BASED ON

- KIM, OH, PARK AND SON, PRD (2021)
- SON, KIM AND OH, IN PREPARATION
- SON, IN PREPARATION

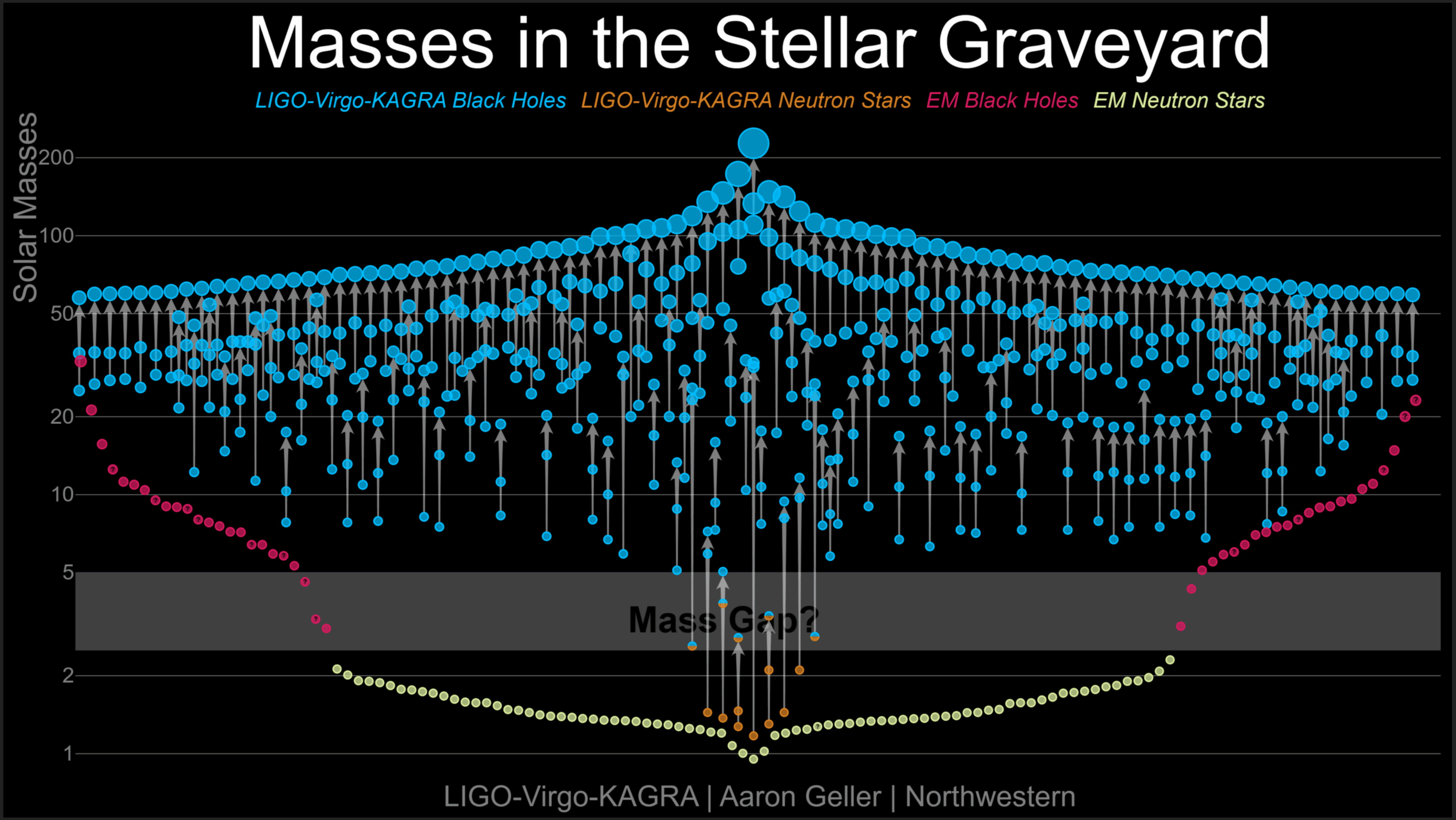
EDWIN J. SON (NIMS)

COMPACT OBJECTS IN HORAVA-LIFSHITZ GRAVITY

http://www.nasa.gov/sites/default/files/pia18848-wisefacepalm.jpg

OVERVIEW

- Mass Gap and Dark Matter
- TOV equation in HL
- Compact Objects in HL
  - Limits on Mass of CO
  - Neutron Stars for some EoSs
  - Fermionic Compact Objects
- Concluding Remarks


Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
EM Black Holes
EM Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
Mass Gap?
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

MASS GAP

(A BRIEF) INTRODUCTION TO

DARK MATTER

[Image: ESO/L. Calçada]
https://www.eso.org/public/unitedkingdom/videos/eso1709c/

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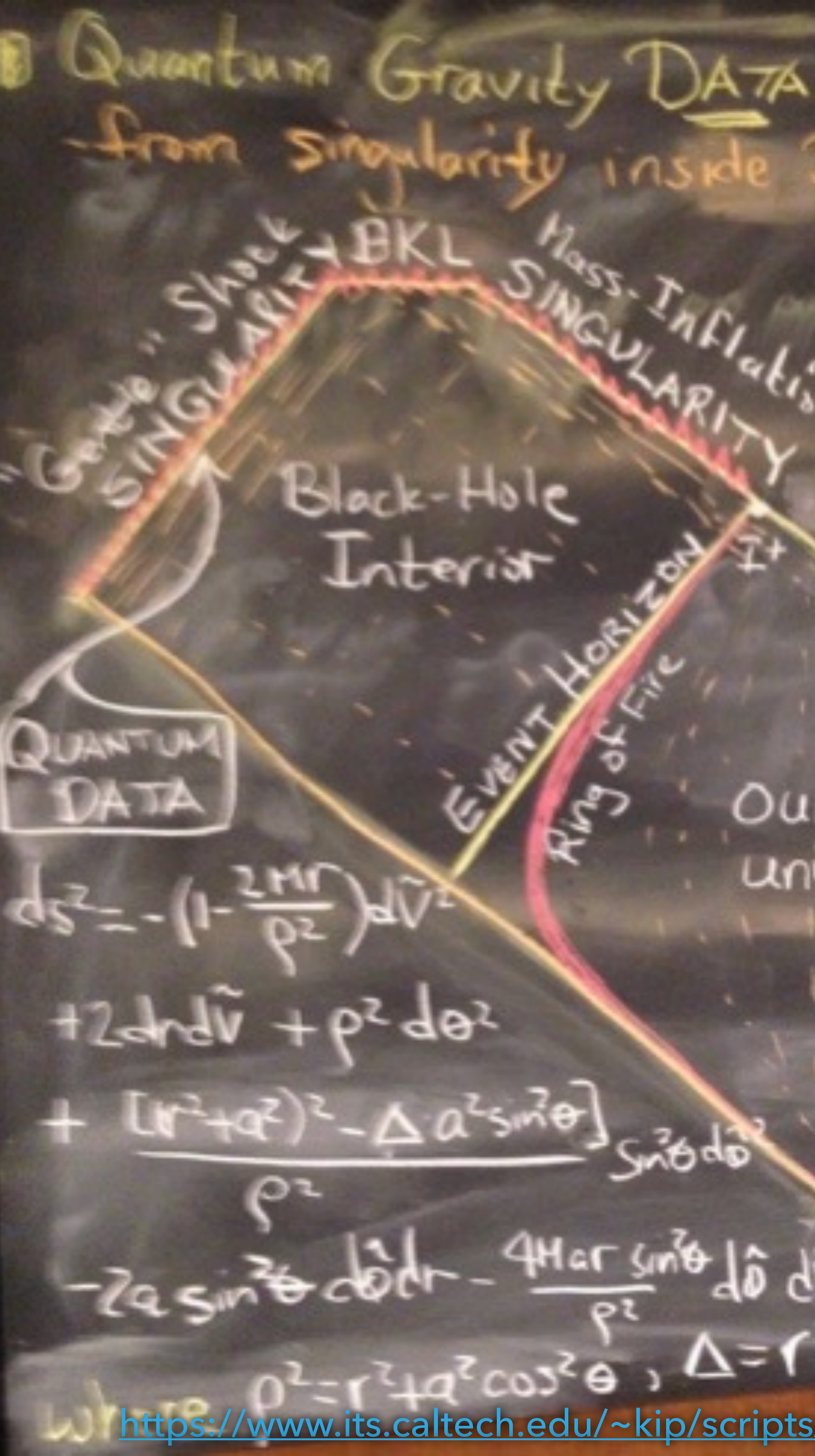


