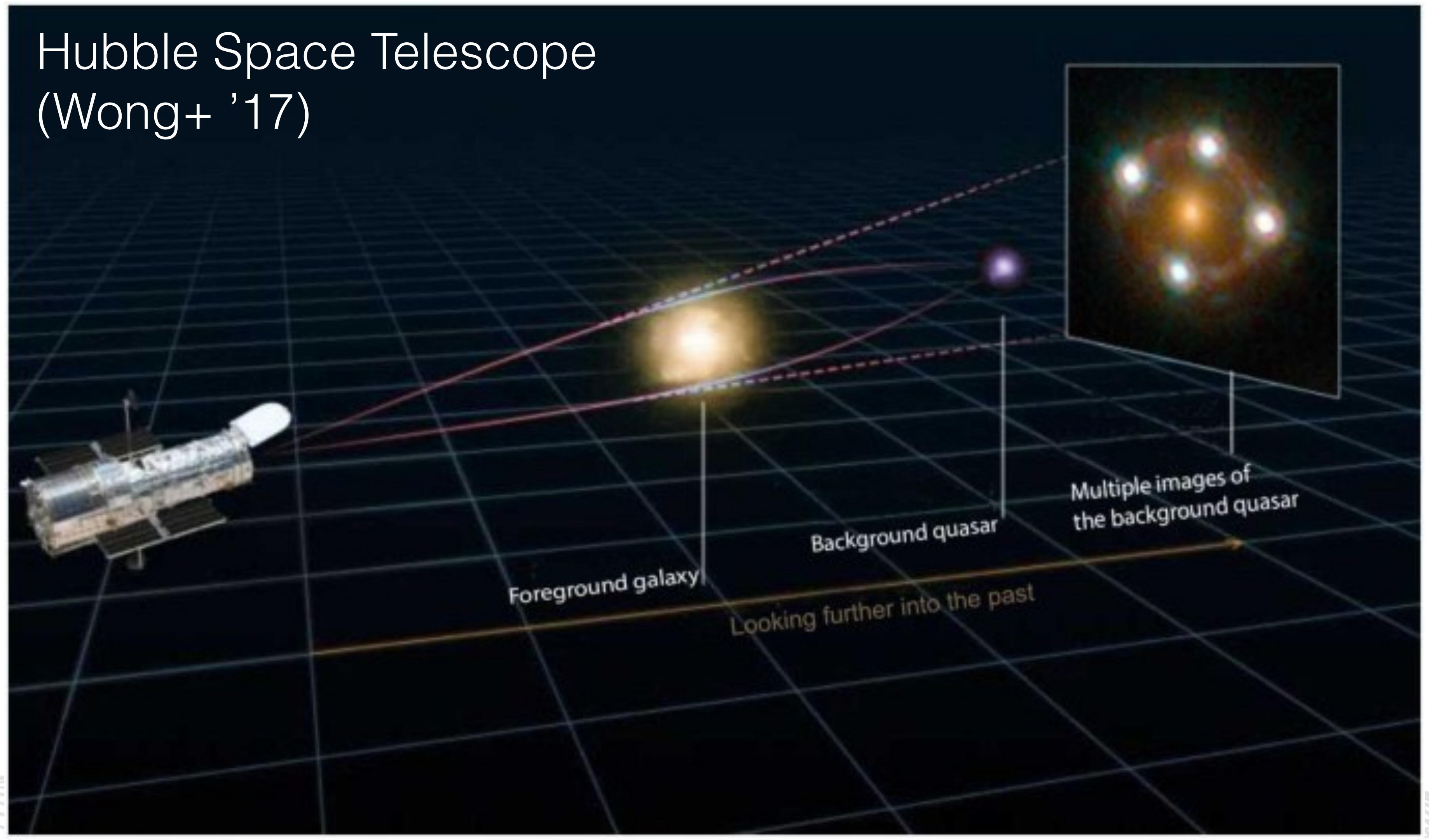


[credit: Nature & Abbott+ '17]

Gravitational lensing of light

- Gravitational lensing of light
  - allows us to observe far distant objects beyond observable horizon.
  - produces multiple images experiencing relative time delay in the arrival to an observer due to
    - the path difference and
    - the different gravitational potentials
  - apparent magnitude of k-th lensed image:
    $$m'_k = m - 2.5\log_{10}(\mu_k),$$
    where μ_k is the magnification factor of k-th image
- Time-delay cosmography
  - utilizes the relative time delay of multiple images to measure H_0.





Gravitational lensing of GWs

- Similar to light, gravitational lensing of gravitational waves (GW lensing, hereafter) can occur.

➡ ***Good candidate of multi-messenger astronomy using GW and EM observations!***

- Lensed GWs in frequency domain: $h_{\mathrm{L}}(f) = F(f)h_{\mathrm{U}}(f)$ [thin lens approximation]
  - $h_{\mathrm{U}}(f)$ and $h_{\mathrm{L}}(f)$: unlensed and lensed GWs in frequency domain, respectively.
  - $F(f)$: **amplification factor**

[Takahashi & Nakamura (ApJ,2003) with $c = G = 1$]

Lens redshift

$$F(f) = \frac{\boxed{D_s}\xi_0^2(1+\boxed{z_L})}{\boxed{D_L}\boxed{D_{LS}}}\frac{f}{i}\int d^2\mathbf{x}\,\exp[2\pi i f\,\boxed{t_d(\mathbf{x},\mathbf{y})}]$$

