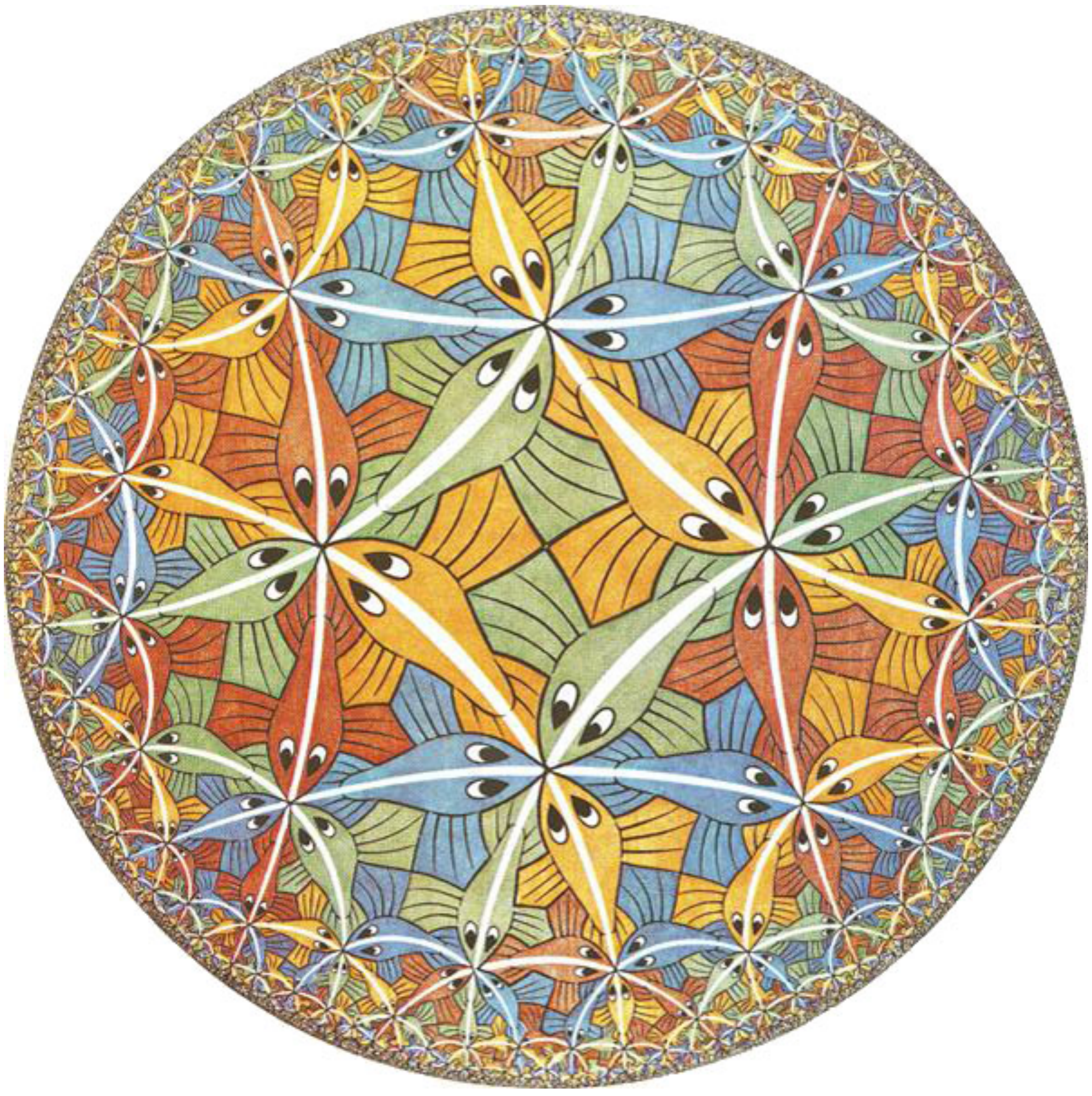
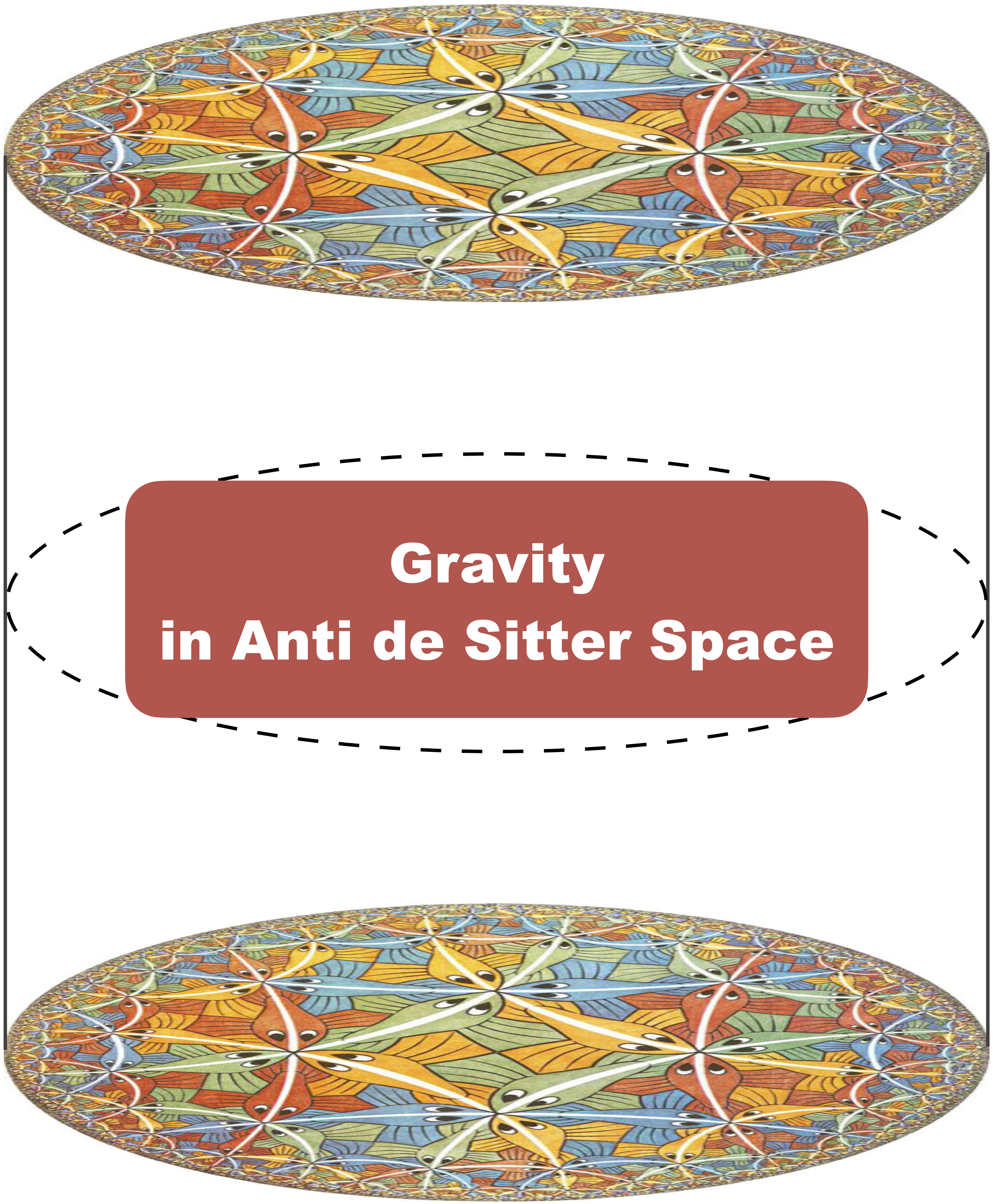
2025년 한국고에너지물리학회 및 핵-입자-천체분과 연합 워크샵