(A BRIEF) INTRODUCTION TO

HL GRAVITY

[Image: Interstellar - Professor Brand's Blackboards]
https://www.its.caltech.edu/~kip/scripts/INTERSTELLAR/BrandBlackBoards/blackboards.html

TOWARDS UV COMPLETE THEORY OF GR

- Horava proposed a UV complete theory of GR by introducing the anisotropic scaling.
- In the IR limit, the Lorentz symmetry should be recovered.
- To deal with the anisotropic scaling, the ADM decomposition is adopted:

$$ds^2 = -N^2c^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\,.$$

DEFORMED HORAVA-LIFSHITZ GRAVITY

- Deformed HL gravity (*softly* broken detailed balance):

$$I_{\mathrm{HL}} = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2}{2\zeta^4}\left(C_{ij} - \frac{\mu\zeta^2}{2}R_{ij}\right)\left(C^{ij} - \frac{\mu\zeta^2}{2}R^{ij}\right) \right.$$
$$\left. + \frac{\kappa^2\mu^2(4\lambda-1)}{32(3\lambda-1)}\left(R^2 + \frac{2(q^{-2} - 2\Lambda_W)}{4\lambda-1}R + \frac{12\Lambda_W^2}{4\lambda-1}\right)\right],$$

where q is introduced to include asymptotically flat solutions and C_{ij} is the Cotton-York tensor,

$$C^{ij} = \varepsilon^{ik\ell}\nabla_k\left(R^j_\ell - \frac{1}{4}R\delta^j_\ell\right).$$

IR LIMIT OF DEFORMED HORAVA-LIFSHITZ GRAVITY

- In the IR limit, GR is recovered with λ=1:

$$I_{\mathrm{EH}} = \frac{1}{16\pi G_N}\int d^4x\sqrt{g}N\left[(K_{ij}K^{ij}-K^2)+R-2\Lambda\right],$$

  where the fundamental coefficients are identified as

$$c = \frac{\kappa^2}{4}\sqrt{\frac{\mu^2(q^{-2}-2\Lambda_W)}{2(3\lambda-1)}},\ G_N = \frac{\kappa^2c^2}{32\pi},\ \Lambda = -\frac{3\Lambda_W^2}{q^{-2}-2\Lambda_W}.$$

KEHAGIAS-SFETSOS BLACK HOLE

- Spherically symmetric metric ansatz:

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,.$$

- Asymptotically flat (Λ_W=0) vacuum solution with λ=1:

$$N^2 = f = 1 + \frac{r^2}{2q^2}\left[1 - \sqrt{1 + 8q^2M/r^3}\right]$$

$$= \frac{2[1 - 2M/r + q^2/r^2]}{1 + 2q^2/r^2 + \sqrt{1 + 8q^2M/r^3}}$$

$$\approx 1 - \frac{2M}{r} + O\left(\frac{q^2M^2}{r^4}\right) \text{ for } \frac{q^2}{r^2}, \frac{M}{r} \ll 1.$$

[X-ray: NASA/CXC/Univ. of Wisconsin-Madison/S. Heinz, et al.; Optical: DSS]
http://chandra.si.edu/photo/2015/cirx1/

DERIVING AND SOLVING

TOV EQUATION

EQUATIONS OF MOTION IN DHL

- Starting with I_{tot}=I_{HL}+I_{mat}, where I_{mat} is the matter action of a perfect fluid and will be specified by choosing a EoS.
- A static, spherically symmetric metric ansatz:

$$ds^2 = -e^{2\Phi(r)}c^2dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

- Equations of motion:

$$\rho = \frac{c^2q^2}{8\pi G_N r^2}\left(q^{-2}r(1-f) + \frac{(1-f)^2}{r}\right)',$$

$$p = \frac{c^4q^2}{8\pi G_N r^4}\left[(1-f)\left(1-f-q^{-2}r^2\right) + 4rf\left(1-f+r^2/2q^2\right)\Phi'\right],$$

$$p' = -\left(\rho c^2 + p\right)\Phi'.$$

TOLMAN-OPPENHEIMER-VOLKOFF EQUATION IN DHL

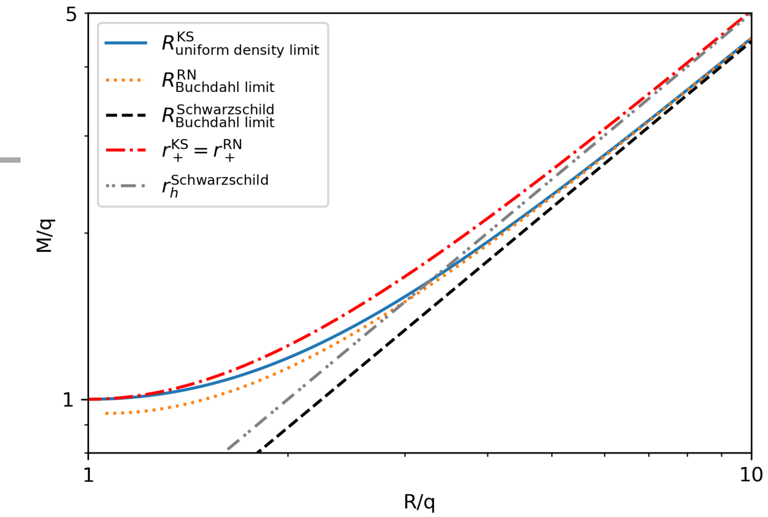
- The function f is solved as
$$f = 1 + \frac{r^2}{2q^2}\left[1 - \sqrt{1 + 8q^2 G_N c^{-2} m(r)/r^3}\right], \qquad m(r) = \int_0^r dr' 4\pi r'^2 \rho(r')\,.$$

- TOV equation in DHL:
$$m' = 4\pi r^2 \rho,$$
$$p' = -\frac{G_N m \rho \alpha}{r^2 \beta} \frac{(1 + p/\rho c^2)\left[1 + 4\pi r^3 p\beta/mc^2 - q^2\tilde{\rho}\right]}{\sqrt{1 + 8q^2\tilde{\rho}}\left[1 - 2G_N m/rc^2 + q^2/r^2\right]},$$
where $\alpha = 2^{-1}\left[1 + 2q^2/r^2 + \sqrt{1 + 8q^2\tilde{\rho}}\right], \beta = 2^{-1}\left[1 + 2q^2\tilde{\rho} + \sqrt{1 + 8q^2\tilde{\rho}}\right]$
and $\tilde{\rho} = G_N c^{-2} m r^{-3}$.

- We can solve TOV equation by considering a specific equation-of-state.

EQUATION-OF-STATE FOR AN ARBITRARY COMPACT OBJECT

- Uniform density case: $\rho = \rho_c =$ const.
- Randomly chosen EoS that satisfies: $\rho(r)$ decreases as the radius r inside the compact object increases and $p(r) \geq 0$.
- Causal limit: $c_s = (dp/d\rho)^{1/2} \leq c$.