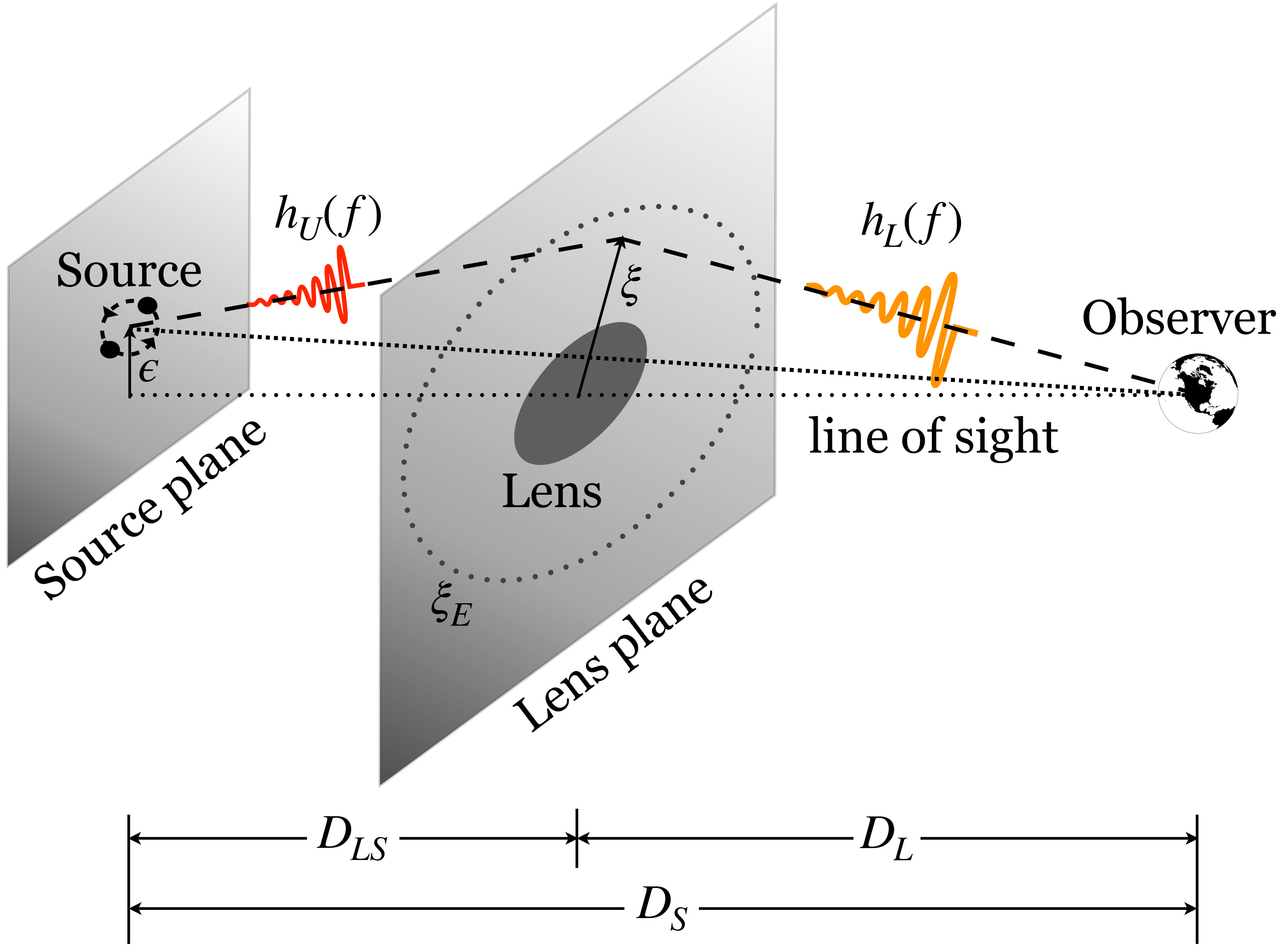
Angular diameter distances

$$\mathbf{x} = \xi/\xi_0$$
$$\mathbf{y} = (\epsilon D_L)/(\xi_0 D_S)$$

Lensing time delay

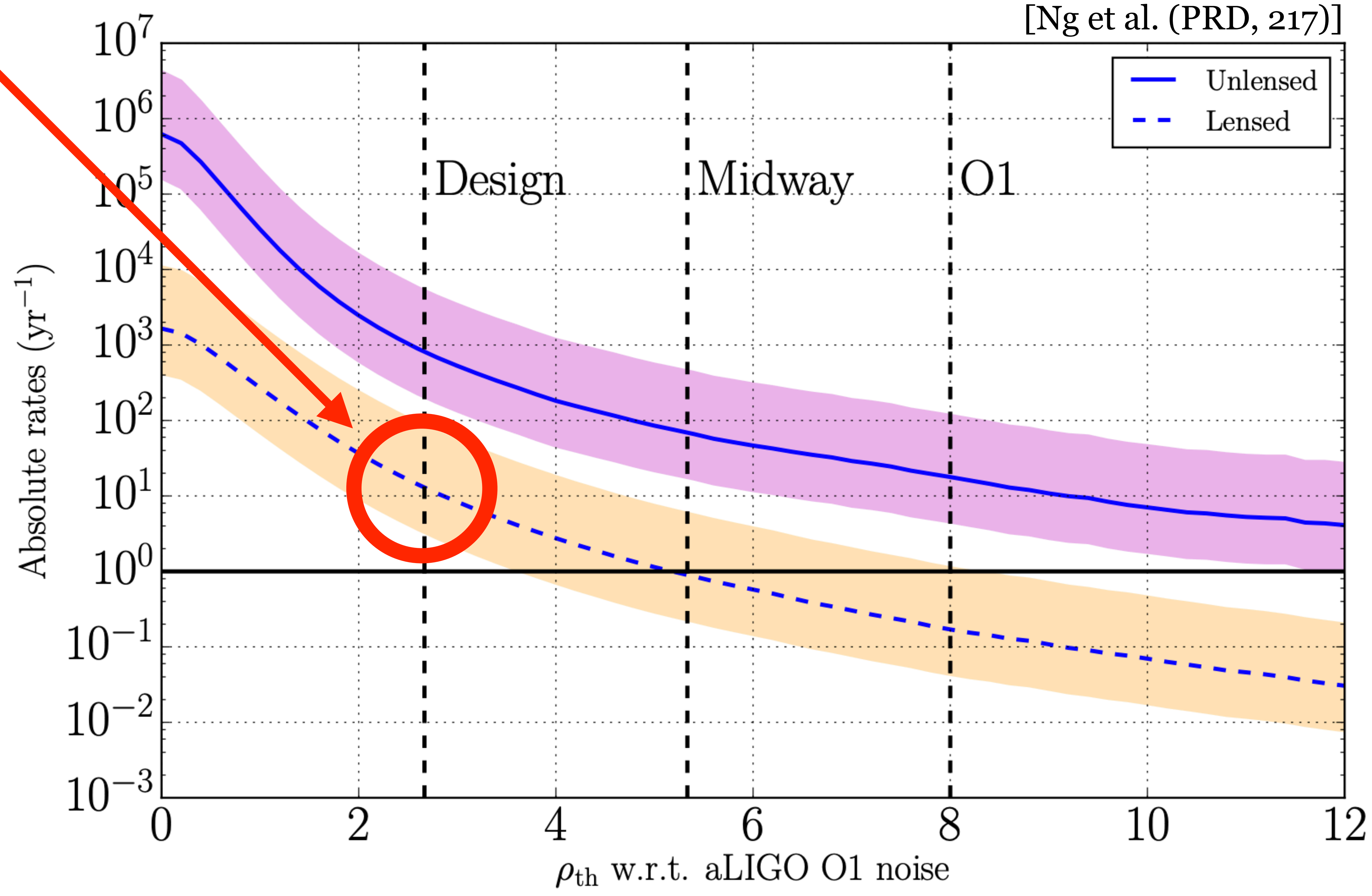
$$t_d(\mathbf{x},\mathbf{y}) = \frac{D_s\xi_0^2}{D_L D_{LS}}(1+z_L)\left[\frac{1}{2}|\mathbf{x}-\mathbf{y}|^2 - \boxed{\psi(\mathbf{x})} + \phi_m(\mathbf{y}))\right]$$