Genuine Multipartite Entanglement as a Diagnostic of Multiboundary Wormholes

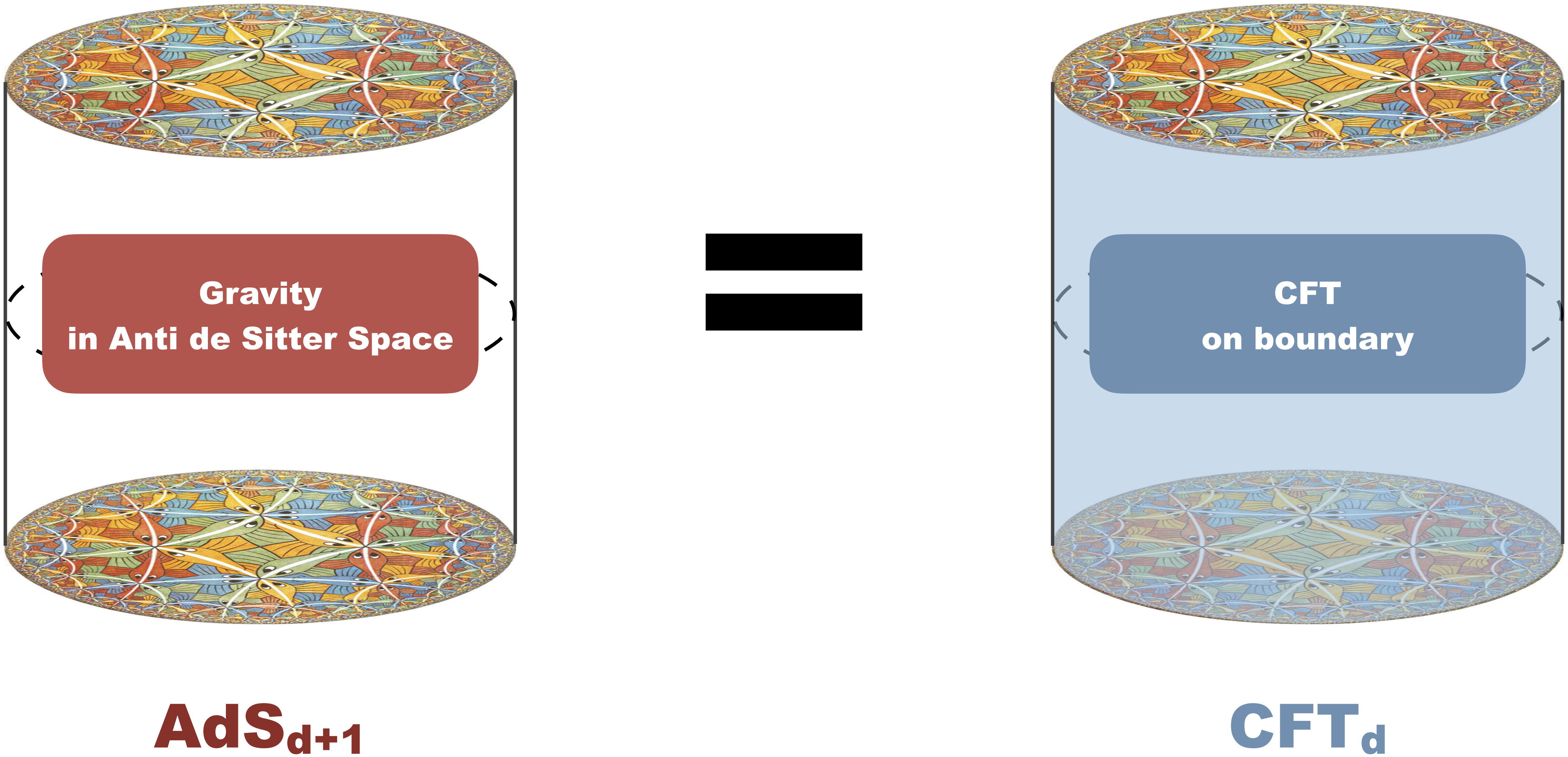
Junggi Yoon
Kyung Hee University

September 26, 2025



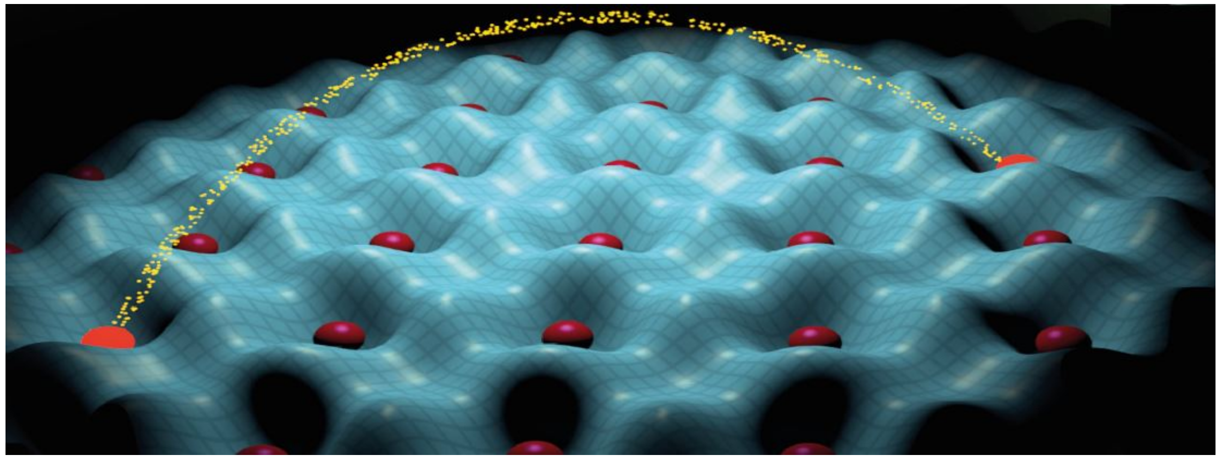

Gravity
in Anti de Sitter Space

Holography or AdS/CFT correspondence



Holography

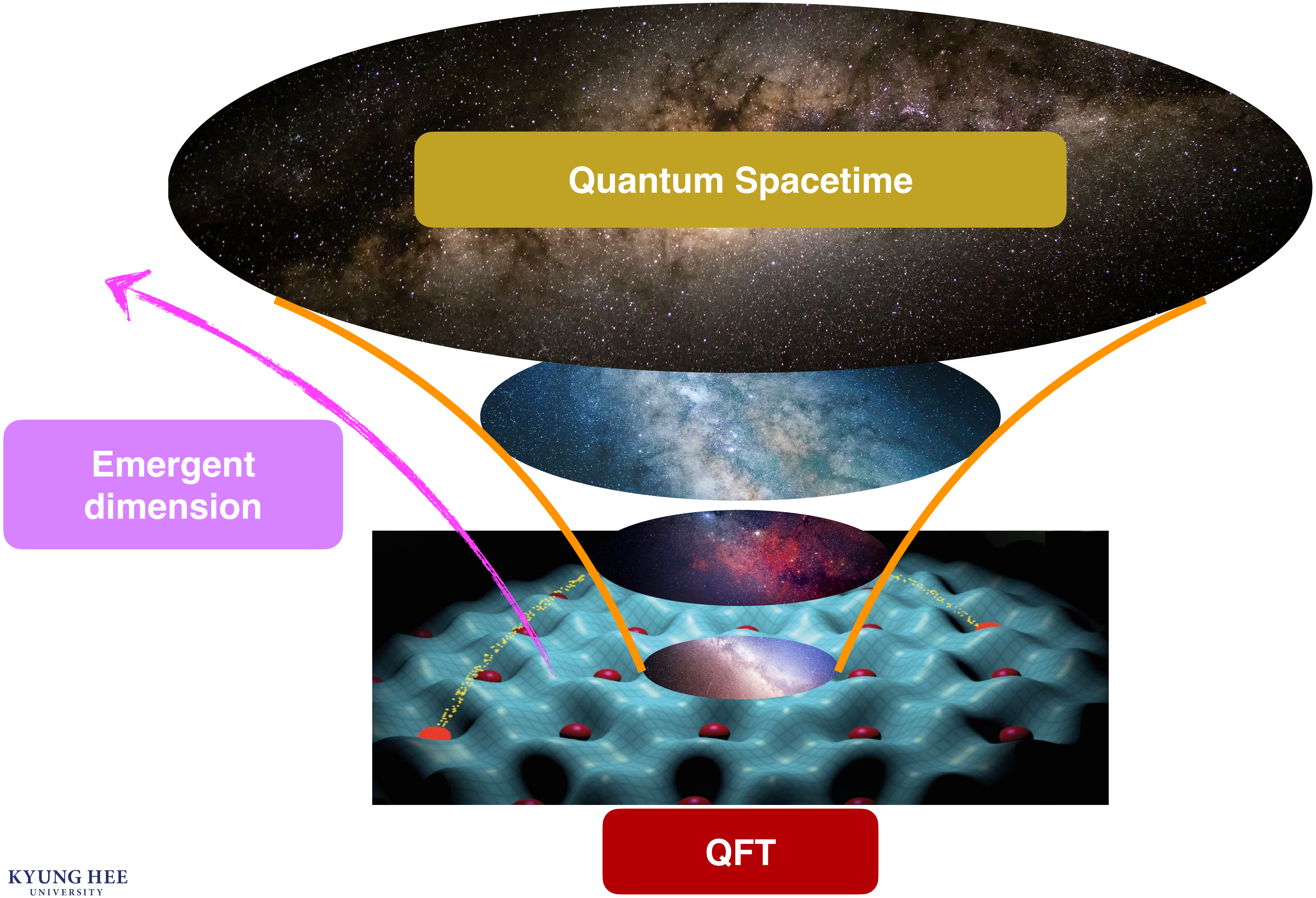
Emergent of quantum spacetime from QFT



QFT

Holography