UNIFORM DENSITY LIMIT

- Following [Buchdahl, Phys. Rev. (1959)], a compact object of radius R with uniform density, $\rho(r) = \rho_c =$ const. for $r < R$, is considered. Then, we have

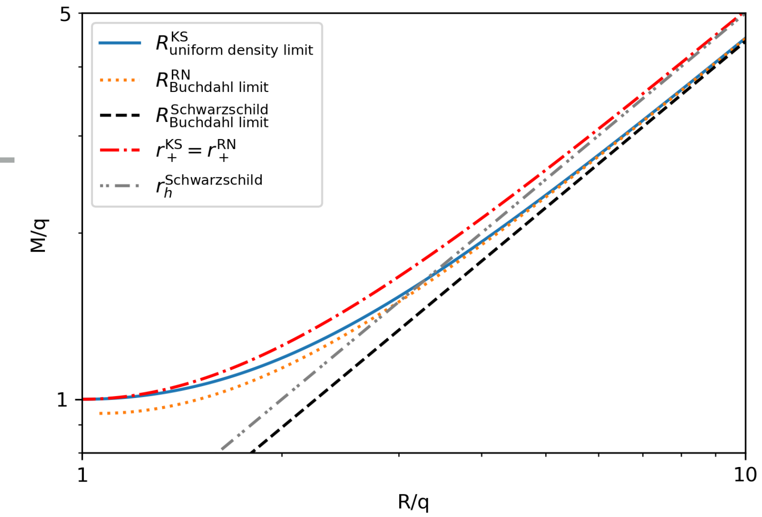
$$\frac{p}{\rho_c} = \frac{\left(1 - \alpha r^2/q^2\right)^{1/2} - \left(1 - \alpha R^2/q^2\right)^{1/2}}{\beta\left(1 - \alpha R^2/q^2\right)^{1/2} - \left(1 - \alpha r^2/q^2\right)^{1/2}},$$

where $\alpha = 2^{-1}[\sqrt{1 + (32\pi/3)q^2\rho_c} - 1]$ and $\beta = 3(1+\alpha)/(1-\alpha)$.

- The uniform density limit is given by

$$\frac{M}{R} < \frac{\left(4 - 9R^2/q^2 + \xi\cos(\theta/3)\right)\left(8 + 9R^2/q^2 + 2\xi\cos(\theta/3)\right)}{729R^2/q^2} \approx \frac{4}{9} + \frac{16}{27}\left(\frac{R}{q}\right)^{-2} + O\left(\frac{R}{q}\right)^{-4}$$

where $\theta = \tan^{-1}\dfrac{162\sqrt{3}R^2\sqrt{64 + 368R^2/q^2 + 144R^4/q^4 + 81R^6/q^6}}{512q^2 + 3024R^2 - 972R^4/q^2 + 729R^6/q^4}$ and $\xi = \sqrt{64 + 252R^2/q^2 + 81R^4/q^4}$



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where $\alpha = 2^{-1}[\sqrt{1 + (32\pi/3)q^2\rho_c} - 1]$ and $\beta = 3(1+\alpha)/(1-\alpha)$.

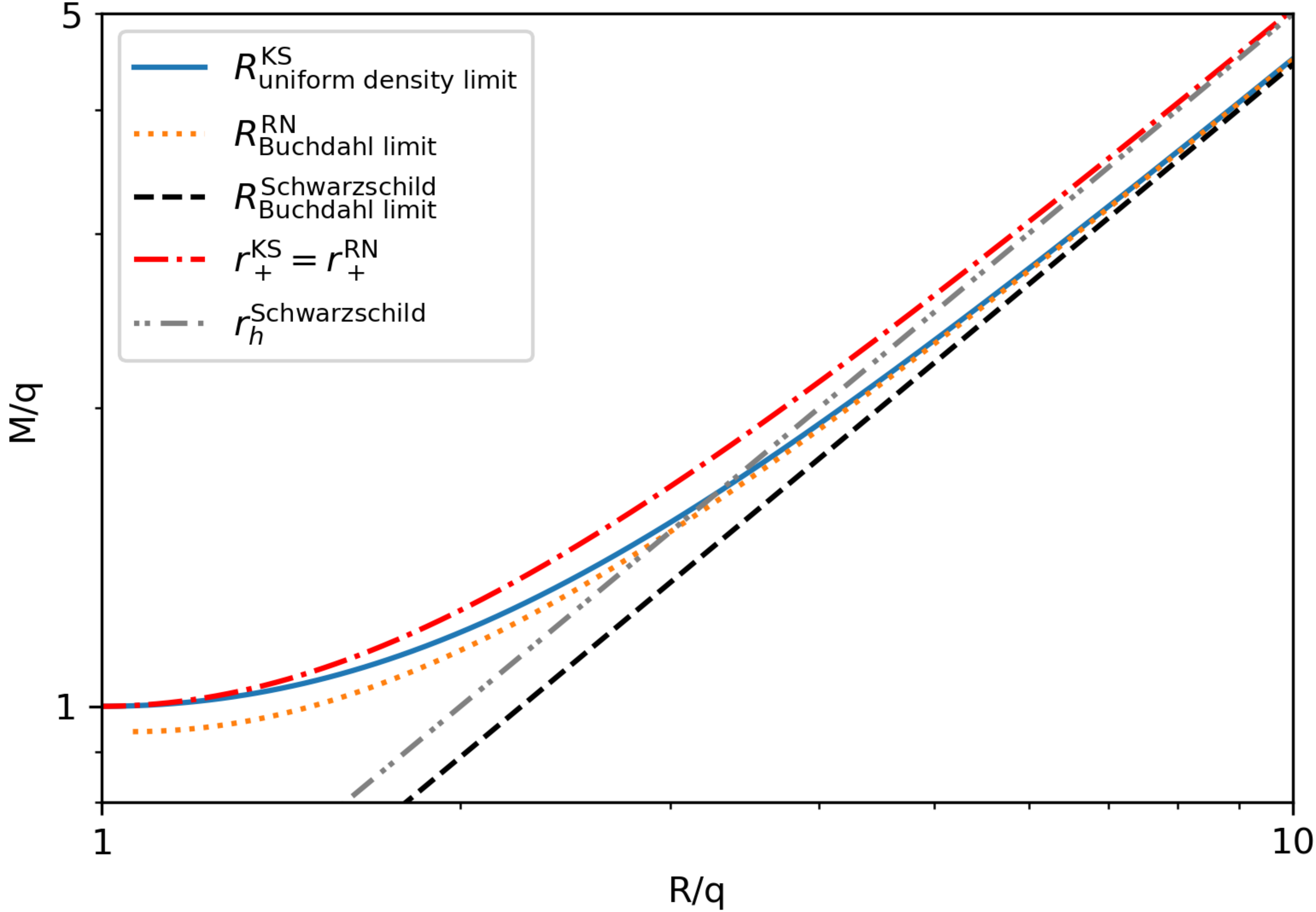
- Buchdahl limit in RN [Dadhich, JCAP (2020)]

$$\frac{M}{R} \leq \frac{4}{9} + \frac{Q^2}{2R^2}$$

- The uniform density limit is given by

$$\frac{M}{R} < \frac{\left(4 - 9R^2/q^2 + \xi\cos(\theta/3)\right)\left(8 + 9R^2/q^2 + 2\xi\cos(\theta/3)\right)}{729R^2/q^2} \approx \frac{4}{9} + \frac{16}{27}\left(\frac{R}{q}\right)^{-2} + O\left(\frac{R}{q}\right)^{-4}$$

where $\theta = \tan^{-1}\dfrac{162\sqrt{3}R^2\sqrt{64 + 368R^2/q^2 + 144R^4/q^4 + 81R^6/q^6}}{512q^2 + 3024R^2 - 972R^4/q^2 + 729R^6/q^4}$ and $\xi = \sqrt{64 + 252R^2/q^2 + 81R^4/q^4}$