Lens potential



Why is GW lensing exciting now?

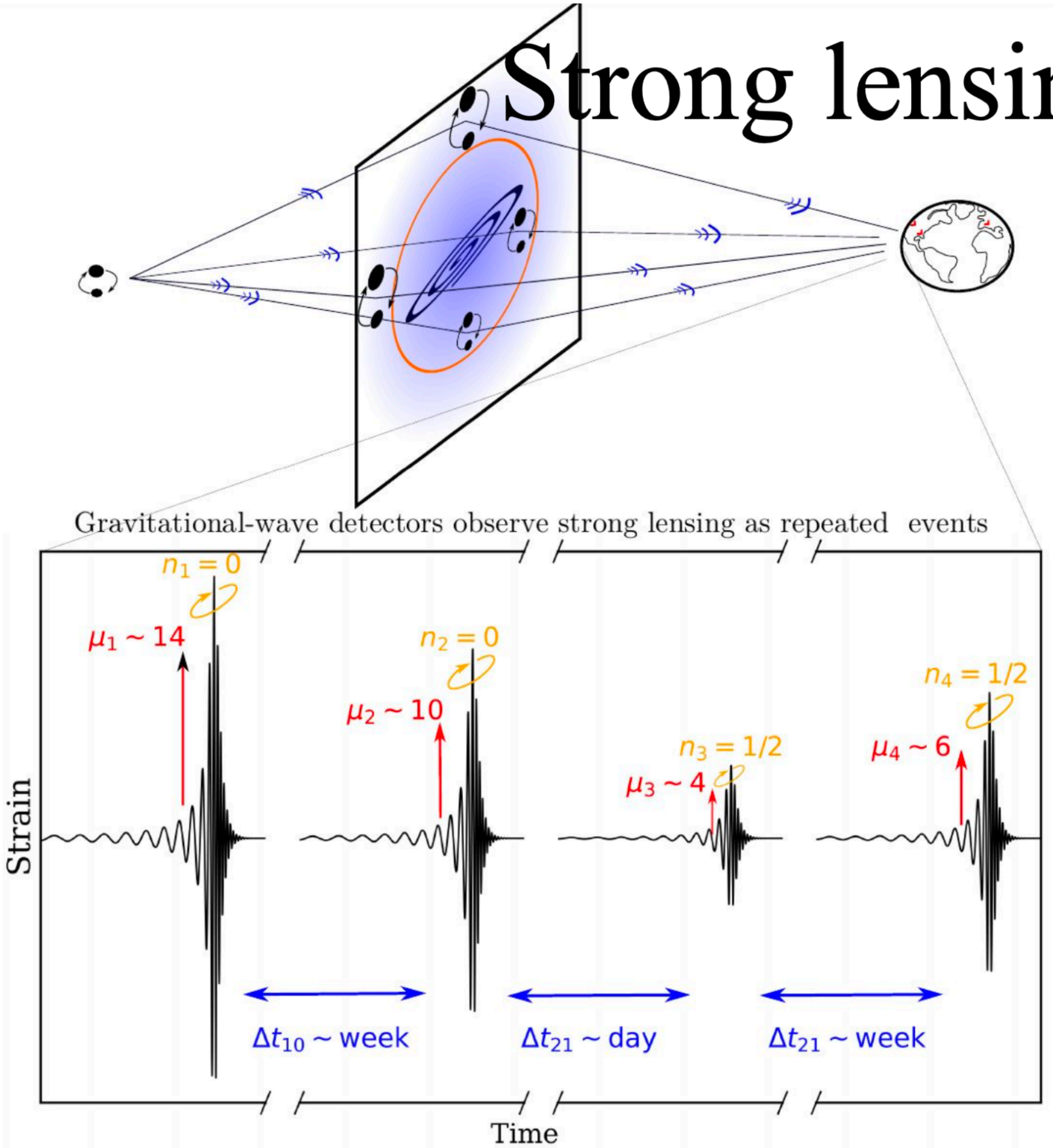
- Forecasts predict strong lensing at a reasonable rate
  Ng et al. (2017), Li et al. (2018), Oguri (2018), Wierda et al. (2021), Xu et al. (2021)