Emergent of quantum spacetime from QFT



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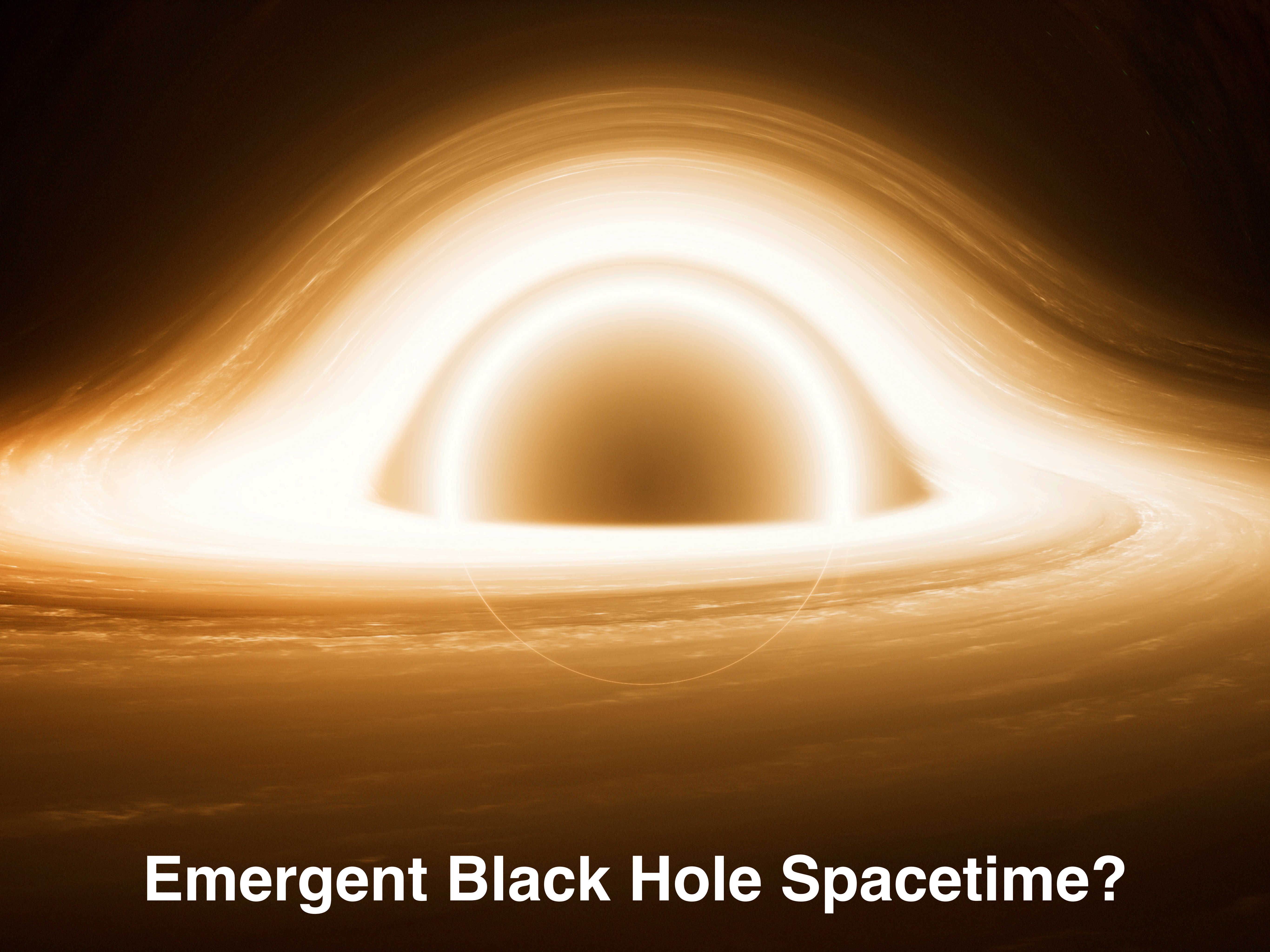
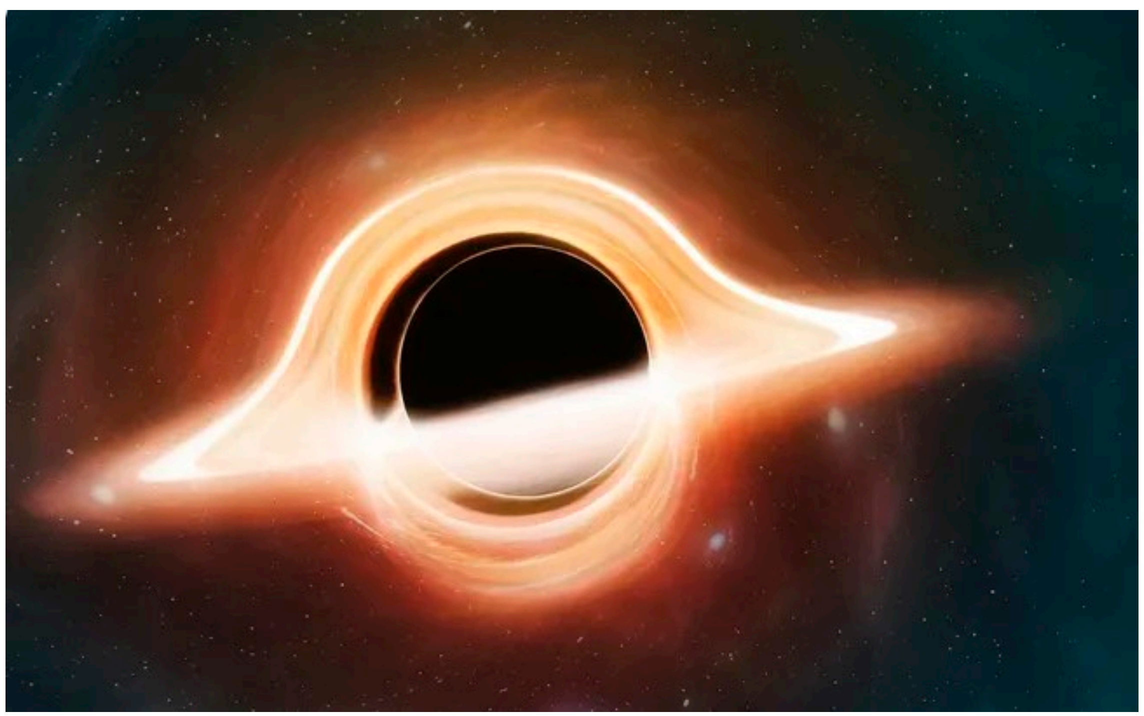


Emergent Black Hole Spacetime?

Holographic Dual of Black Hole

TFD state: Entangled State of Two CFTs

$$|TFD(\beta)\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\frac{\beta}{2}E_n} |E_n\rangle \otimes |E_n\rangle$$