$R^{\mathrm{KS}}_{\text{uniform density limit}}$
$R^{\mathrm{RN}}_{\text{Buchdahl limit}}$
$R^{\mathrm{Schwarzschild}}_{\text{Buchdahl limit}}$
$r_+^{\mathrm{KS}} = r_+^{\mathrm{RN}}$
$r_h^{\mathrm{Schwarzschild}}$
M/q
R/q
5
1
1
10

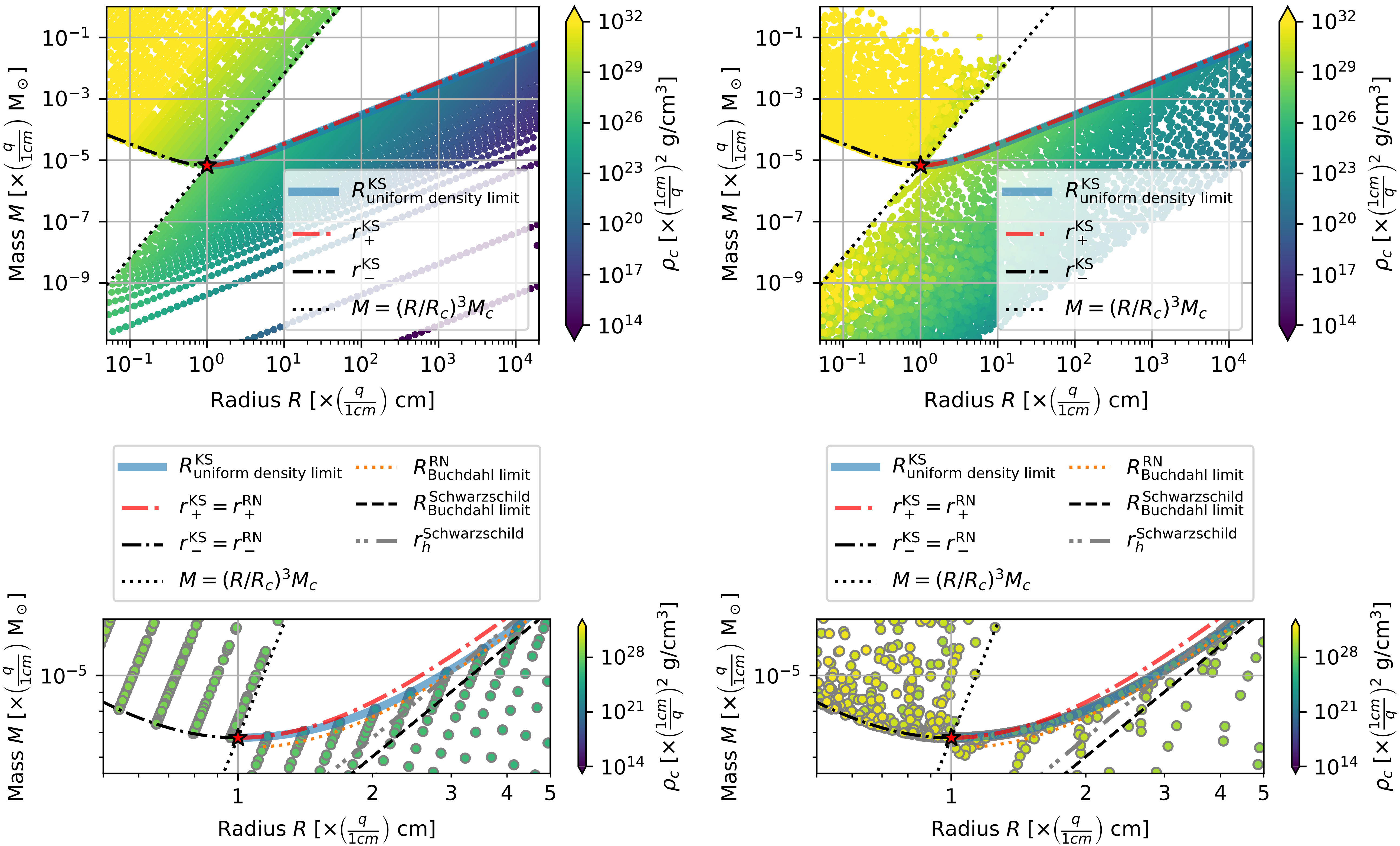
SOLVING TOV NUMERICALLY

- In order to solve the TOV equation numerically, we adopt the explicit Runge-Kutta method of order 8.[E. Hairer, S. P. Norsett G. Wanner, "Solving Ordinary Differential Equations I: Nonstiff Problems", Sec. II.]

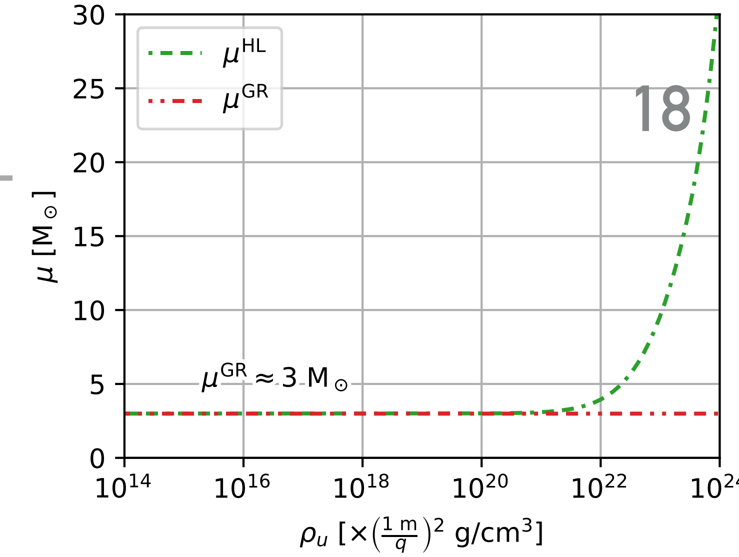
- The radius R of a compact star is determined by $\rho(R) = 0$ or $p(R) = 0$.

NUMERICAL SIMULATION

- The cases of the uniform density and random EoS are simulated.
- Both cases show that the uniform density limit obtained by the analytical calculation gives the upper bound of the possible masses of compact objects with a given size.
- The uniform density limit curve meets the Buchdahl limit for the compact objects significantly larger than the scale parameter q.



Mass M [$\times\left(\frac{q}{1cm}\right)$ M$_\odot$]
Radius R [$\times\left(\frac{q}{1cm}\right)$ cm]
ρ_c [$\times\left(\frac{1cm}{q}\right)^2$ g/cm^3]
$R^{\mathrm{KS}}_{\text{uniform density limit}}$
r^{KS}_+
r^{KS}_-
$M = (R/R_c)^3 M_c$
$r^{\mathrm{KS}}_+ = r^{\mathrm{RN}}_+$
$r^{\mathrm{KS}}_- = r^{\mathrm{RN}}_-$
$R^{\mathrm{RN}}_{\text{Buchdahl limit}}$
$R^{\text{Schwarzschild}}_{\text{Buchdahl limit}}$
$r^{\text{Schwarzschild}}_h$