- Can probe new studies of astrophysics, cosmology, and fundamental physics
  - **Study the origin of black holes**
    Hannuksela, et al. (2020)
  - **Tests of fundamental physics**
    Collet & Bacon (2017; speed of GWs), Liu & KK (2024; millilensed GWs vs precessing GWs)
  - **Study the expansion of the Universe**
    Baker & Trodden (2017), Liao, et al. (2017), Hannuksela, et al. (2020), Seo & KK, et al. (in prep.)
  - **Study microlens populations** *Lai et al. (2018), Jung et al. (2019)*
  - **Study wave optics** *Cheung et al. (2020)*

Search for strong lensing of gravitational waves

Strong lensing challenge



Strong lensing:
- Identify “repeated events” in the data
- Targets: Galaxies, galaxy clusters

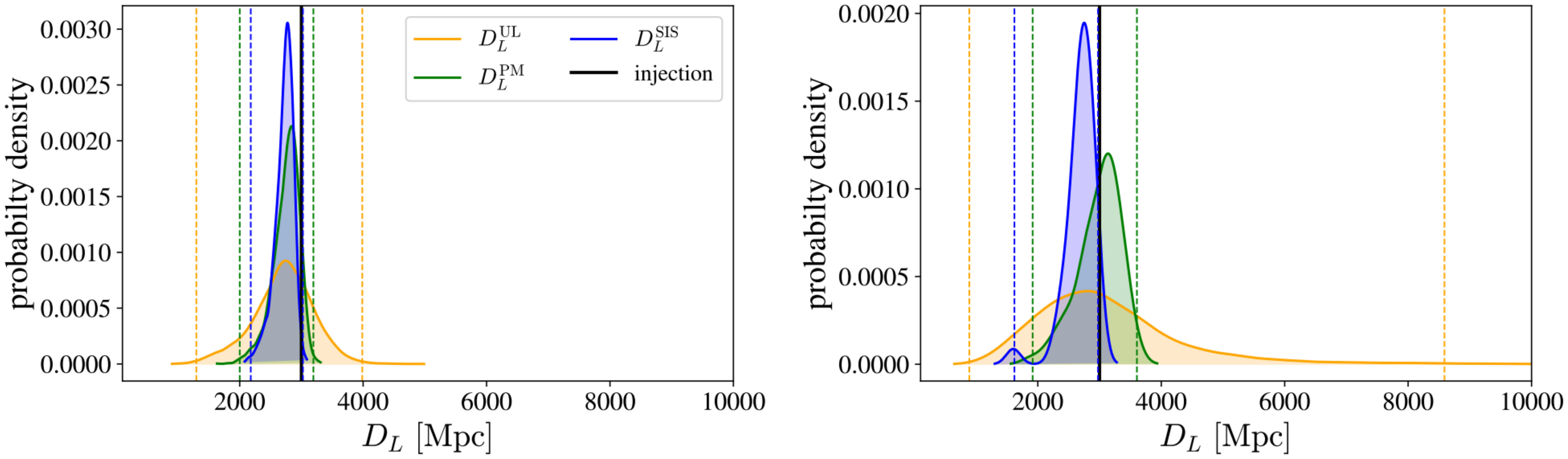
Precise measurement of gravitational-wave image properties

[credit: Otto Hannuksela]

Luminosity distance estimation for Binary black holes with strongly lensed GWs

[KK, E. Seo, C. Kim (PRD, 2024)]

- Luminosity distance d_L: a direct observable of GW observations
- Time delay of strongly lensed GWs: days — months depending on lens mass and alignment
  - likely to identify multiple lensed GW events as multiple independent events.
- Demonstrate estimating d_L to BBHs with strongly lensed GW signals.
  - Combine results from parameter estimations on two individual lensed GWs.
- Conclude about ***a factor of a few better precision*** compared to that can be expected without lensing.



(a) HLV design PSDs with zero noise

(b) HLV O3a PSDs with a Gaussian noise realization

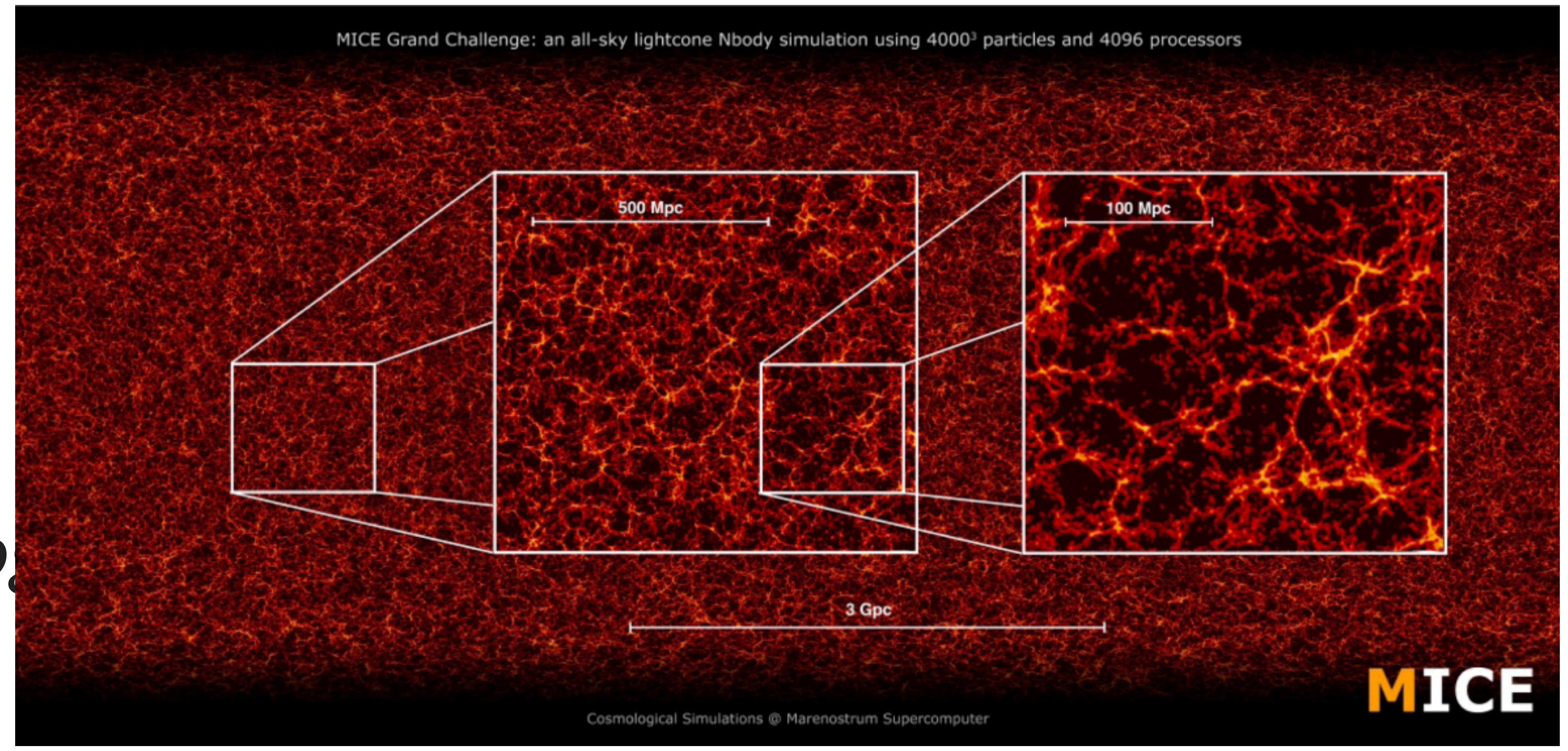
Case A: HLV design PSDs with zero-noise				
Signal	$\mathcal{W}_{99}$ [Mpc]	$\mathcal{R}_{99}$	$\mathcal{W}_{67}$ [Mpc]	$\mathcal{R}_{67}$
Unlensed	2,688	1.00	923	1.00
Lensed (PM)	1,193	2.25	405	2.28
Lensed (SIS)	845	3.18	286	3.23