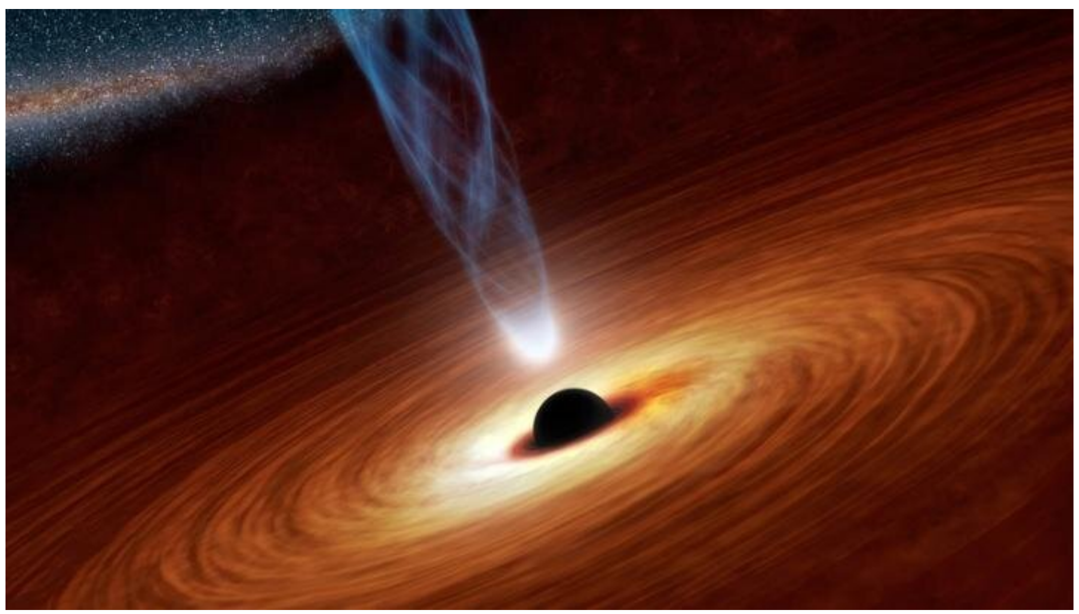
Infinite Temperature

$$|TFD(0)\rangle = \frac{1}{\sqrt{d}} \sum_n |E_n\rangle \otimes |E_n\rangle$$

: Maximally entangled state

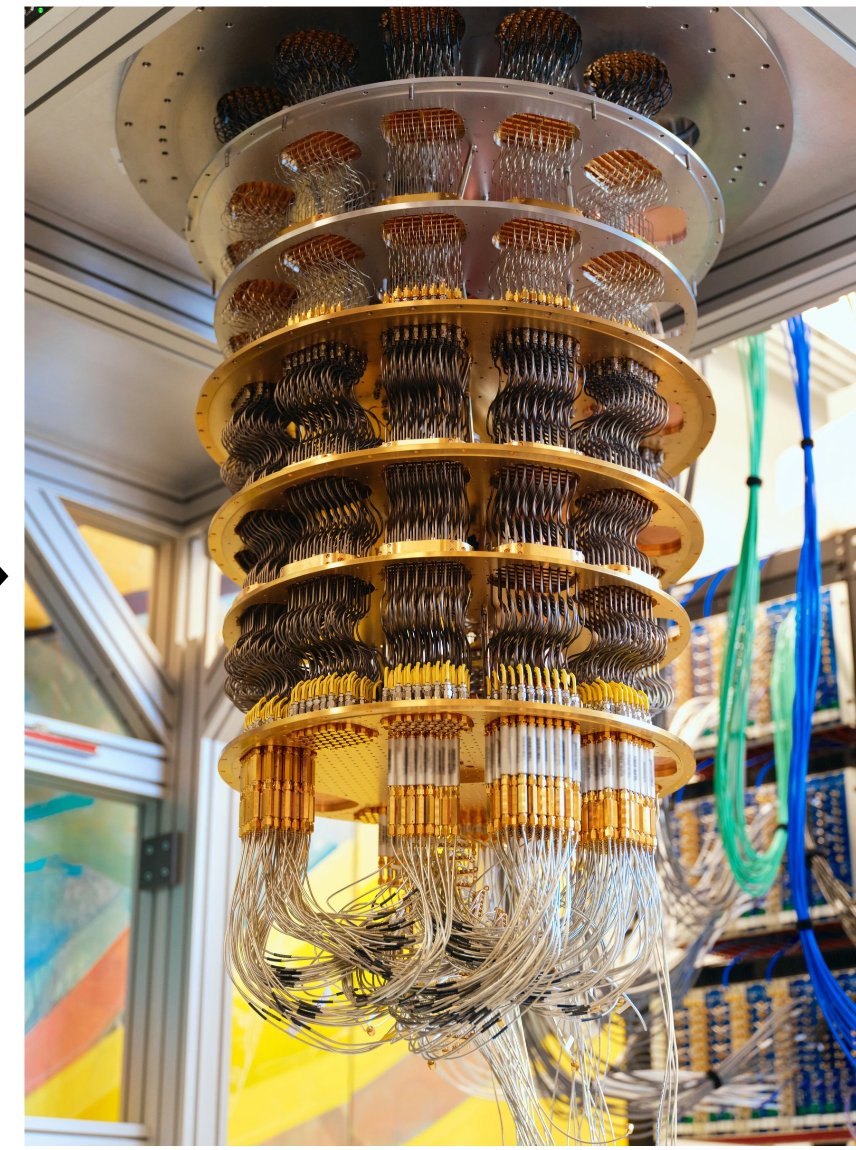
Black Hole in the LAB

홀로그래피를 통한 블랙홀 양자 시뮬레이션



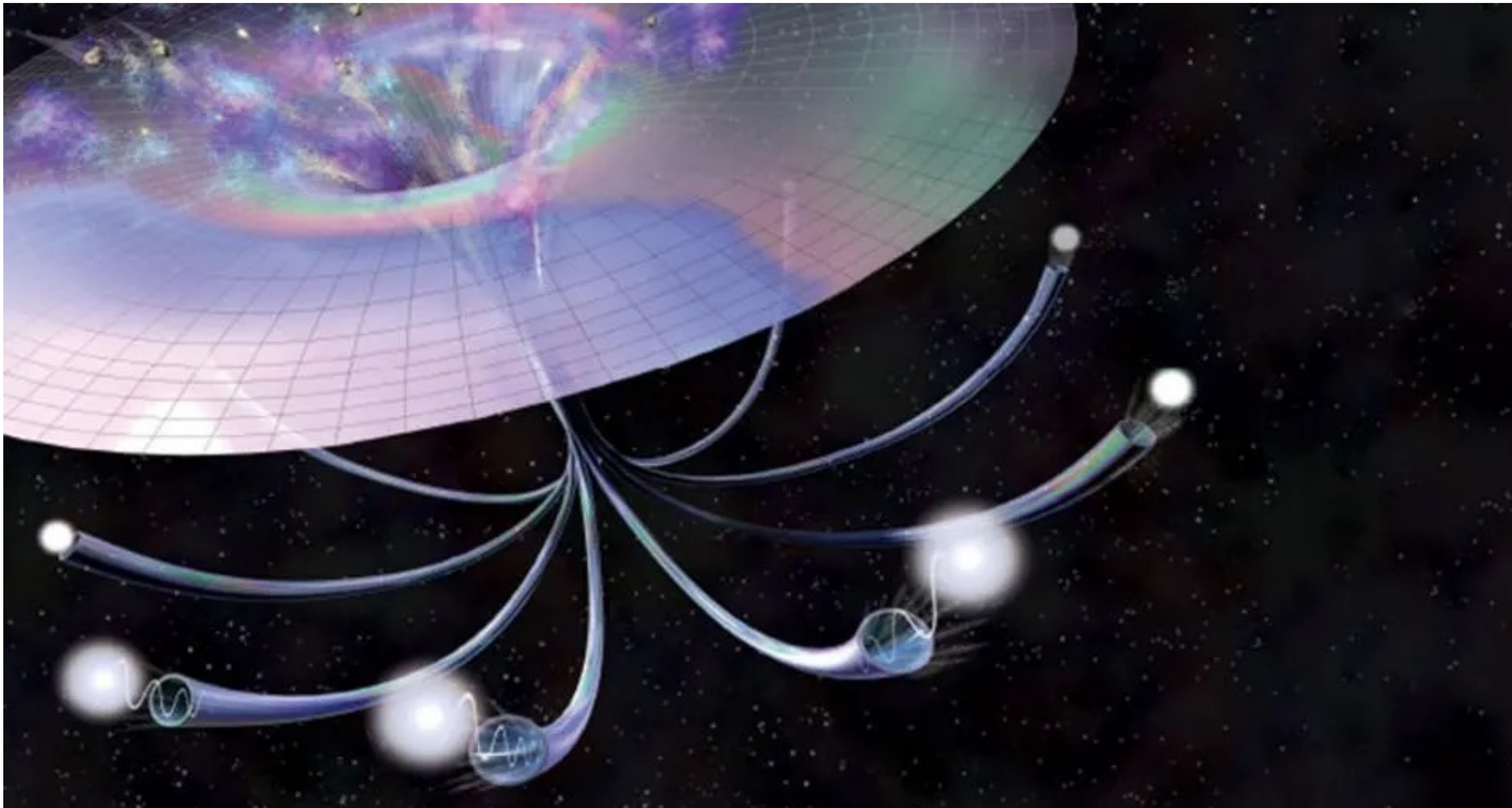
블랙홀

홀로그래피?



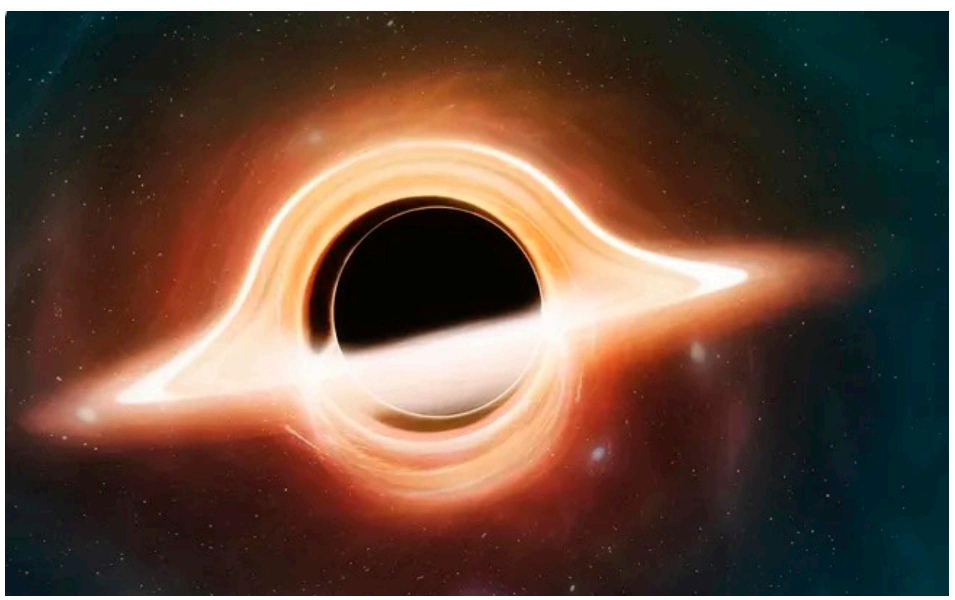
양자 컴퓨터

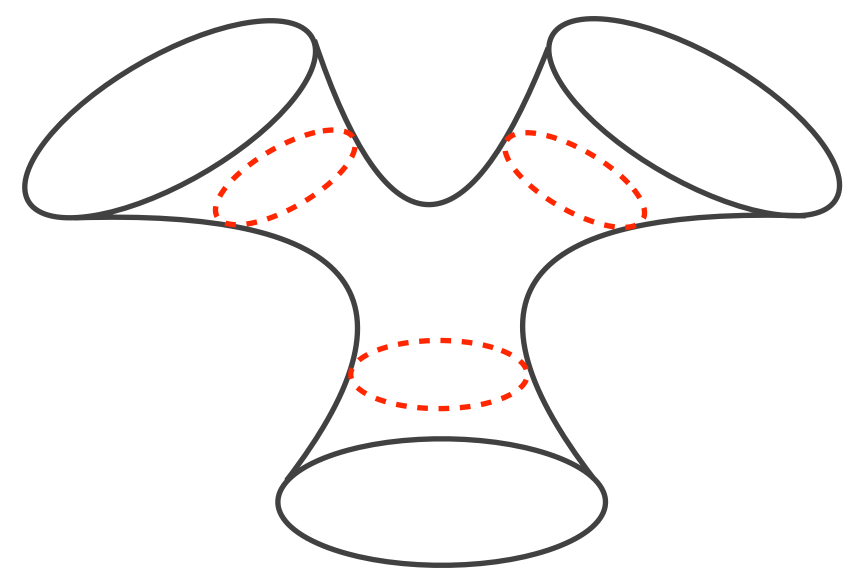
Multiboundary Wormhole



Motivation

What is the quantum state of multiboundary wormhole?





$$|TFD(\beta)\rangle = \frac{1}{\sqrt{Z}}\sum_n e^{-\frac{\beta}{2}E_n}|E_n\rangle \otimes |E_n\rangle$$

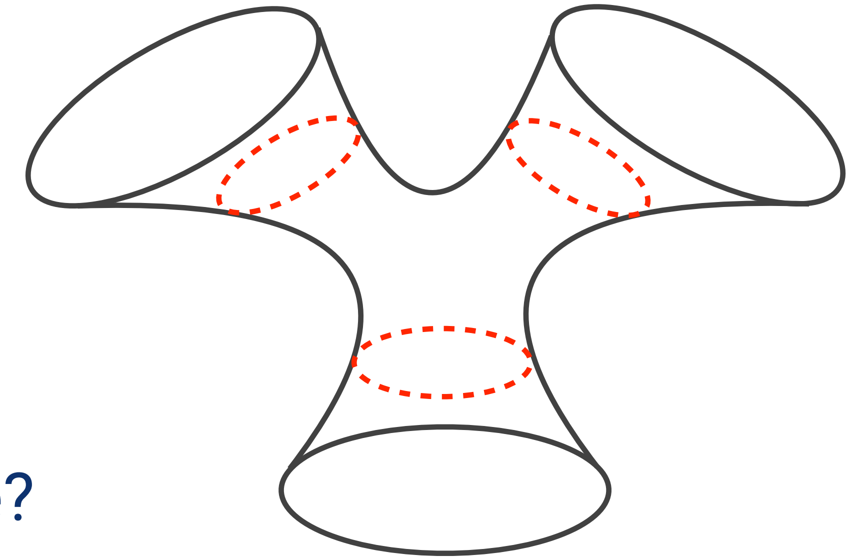
?

Multiboundary Wormhole

What is the quantum state of multiboundary wormhole?