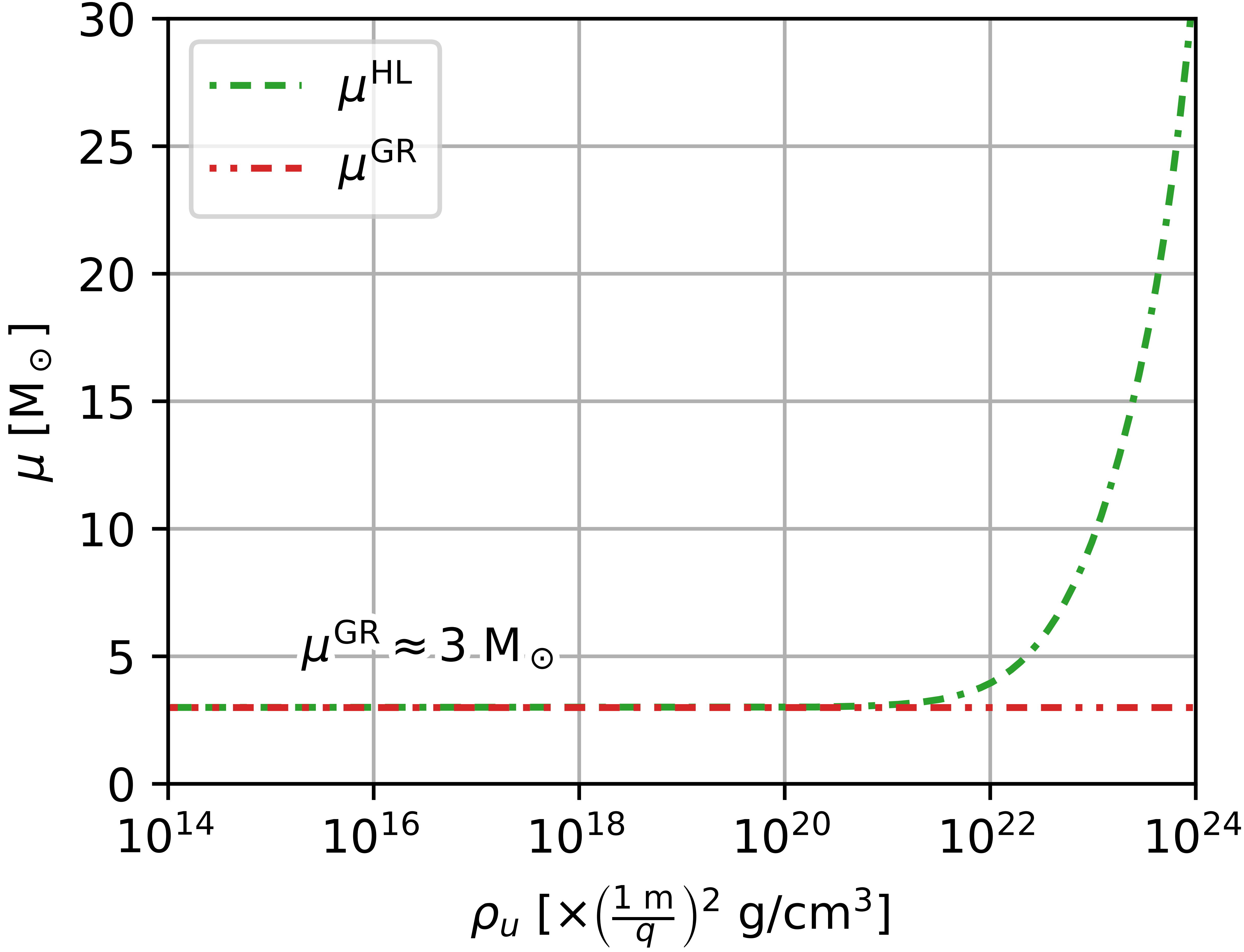
SOUND SPEED LIMIT

- In GR, the speed of sound $c_s = dp/d\rho$ inside a compact object has to be subluminal, which raises the causal limit given by[C. E. Rhoades, Jr. and R. Ruffini, PRL 32, 324 (1974); V. Kalogera and G. Baym, APJL 470, L61 (1996)]

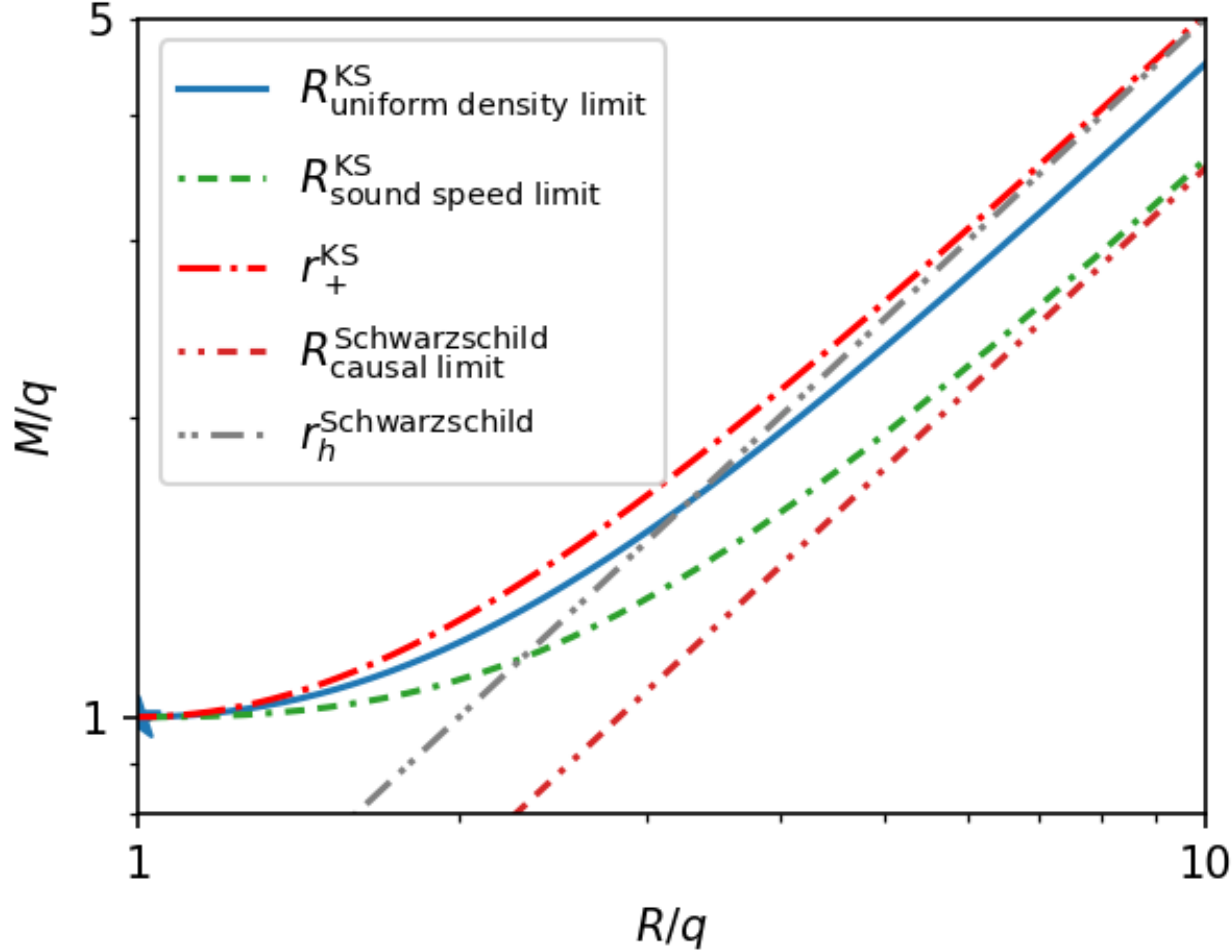
$$M_{\mathrm{max}} = \mu \left(\frac{\rho_u}{5 \times 10^{14}\ \mathrm{g/cm}^3} \right)^{-1/2},$$

where $\mu^{\mathrm{GR}} \approx 3\ \mathrm{M}_\odot$ and ρ_u is the fiducial density that is the maximum density of a known EoS.

- In HL, μ^{HL} can be larger than $3\ \mathrm{M}_\odot$ for large ρ_u.