Case B: HLV O3a PSDs with a Gaussian noise				
Signal	$\mathcal{W}_{99}$ [Mpc]	$\mathcal{R}_{99}$	$\mathcal{W}_{67}$ [Mpc]	$\mathcal{R}_{67}$
Unlensed	7,700	1.00	2,074	1.00
Lensed (PM)	1,689	4.56	637	3.26
Lensed (SIS)	1,354	5.69	348	5.95

- ***We extend this study to the measurement of H_0 using strongly lensed dark sirens.***

Data: Galaxy catalog

- MICE-Grand Challenge light-cone halo and galaxy catalog (MICECATv2)
  - a simulated galaxy catalog covers 5,000 deg^2 of the sky (R.A.=[0°,90°], Dec.=[0°,90°]) and redshift (z) range $0.07 < z < 1.42$.
  - contains ~ 500 million galaxies
    → *randomly select 64 million galaxies to mimic real galaxy catalog*
  - provides luminosities and apparent magnitudes of galaxies, as well as true and observer-frame redshifts.



[credit: Fosalba+'15]

- Simulation of galaxy lensing
  - Singular-Isothermal-Ellipsoid (SIE) lens model → double and quadruple images
- Completeness of catalog
  - take into account the discrepancy between real distribution and observable distribution of galaxies:
    - complete unlensed/lensed catalog (UGC0/LGC0) vs. imcomplete unlensed/lensed catalog (UGC1/LGC1)

Data: Dark siren

- Simulate unlensed/lensed GWs from binary black holes (BBHs)
  - unlensed GWs: use IMRPhenomXPHM waveform model with adopting the prior ranges of parameters and hyperparameters of BBHs inferred from the LIGO-Virgo O3 population analysis [Abbott+ '23].
  - lensed GWs: regarding SIE lens model, adopt the Salpeter model for the velocity dispersion (σ_v) and the Rayleigh model for the axis ratio (q) → compute the amplification factor $F(f)$ using the formalism described in Seo+'24 (ApJ)
  - noise realization: O4a design sensitivities of LIGO-Livingston, LIGO-Hanford, and Virgo
- Signal-to-noise ratio (SNR) threshold: network SNR > 8
- Each simulated BBH is assigned to a galaxy randomly selected from the UGC0

Estimation of H_0 from single lensed dark siren

- Hubble-Lemaître law, $H_0 = v_r/D$, where
  - $v_r \approx cz_s$: the recessional velocity of the source galaxy at z_s
  - D: proper distance to the source galaxy
    - In a flat-ΛCDM universe w/ $a_0 = 1$, D is equivalent to the comoving distance D_C and can be related to the luminosity distance d_L and z_s, i.e., $D \equiv D_\mathrm{C} = d_L/(1+z_s)$.
- Assume that if a source-lens system is uniquely specified, we can take z_s from the UGC0.
  - Uncertainties in the redshift of source galaxy, σ_z
    - photometric: $\sigma_z = 0.033(1+z_s)$; spectroscopic: $\sigma_z = 0.001(1+z_s)$
- Delensing
  - recovering the true d_L from the effective luminosity distances of j-th image $d_{L,j}^{\mathrm{eff}} = d_L/\mu_j$, where μ_j is the magnification factor of j-th image signal [Janquart+'21&'23; Kim+'24; Seo+'24]