What is generalization of TFD state?

$$|TFD(\beta)\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\frac{\beta}{2}E_n} |E_n\rangle \otimes |E_n\rangle$$



What is generalization of maximally entangled state?

$$|TFD(0)\rangle = \frac{1}{\sqrt{d}} \sum_n |E_n\rangle \otimes |E_n\rangle$$

A New Genuine Multipartite Entanglement Measure: from Qubits to Multiboundary Wormholes

2411.11961

Vinay Malvimat
(Kyung Hee University)

Jaydeep Kumar Basak
(GIST)

Probing the Hierarchy of Genuine Multipartite Entanglement with Generalized Latent Entropy

2510.19922





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(GIST)

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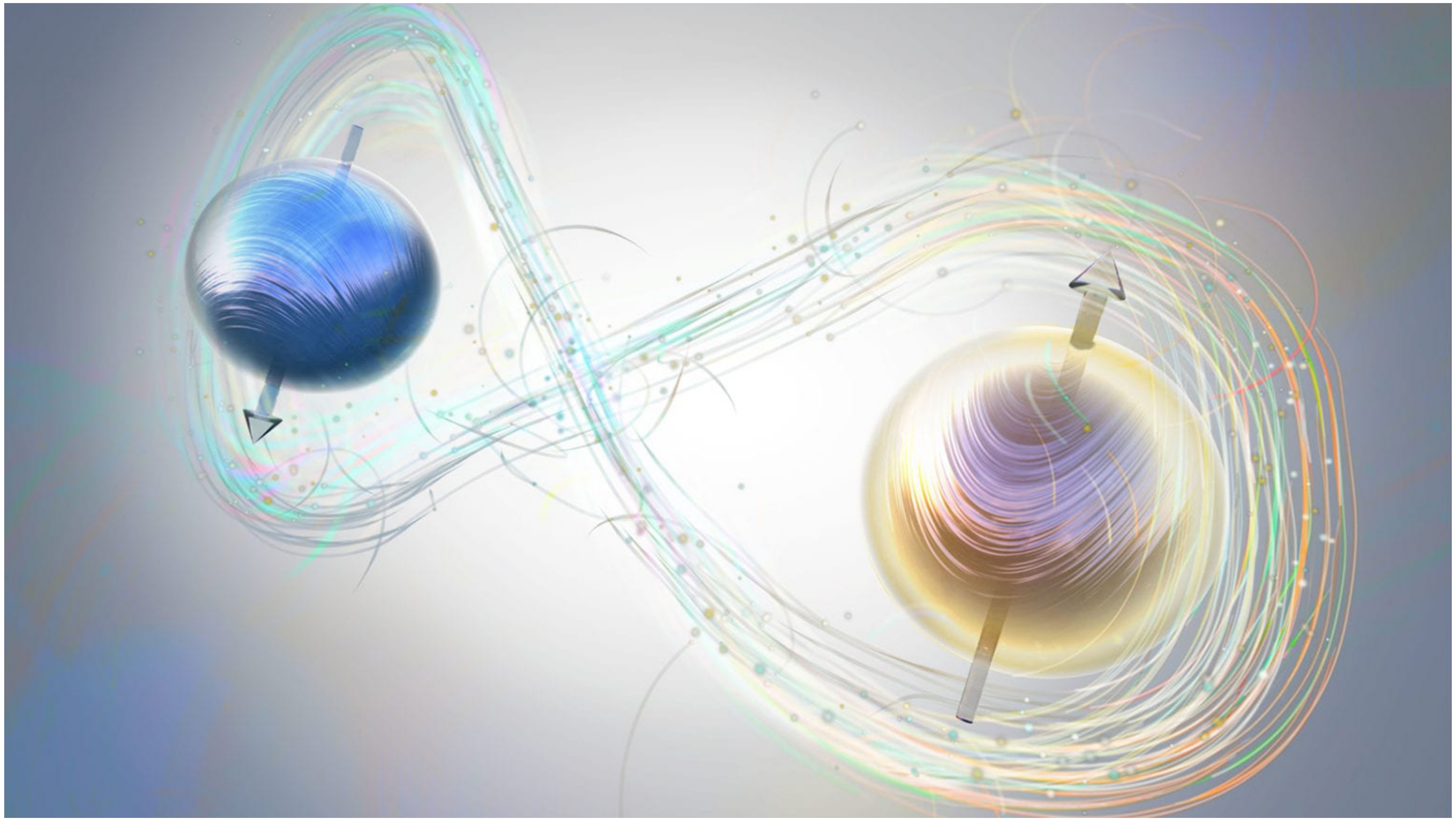
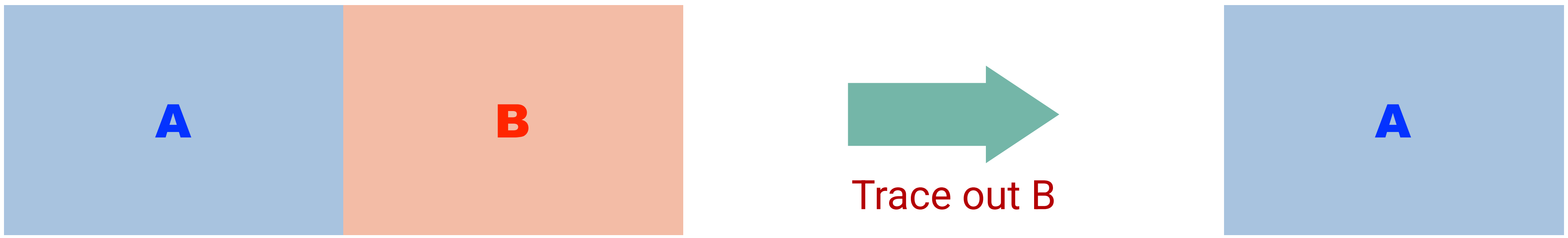
Bell State

The Simplest State $\beta = 0$ TFD State



$$|\mathrm{Bell}\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big)$$



$$|\text{Bell}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$\rho_A = \text{Tr}_B(\rho) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

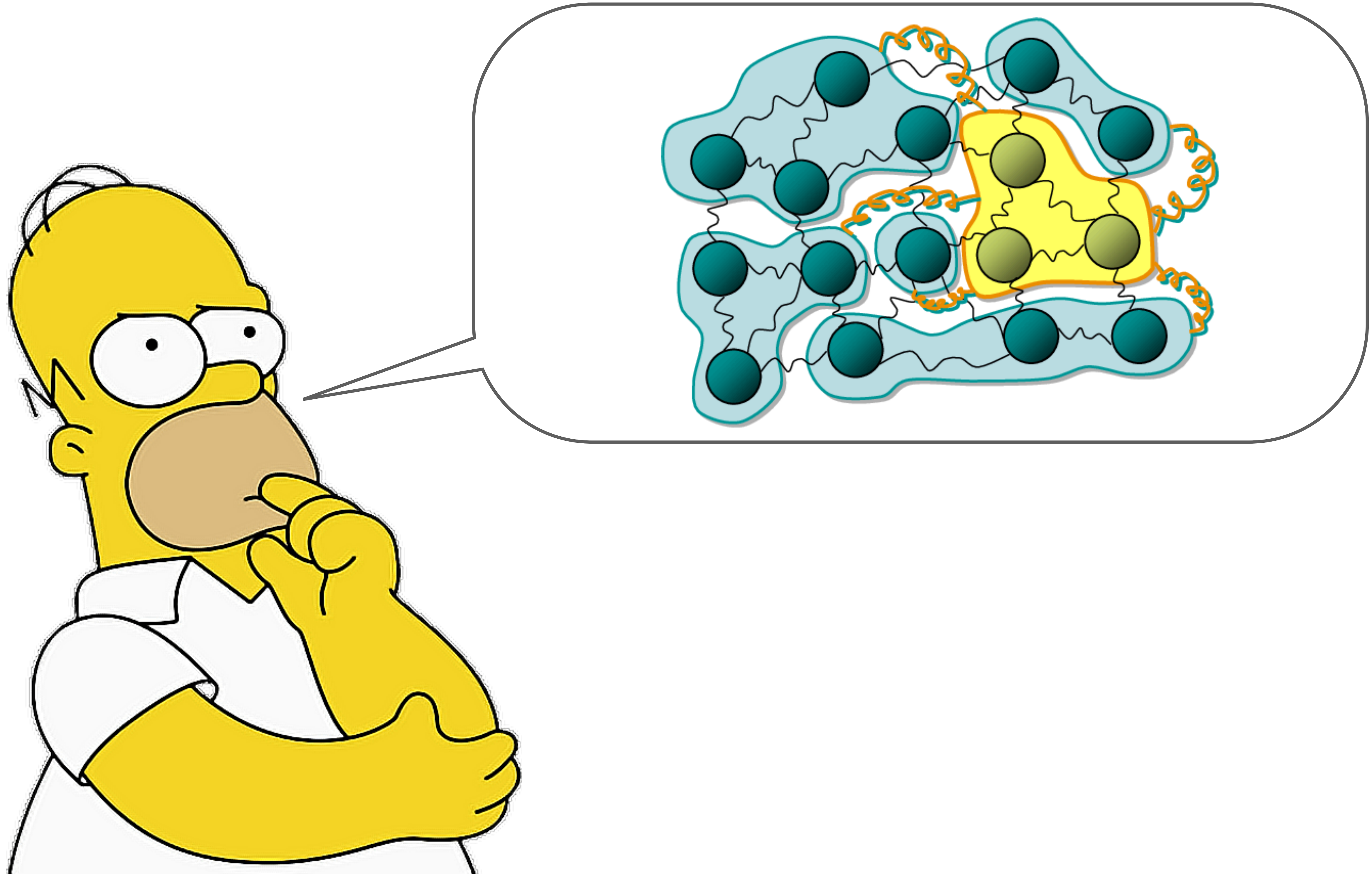
: maximally mixed state

MAXimally mixed state saturates the entanglement entropy.

$$S_A = -\,\text{Tr}(\rho_A \log \rho_A)$$

Good Measure
for bi-partite Entanglement

How about Multi-partite Entangled State?



Which state is more "multi-partite entangled"?

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}\big(|000\rangle + |111\rangle\big) \qquad vs \qquad |\mathrm{W}\rangle = \frac{1}{\sqrt{3}}\big(|100\rangle + |010\rangle + |001\rangle\big)$$

or, are there other states which is more entangled?

Generalization to Multipartite Case

Need New Measure for Multi-partite Entanglement

Maximally Entangled State

$$|\Psi\rangle = \frac{1}{\sqrt{d}} \sum_n |n\rangle \otimes |n\rangle$$

→ **Maximally Multi-Entangled State**

A	B

A_1	A_2	$\cdots$	A_n

Measure for Bi-partite Entanglment

$$S_A = -\,\mathrm{Tr}(\rho_A \log \rho_A)$$

→ **Measure for Multi-partite Entanglment**

We need new MEASURE for

Multi-partite Entanglement!!