30
25
20
15
10
5
0
μ^{HL}
μ^{GR}
$\mu^{\mathrm{GR}} \approx 3\,\mathrm{M}_\odot$
$\mu\ [\mathrm{M}_\odot]$
10^{14}
10^{16}
10^{18}
10^{20}
10^{22}
10^{24}
$\rho_u\ [\times\left(\frac{1\ \mathrm{m}}{q}\right)^2\ \mathrm{g/cm^3}]$



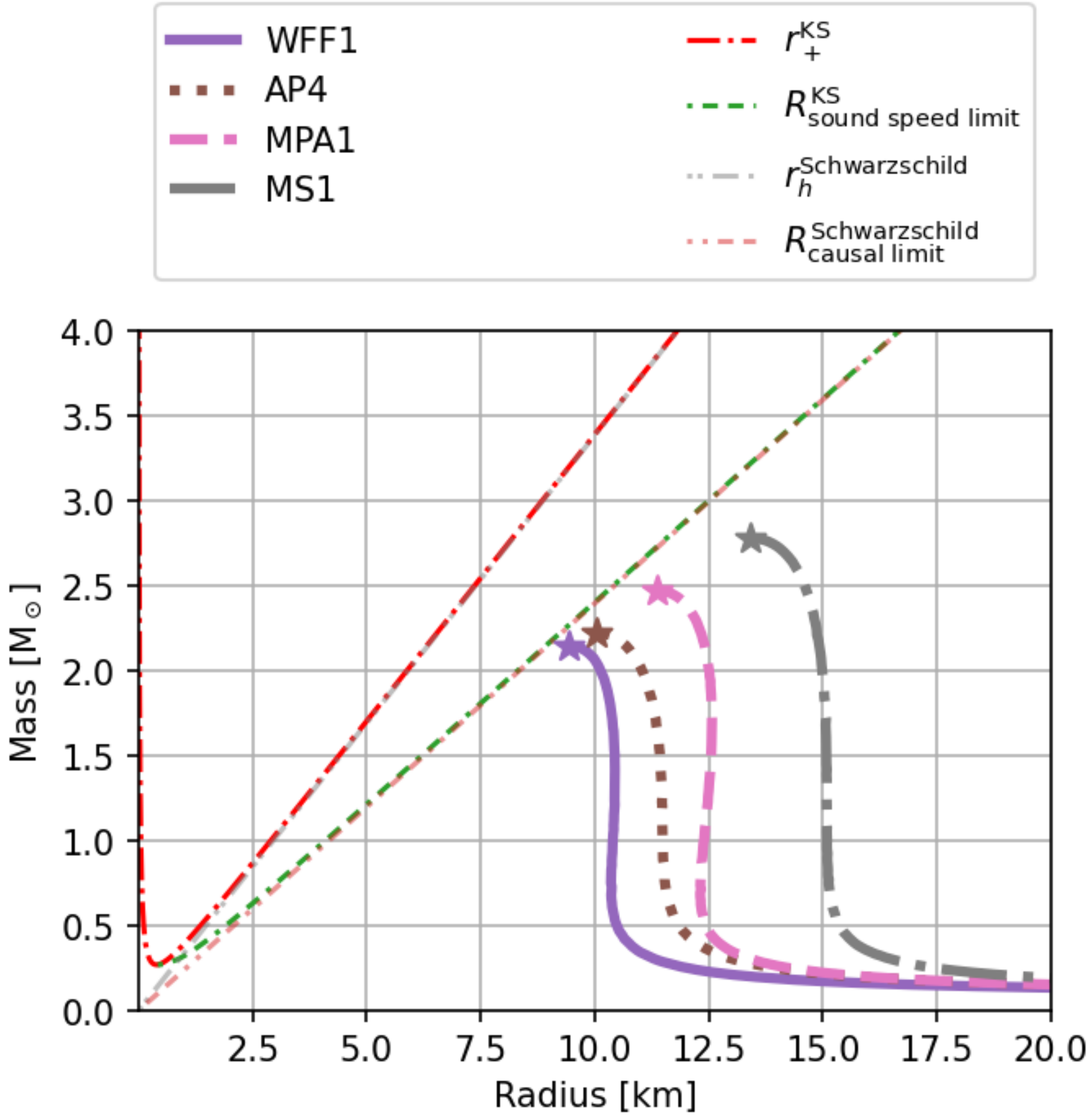
5
1
1
10
M/q
R/q
$R^{\mathrm{KS}}_{\text{uniform density limit}}$
$R^{\mathrm{KS}}_{\text{sound speed limit}}$
r^{KS}_{+}
$R^{\text{Schwarzschild}}_{\text{causal limit}}$
$r^{\text{Schwarzschild}}_{h}$

SELECTED EQUATION-OF-STATE MODELS

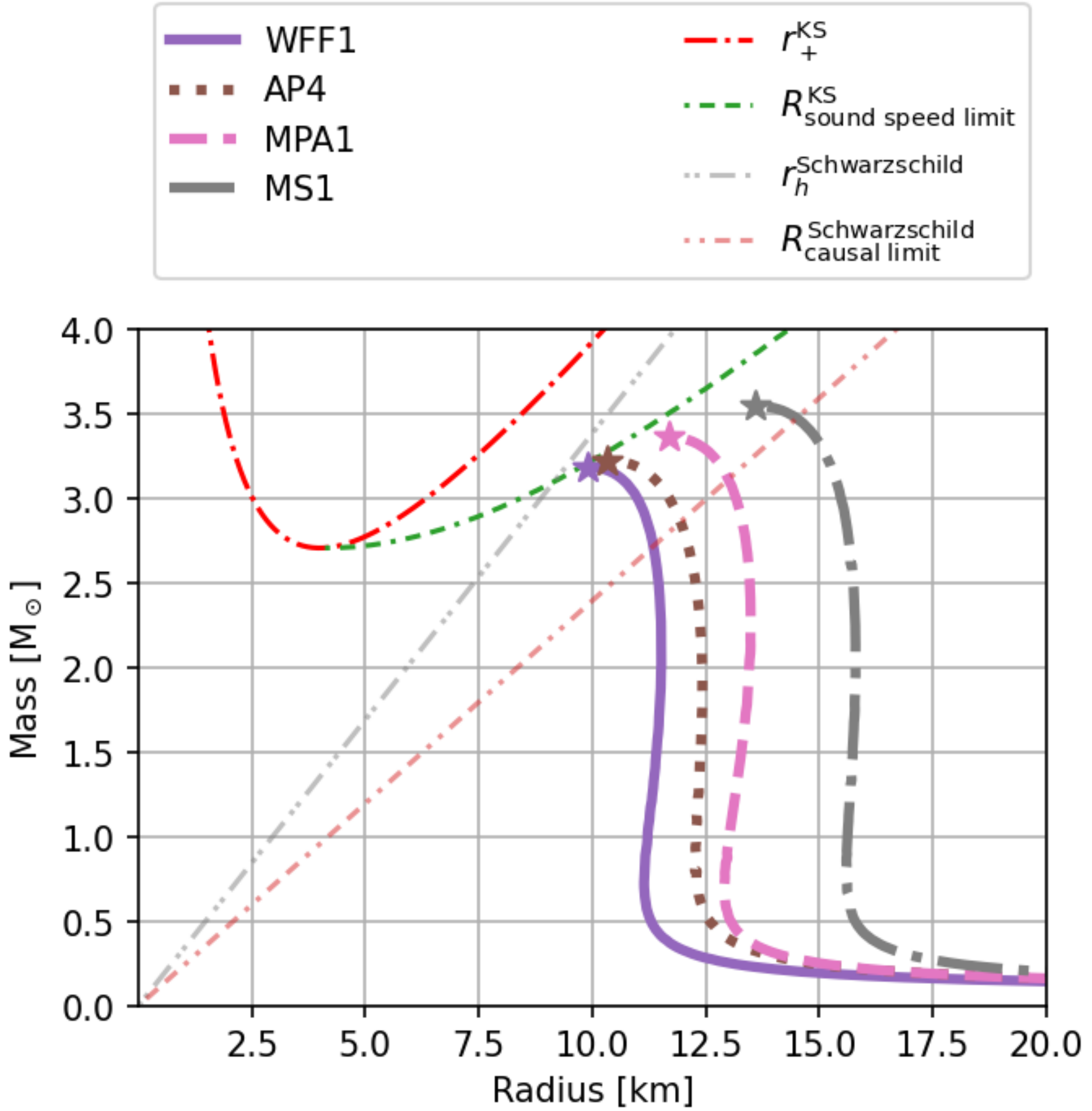
- We select four EoS models which cover the observed maximum mass, $\sim 2M_{\odot}$ [P. B. Demorest et al., Nature (2010); M. Linares et al., ApJ (2018)]
- APR4: derived by variational method but with a specific nucleon potential model [A. Akmal et al., PRC (1998)]
- MPA1: derived by relativistic Brueckner-Hartree-Fock theory [H. Müther et al., PLB (1987)]
- MS1: derived by relativistic mean field theory [H. Müller et al., NPA (1996)]
- WFF1: derived by variational method but with a specific nucleon potential model [R. B. Wiringa et al., PRC (1988)]
- For the crust structure, we impose Skyrme-Lyon model.[F. Douchin et al., A&A (2001)]