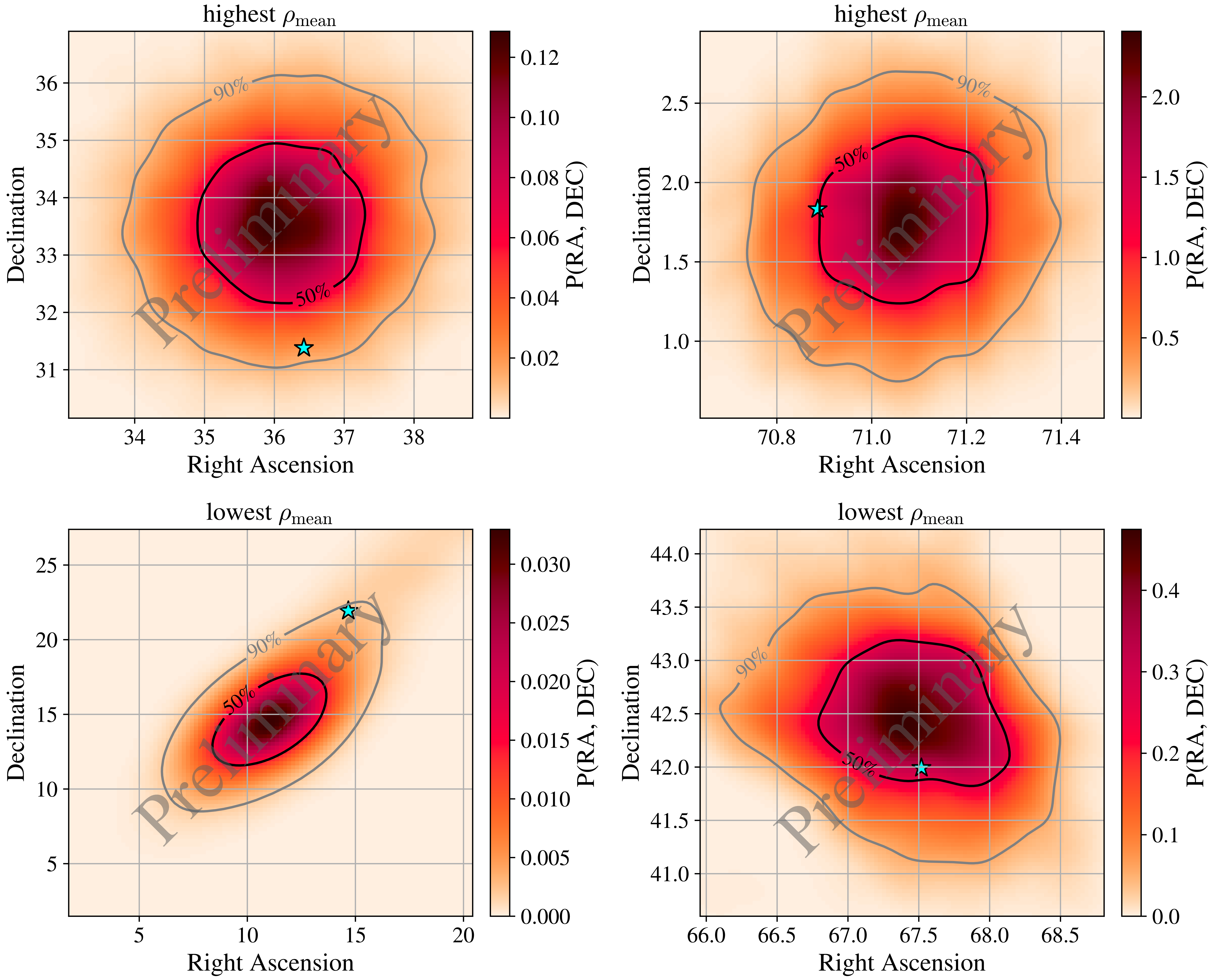
Estimation of H_0 from single lensed dark siren

- **Sky localization between double and quadruple images**
  - using quadruple images is better than using double images → beneficial in specifying source-lens galaxy system!

System	mean SNR (ρ_{mean})	$\Delta\Omega_{90\%}$ [deg²]	N_{LG}	N_{LG}^{LGW}
Double 1	22.0	17.5	306	2
Double 2	8.1	178.8	1971	2
Quadruple 1	30.0	0.97	1	1
Quadruple 2	9.0	4.35	13	1

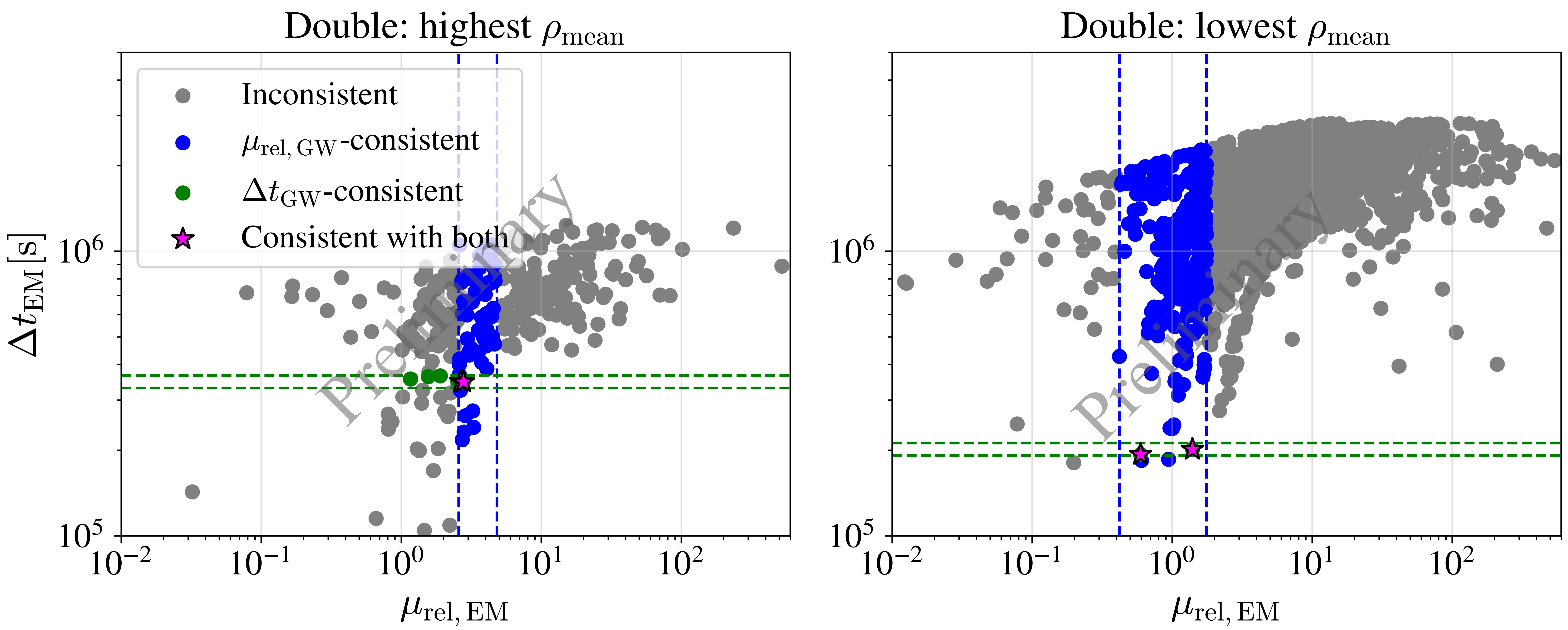
* $\rho_{\mathrm{mean}} = \langle \rho_j \rangle$