We propose new MEASURE for

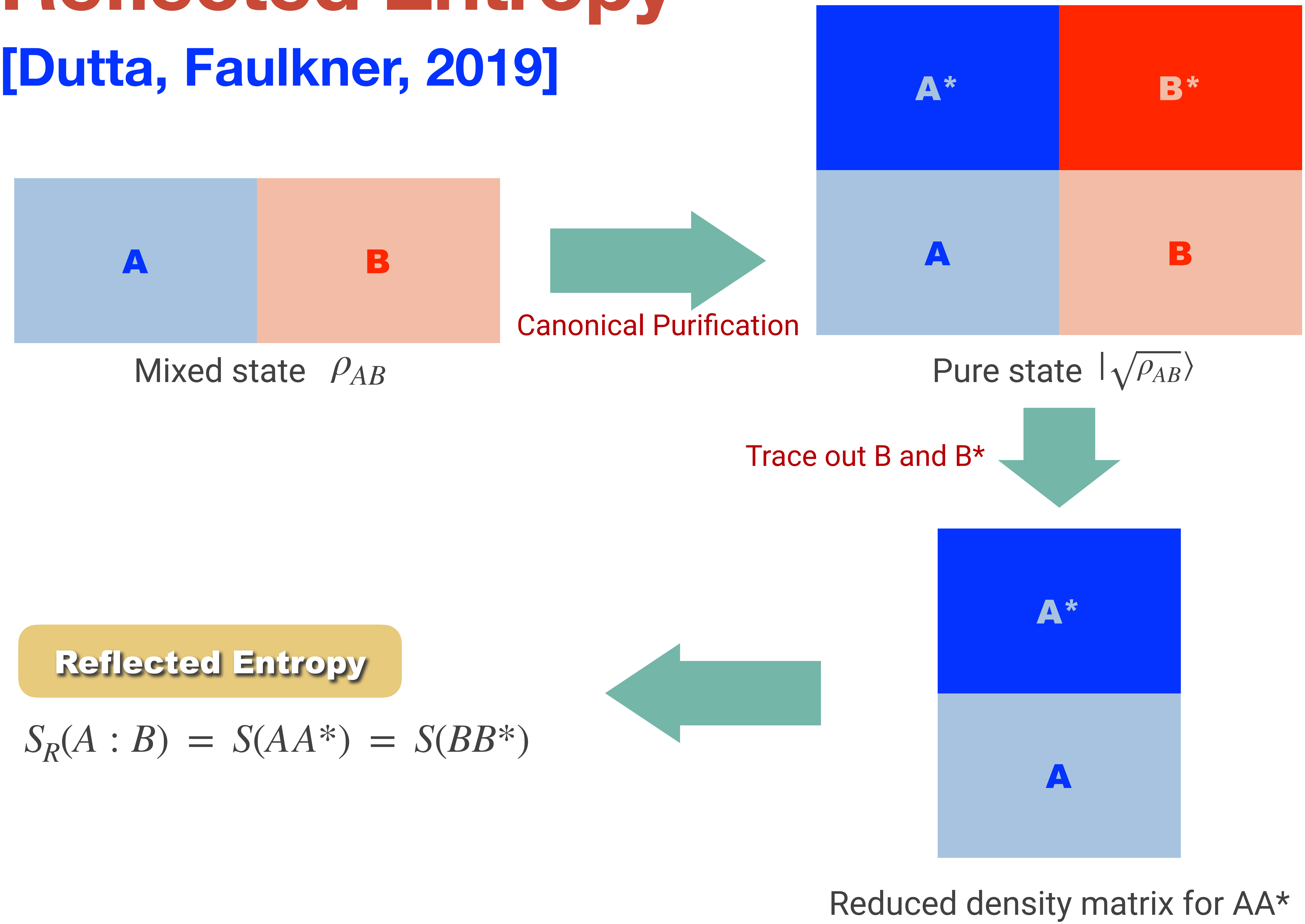
Multi-partite Entanglement.

"L-Entropy" of subsystem A and B

$$\ell_{AB} \equiv 2\min[S(A), S(B)] - S_R(A:B)$$

Reflected Entropy

[Dutta, Faulkner, 2019]



$$S_R(A:B) = S(AA^*) = S(BB^*)$$

Averaged L-Entropy

New Measure for Multi-partite Entanglement

✳ The bound for the reflected entropy

$$2\min[S(A), S(B)] \geqq S_R(A:B) \geqq I(A:B)$$

$$\ell_{AB} \equiv 2\min[S(A), S(B)] - S_R(A:B) \geqq 0$$

✳ For n-partite system, calculate L-entropy of all possible choices of two parties.

n-partite system

Averaged “L-Entropy”

$$\ell_{av} \equiv \prod_{i<j} \left[\ell_{A_i A_j}\right]^{\frac{2}{n(n-1)}}$$

Criteria for Multipartite Entanglement

Genuine Multipartite Entanglement Measure $\mathscr{E}$ (GME)

[Ma, Chen, Chen, Spengler, Gabriel, and Huber, 2011] [Xie, Eberly, 2021]

I. $\mathscr{E} = 0$ for fully-seperable or bi-seperable state

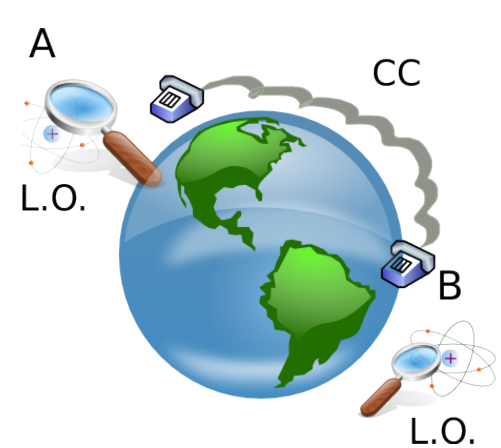
$|000\rangle$ $\quad$ $|\mathrm{Bell}\rangle \otimes |0\rangle$

II. $\mathscr{E} > 0$ for non-biseperable state

III. $\mathscr{E}$: invariant under Local Unitary operation.

IV. $\mathscr{E}$: Non-increasing under LOCC [Entanglement Monotone]

Local Operations and Classical Communication



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III. $\mathscr{E}$: invariant under Local Unitary operation.

IV. $\mathscr{E}$: Non-increasing under LOCC [Entanglement Monotone]

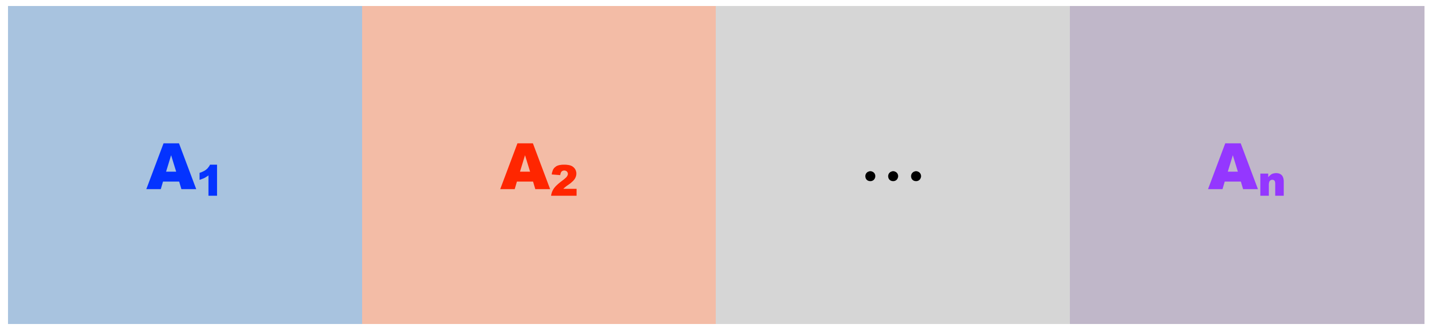
Local Operations and Classical Communication

Our averaged L-entropy satisfies this criteria.

Maximally Multi-entangled State

The Bound of L-entropy

n-partite system



$d_1 = d_2 = \cdots = d_n = d$

✴ In general , the L-entropy is bounded by $2\log[d]$

$$\ell_{av} \leq 2\log[d]$$

✴ Depending on n and d, the bound is not saturated.

For tri-partite system ($n = 3$), the averaged L-entropy is bounded by $\log[d]$

which is the averaged L-entropy of (generalized) GHZ state

$$\ell_{av} \leqq \ell_{GHZ} = \log[d] < 2\log[d] \qquad |\psi\rangle_{GGHZ} = \frac{1}{\sqrt{d}}\sum_{j=1}^{d} |j_A j_B j_C\rangle$$

Which states saturate

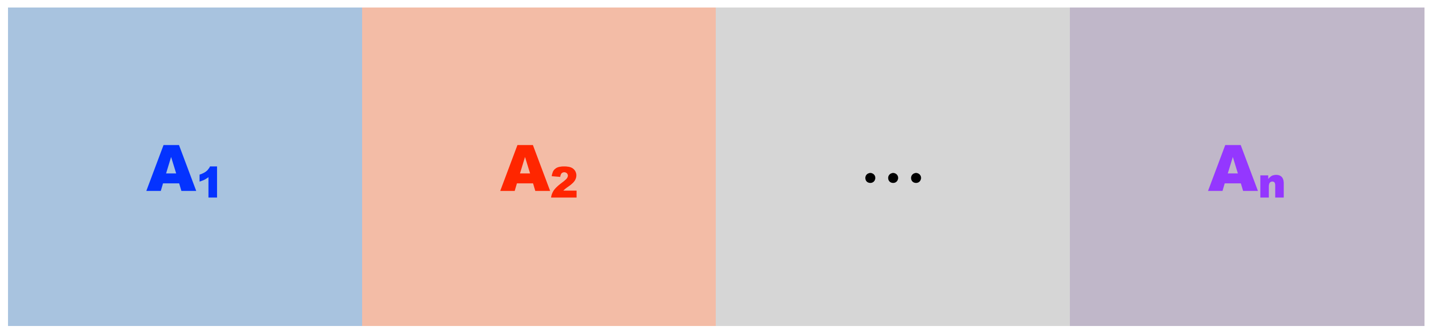
the bound of L-entropy?

$$\ell_{av} \leq 2\log[d]$$

k-Uniform State

Saturates the bound of the L-entropy

n-partite system



✳ ***k*-uniform state**: In n-partite system, the reduced density matrix of any k numbers of subsystems is maximally mixed.