MASS-RADIUS PROFILE OF NEUTRON STARS

$q = 400$ m



$q = 4\,000$ m



EQUATION-OF-STATE FOR A FREE FERMION GAS

- The EoS for a free fermion gas at zero temperature:

$$\rho = \frac{1}{\pi}\int_0^{k_F} k^2\sqrt{m_f^2 + k^2}dk = \frac{m_f^4}{8\pi^2}[(2\eta^3 + \eta)(1+\eta^2)^{1/2} - \sinh^{-1}(\eta)]$$

$$p = \frac{1}{3\pi^2}\int_0^{k_F} \frac{k^4}{\sqrt{m_f^2 + k^2}}dk = \frac{m_f^4}{24\pi^2}[(2\eta^3 - 3\eta)(1+\eta^2)^{1/2} + 3\sinh^{-1}(\eta)],$$

  where $\eta \equiv k_F/m_f$.

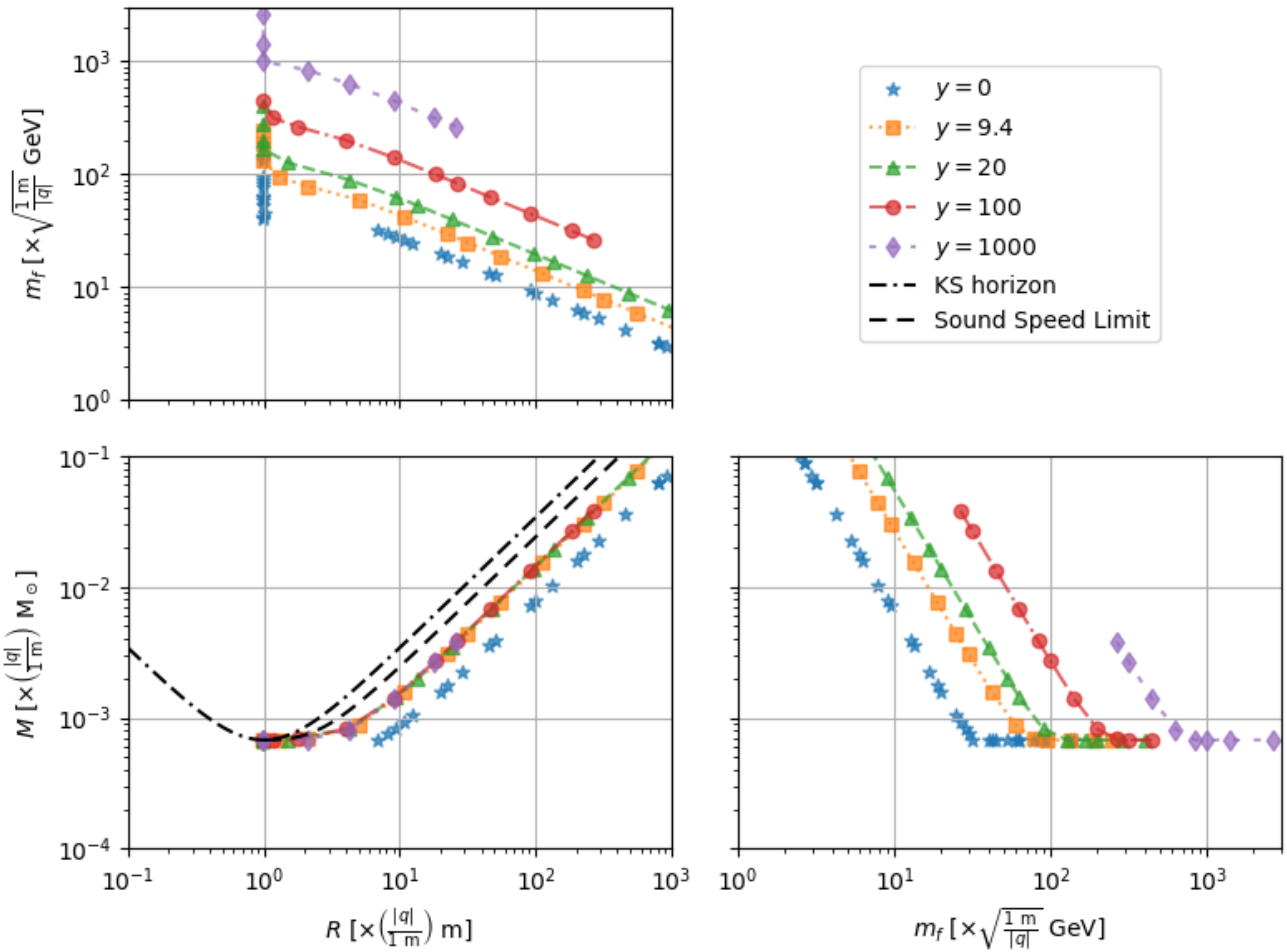
- An interaction term $(m_f^2/3\pi^2)^2 y^2 \eta^6$ can be added to both equations, where $y = m_f/m_I$ determining the interaction strength when the amount of interaction energy m_I is given.