“Double images”   “Quadruple images”



Estimation of H_0 from single lensed dark siren

- **Consistency test between source-lens galaxy systems in EM and GW observations**
  - Ex. Double 1 and Double 2 cases resulting in $N_{LG}^{LGW} = 2$

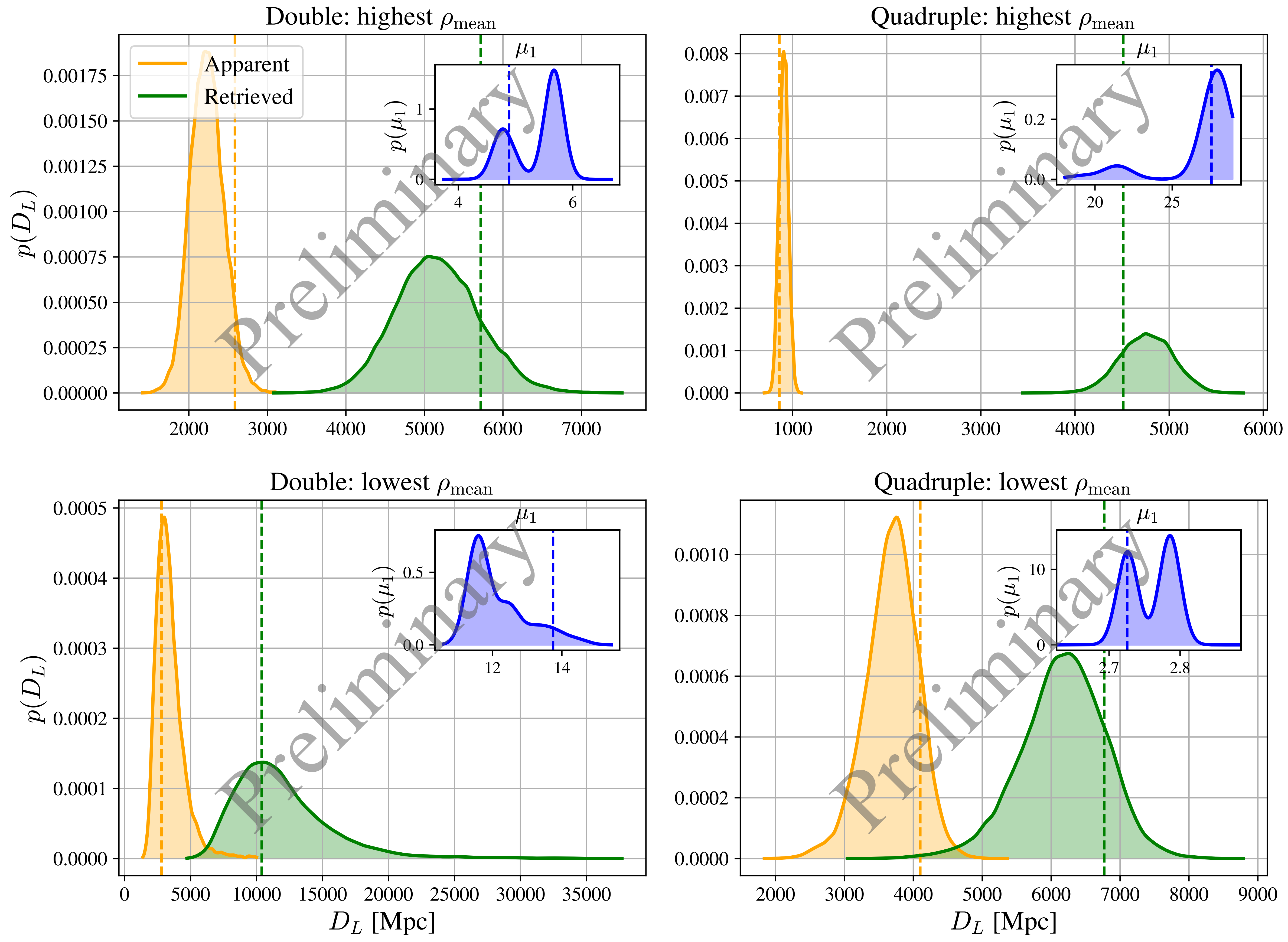


- blue and green dashed lines: constrained boundaries of μ_{rel} and Δt inferred from lensed GW
- blue and green dots: consistent systems of μ_{rel} and Δt between EM and GW
- magenta dots: consistent with both constraints on μ_{rel} and Δt
- gray dots: inconsistent systems between EM and GW