$$\rho_{A_1A_2\cdots A_k} = \frac{1}{d^k}\mathbb{1}_{A_1\cdots A_k} = \frac{1}{d^k}\mathbb{1}_{A_1} \otimes \cdots \otimes \mathbb{1}_{A_k}$$: **Maximally mixed state**

✳ ***k*-uniform state** has maximum L-entropy $2\log[d]$ $(k \geqq 2)$

$$\ell_{av}(k\text{-uniform}) = 2\log[d]$$

✳ In n-partite system, k-uniform state can exists only if $k \leqq \lfloor\frac{n}{2}\rfloor$ [necessary condition]

Ex) There is no k-uniform state $(k \geqq 2)$ in tri-partite system $(n = 3)$

$$\ell_{av} \leqq \ell_{GHZ} = \log[d] < 2\log[d]$$

Example of k-uniform State

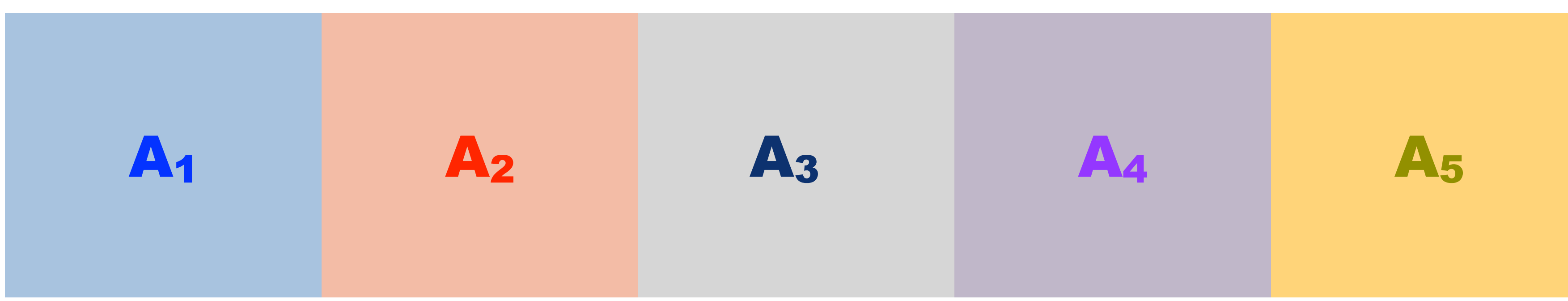
k-uniform state saturate the L-entropy ($k \geqq 2$)

$$|\mathrm{Bell}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

: 1-uniform state

$$|\psi_1\rangle = \frac{1}{\sqrt{8}}\left(|00000\rangle + |01100\rangle + |10001\rangle + |11101\rangle - |00111\rangle - |01011\rangle - |11010\rangle - |11101\rangle\right)$$

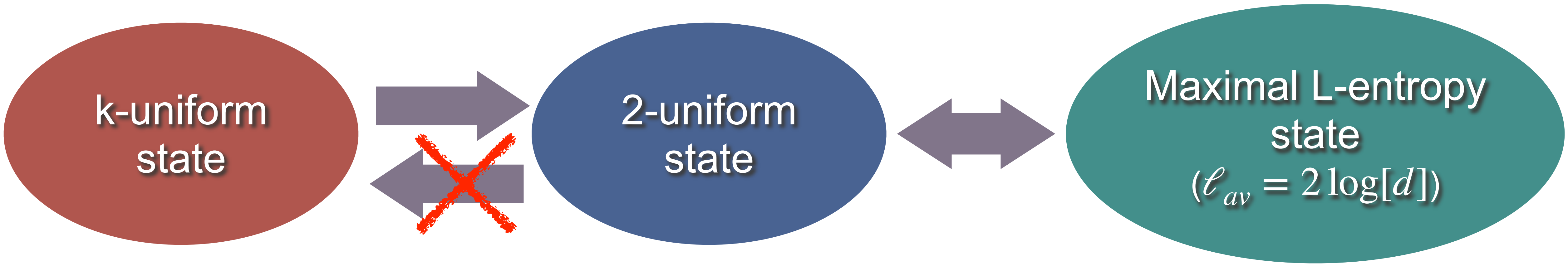
: 2-uniform state



$$\rho_{A_1A_2} = \frac{1}{4}\mathbb{1}_4$$

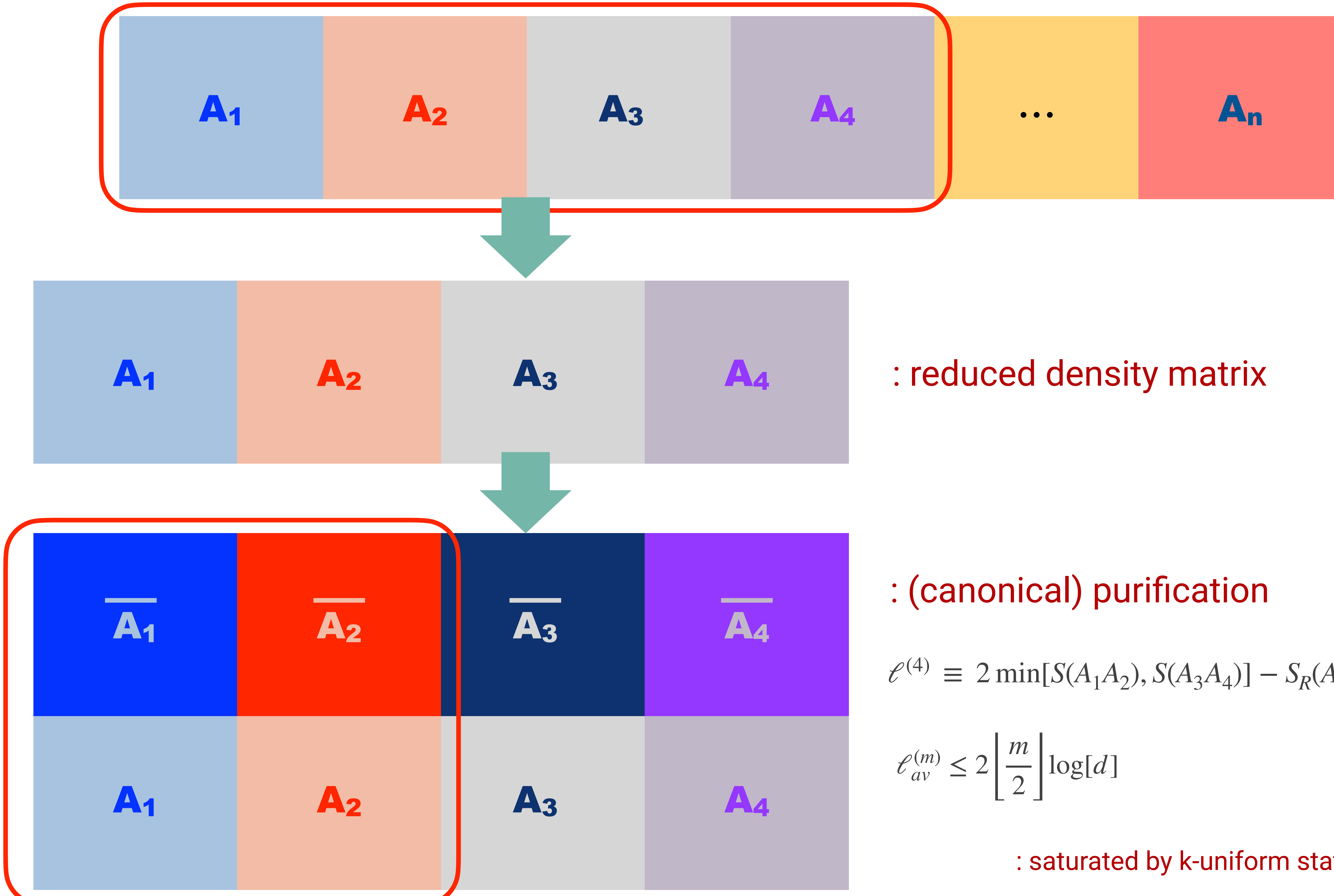
2-Uniform State

L-entropy can capture 2-uniform state



Generalized L-Entropy

How to distinguish k-uniform states?



$$\ell^{(4)} \equiv 2\min[S(A_1A_2), S(A_3A_4)] - S_R(A_1A_2 : A_3A_4)$$

$$\ell_{av}^{(m)} \leq 2\left\lfloor \frac{m}{2} \right\rfloor \log[d]$$

: saturated by k-uniform state ($k \geqq m$)

Maximally Mixed State

$$|\Psi\rangle = \frac{1}{\sqrt{d}} \sum_n |E_n\rangle \otimes |E_n\rangle$$

Introduce **Temperature**

Thermofield Double(TFD) State

(Canonical Purification of Thermal State)

$$|TFD(\beta)\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\frac{\beta}{2}E_n} |E_n\rangle \otimes |E_n\rangle$$

Black Hole in Gravity

What is Finite Temperature version of Multi-entangled State?

Thermal Pure Quantum (TPQ) State

Pure state reproducing Thermal Expectation Value

[Sugiura and Shimizu, 2013]

- ✳ In a given Hilbert space $\mathscr{H}$, we choose a random state $|\psi\rangle$.

  Then, we define the TPQ state $|\Psi_\beta\rangle$ by

$$|\Psi\rangle \equiv e^{-\frac{\beta}{2}H}|\psi\rangle$$

- ✳ The random average of the expectation value with respect to the TPQ state yields the thermal expectation value.

$$\frac{\overline{\langle\Psi_\beta|\mathcal{O}|\Psi_\beta\rangle}}{\overline{\langle\Psi_\beta|\Psi_\beta\rangle}} = \frac{1}{Z(\beta)}\mathrm{Tr}\left(\mathcal{O}\, e^{-\beta H}\right)$$

$$\frac{\overline{\langle \Psi_\beta | \mathcal{O} | \Psi_\beta \rangle}}{\overline{\langle \Psi_\beta | \Psi_\beta \rangle}} = \frac{1}{Z(\beta)} \mathrm{Tr}\left(\mathcal{O}\, e^{-\beta H}\right)$$

This result looks nice.

One might think it is similar to TFD state.