FERMIONIC COMPACT OBJECTS IN GR

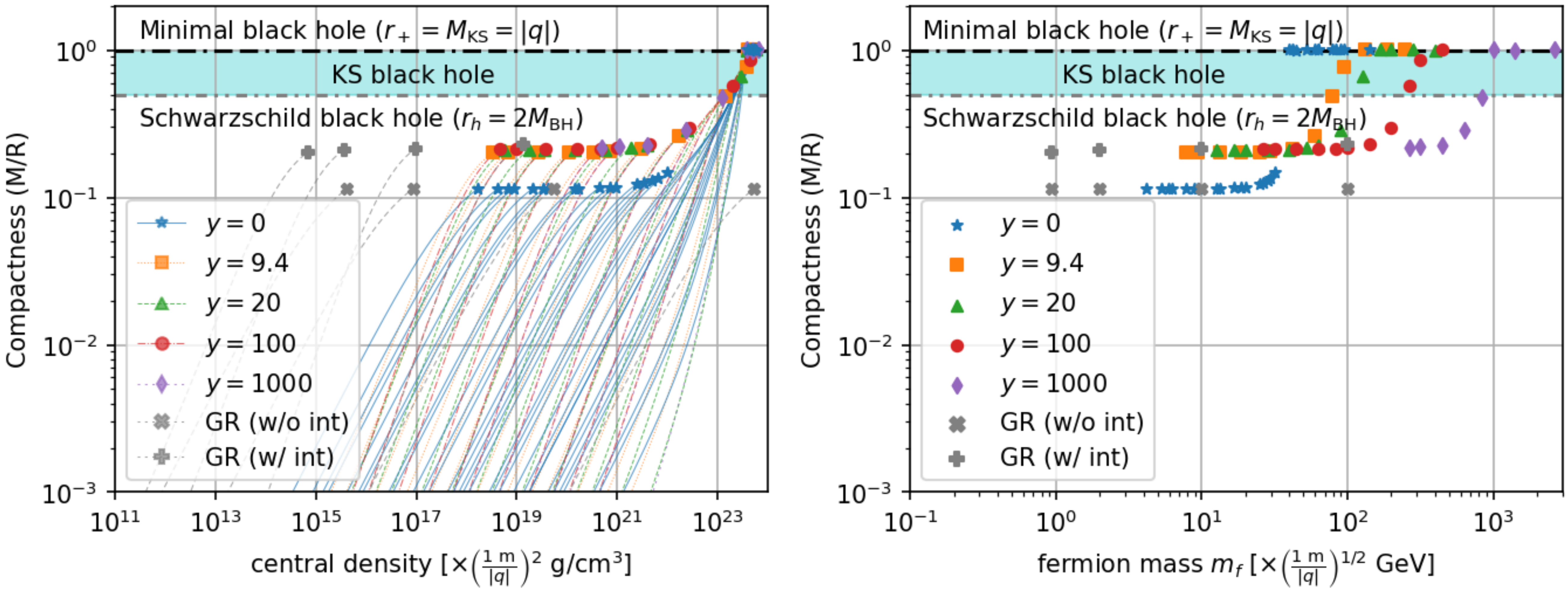
TABLE I. Maximum masses M_{max} and radii R_{min} for various cold compact stars made of a free Fermi gas.

Fermion mass	$M_{max}(M_\odot)$	R_{min}	Comment
100 GeV	10^{-4}	1 m	neutralino star (cold dark matter)
1 GeV	1	10 km	neutron star
1 GeV/0.5 MeV	1	10^3 km	white dwarf
10 keV	10^{10}	10^{11} km	sterile neutrino star
1 keV	10^{12}	10^{13} km	axino star (warm dark matter)
1 eV	10^{18}	10^{19} km	neutrino star
10^{-2} eV	10^{22}	10^{23} km	gravitino star

MAXIMUM MASSES FOR VARIOUS y AND m_f

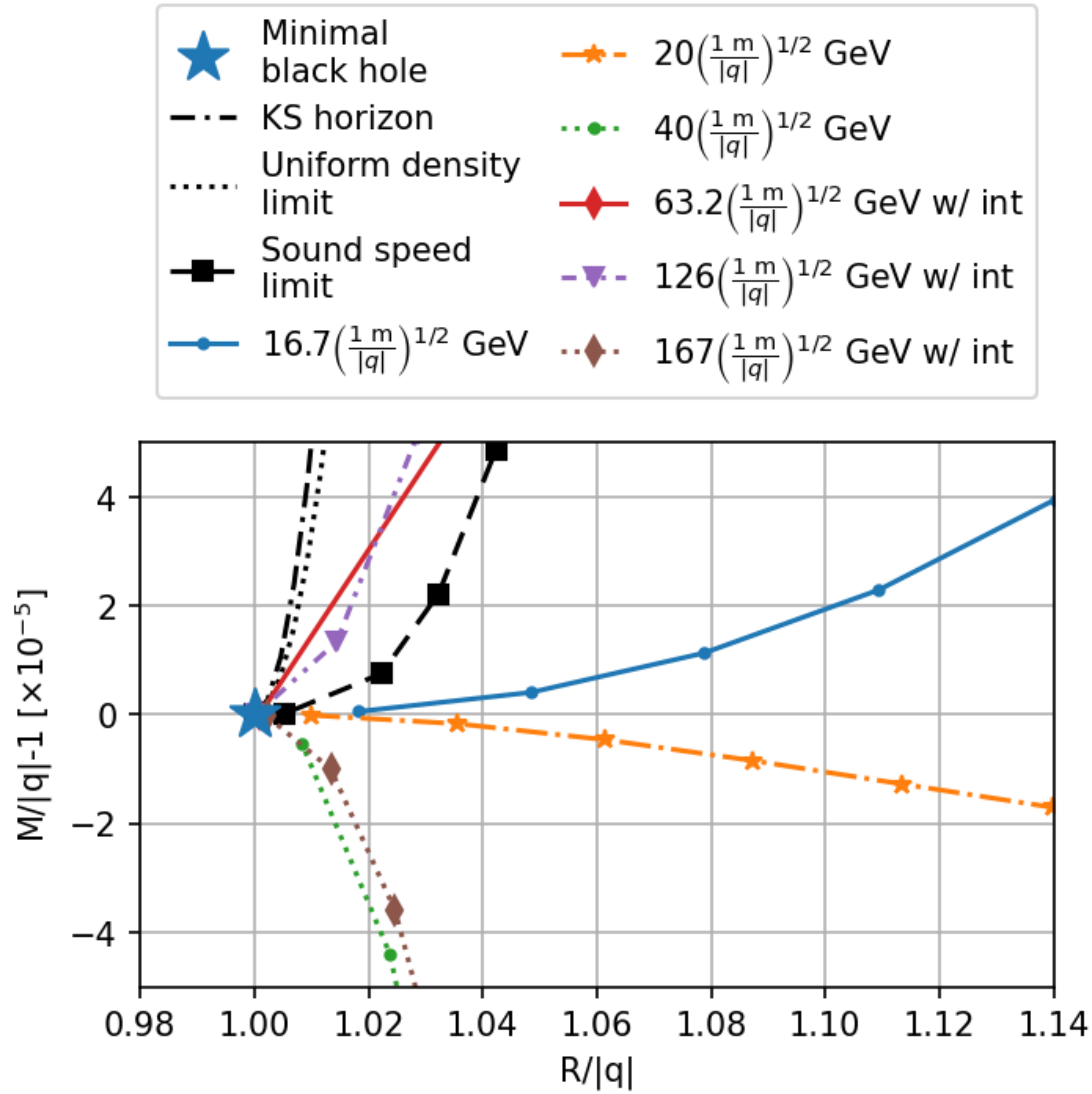


COMPACTNESS OF FERMIONIC COMPACT OBJECTS



FERMIONIC COMPACT OBJECTS NEAR MBH

- Fermionic Compact Objects for several m_f
- Minimal BH (MBH): $R = M = q$



CONCLUDING REMARKS

SUMMARY

- KS solution approximates to Schwarzschild spacetime at the asymptotic region but it is similar to the Reissner-Nordstrom BH near the horizon and the HL parameter q plays the role of the electric charge in RN solution.
- The primordial BH (PBH) is a candidate of dark matters and some PBHs may be the extremal BHs (EBHs), which do not evaporate and may be tiny for small q.
- Near the EBH, there exist equilibrium states of compact objects, which is not seen in GR.
- If the equilibrium states pass stability tests, a new type of compact objects may be predicted.
- The new compact objects are more compact than neutron stars and hardly observable for small q, which suggests that this new equilibrium may be a candidate of dark matters.

BACKUP SLIDES

STABILITY OF COMPACT OBJECTS

[Image from: Shapiro & Teukolsy (1983)]

- A criterion:
  - $\dfrac{dM}{d\rho_c} > 0$ for stable equilibrium
  - $\dfrac{dM}{d\rho_c} < 0$ for unstable equilibrium
- More stability tests exist:
  - For example, the equilibrium between D and E is unstable because $\omega_0^2 < \omega_1^2 < 0$, whereas $0 < \omega_0^2 < \omega_1^2 < \cdots$ between B and C.