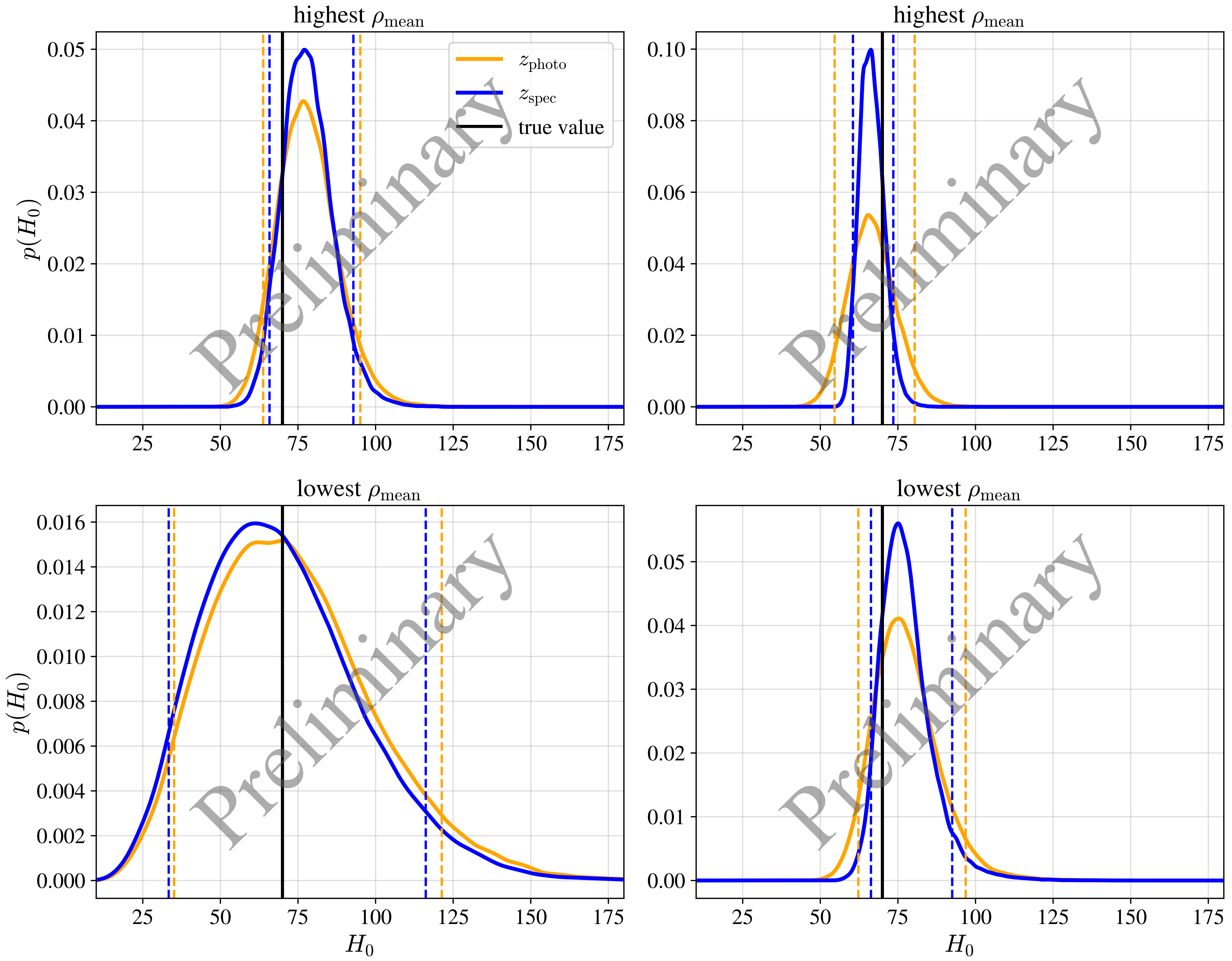
Estimation of H_0 from single lensed dark siren

- *Quadruple* images are beneficial in *better delensing* than double images.

"Double images" "Quadruple images"



"Double images" "Quadruple images"



- Thus, H_0 estimation is more precise and less biased when using quadruple images of higher ρ_{mean} GWs w/ spectroscopic z of galaxies.

Estimation of H_0 from multiple lensed dark sirens

- If *multiple* lensed dark sirens originating from *different* lensing systems are detected…
  - We need to construct a joint likelihood, multiplied by the H_0 prior, to obtain a combined H_0 posterior distribution.
- To do that, we should consider
  - ***Line-of-Sight (LoS) redshift priors***
    : for the redshift information of source galaxies cross-matching the sky localizations of detected dark sirens
  - ***Selection effects***
    : for the avoidance of Malmquist bias caused by missing fainter galaxies or weaker dark sirens

Formula for the posterior of H_0

- The posterior of H_0 for the set of BBHs resulting in lensed dark sirens ($\{x_{\mathrm{LGW}}\}$) and the set of the distance to the source of lensed dark sirens ($\{D_{\mathrm{LGW}}\}$):

"prior terms"

"joint likelihood"

$$
\begin{aligned}
p(H_0|\{x_{\mathrm{LGW}}\},\{D_{\mathrm{LGW}}\}) \propto\ & \boxed{p(H_0)p(N_{\mathrm{det,lens}}|H_0)} \\
\times & \boxed{\left[\int\!\!\int\!\!\int\!\!\int p(D_{\mathrm{LGW}}|z_s,\theta',\theta_l,z_l,H_0)p(\theta_l|\Omega_j,H_0)p(\theta'|H_0)\sum_j^{N_{\mathrm{pix}}} p_{\mathrm{Lgal}}(z_s,z_l|\Omega_j,H_0)\,d\theta_l d\theta' dz_l dz_s\right]^{-N_{\mathrm{det,lens}}}} \\
\times & \boxed{\prod_i^{N_{\mathrm{det,lens}}}\left[\int\!\!\int\!\!\int\!\!\int \sum_j^{N_{\mathrm{pix}}} p(\{x_{\mathrm{LGW},i}\}|\Omega_j,z_s,\theta_l,\theta',z_l,H_0)p(\theta_l|\Omega_j,H_0)p(\theta'|H_0)p_{\mathrm{Lgal}}(z_s,z_l|\Omega_j,H_0)\,d\theta_l d\theta' dz_l dz_s\right]}
\end{aligned}
$$

"LoS redshift prior"

- Estimation of H_0 using the above equation is on-going…

Discussions

- Biases in a galaxy-catalog-based measurement of H_0
  - systematic biases in lens reconstruction due to incorrect lens model
  - statistical biases due to
    - small number of observations of lensed dark sirens
    - selection effect (a.k.a. Malmquist bias) in both GW and EM observations

- Possible future works
  - using real galaxy/galaxy cluster catalog
  - considering next generation terrestrial/space GW detectors like Einstein Telescope or LISA
  - incorporating time-delay cosmography

Thank you
for
your attention!