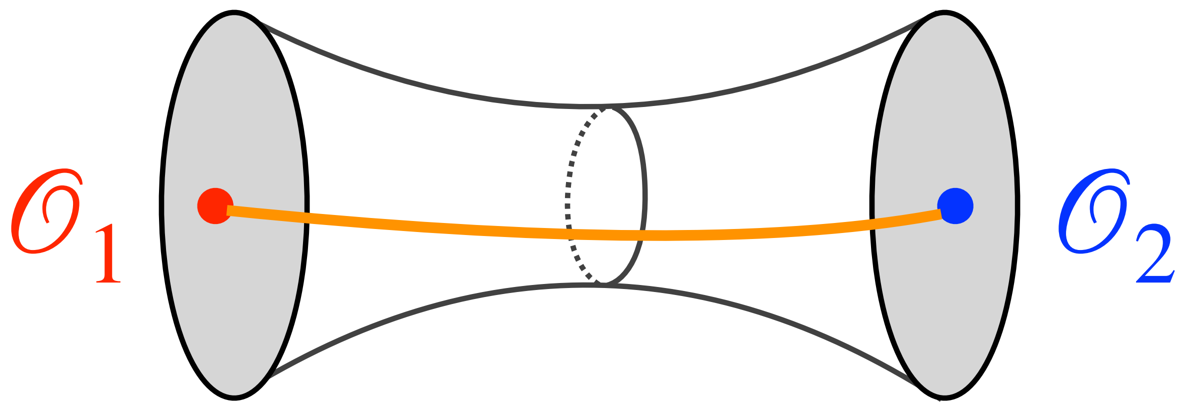
$$\langle TFD(\beta) | \mathcal{O} | TFD(\beta) \rangle = \frac{1}{Z(\beta)} \mathrm{Tr}\left(\mathcal{O}\, e^{-\beta H}\right)$$

But, it is different.

TPQ state $|\Psi_\beta\rangle \in \mathscr{H}$ *VS* $|TFD(\beta)\rangle \in \mathscr{H} \otimes \mathscr{H}$

: Not purification of thermal state

: purification of thermal state



Let's consider the random state

in enlarged Hilbert space?

TPQ-like State in Enlarged Hilbert Space

- For n-partite system, consider a random state in the n copy of Hilbert space:

$$|\psi\rangle \in \underbrace{\mathscr{H} \otimes \cdots \otimes \mathscr{H}}_{n}$$

Define TPQ-like state:

$$|\Psi_\alpha\rangle \equiv \prod_{i=1}^{n} e^{-\frac{\beta}{2}H^{(k)}} |\psi\rangle \in \underbrace{\mathscr{H} \otimes \cdots \otimes \mathscr{H}}_{n}$$

- Then, the random average of the expectation value still reproduce the thermal one!

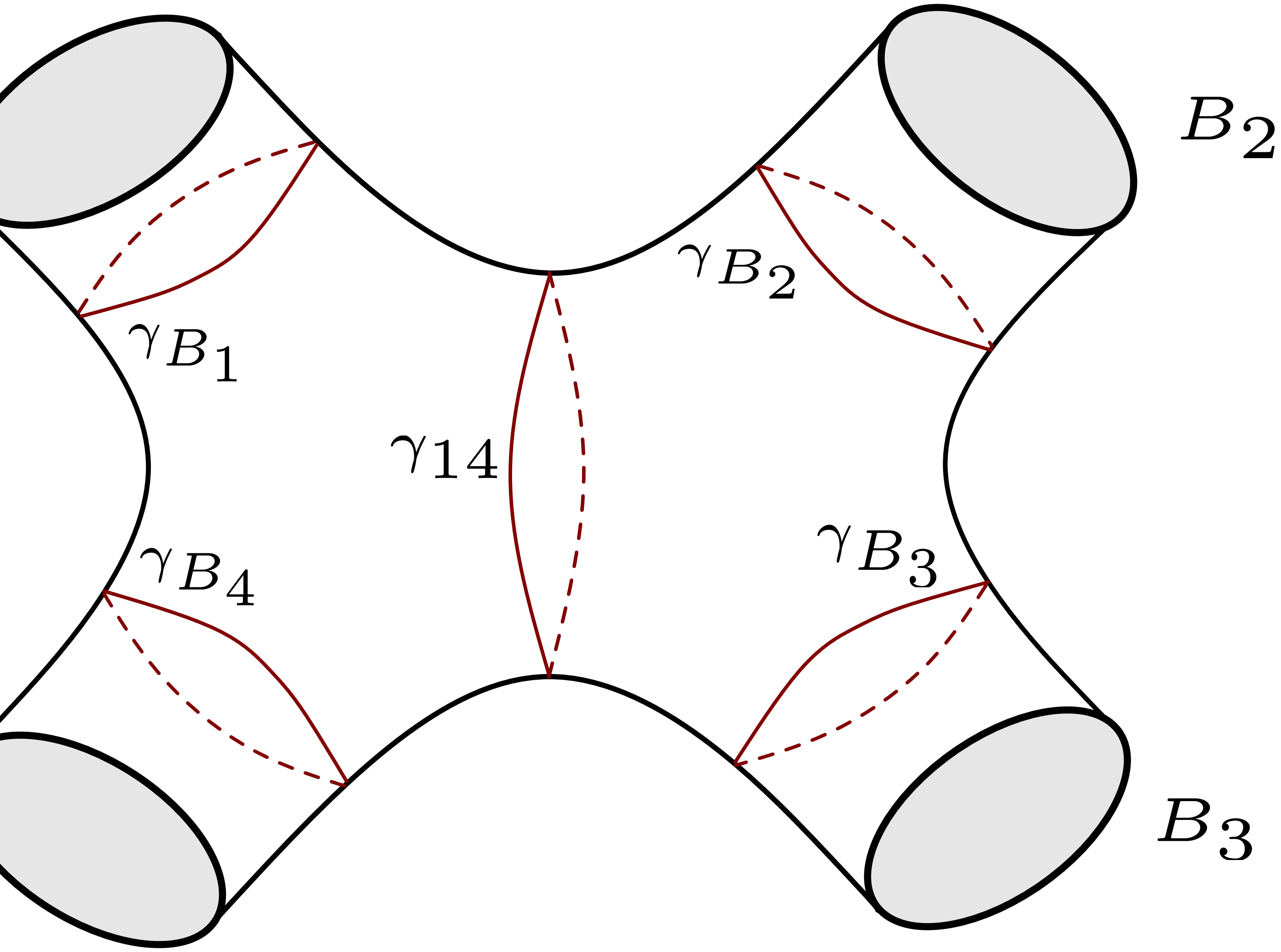
$$\frac{\overline{\langle\Psi_\beta|\mathscr{O}_j|\Psi_\beta\rangle}}{\overline{\langle\Psi_\beta|\Psi_\beta\rangle}} = \frac{1}{Z(\beta)}\mathrm{Tr}\left(\mathscr{O}_j\, e^{-\beta H}\right)$$

when $\mathscr{O}_j$ acts only on j_{th} Hilbert space.

Holographic Dual of TPQ-like State

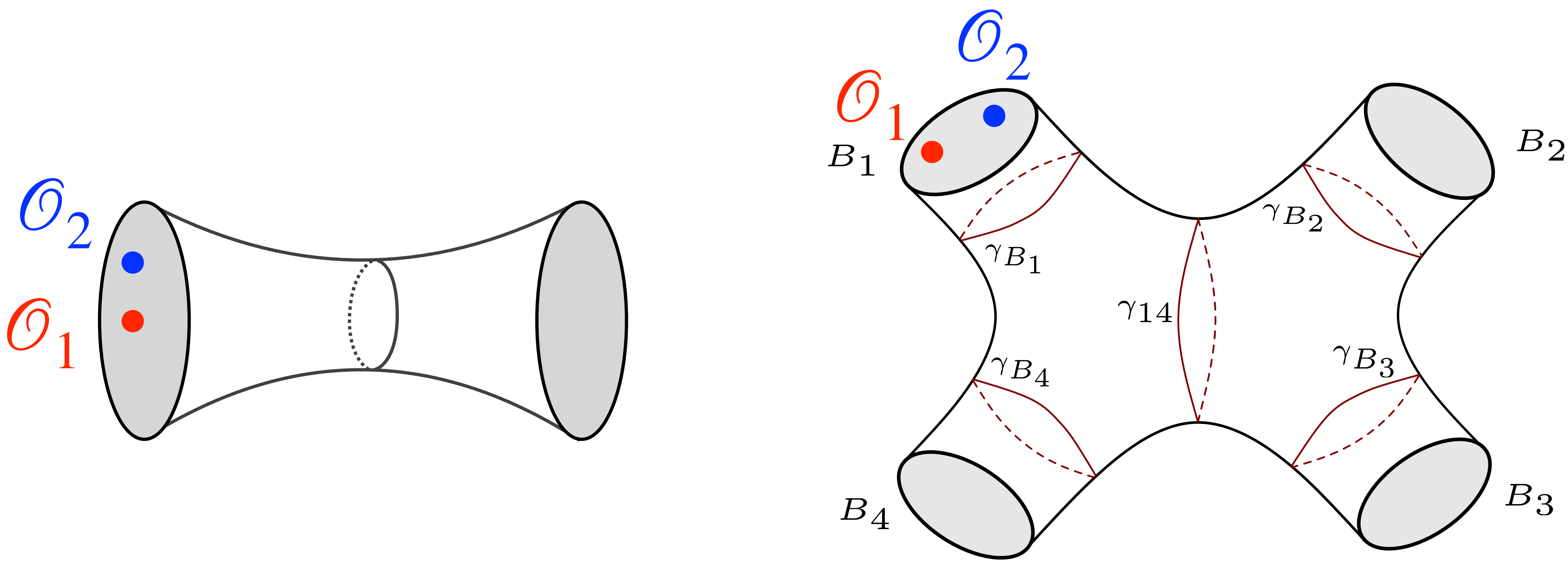
Microstate of Black Hole or Multi-boundary Wormhole?

$$|\Psi_{\alpha}\rangle \equiv \prod_{i=1}^{n} e^{-\frac{\alpha}{2}H^{(k)}}|\psi\rangle$$



Factorization Problem

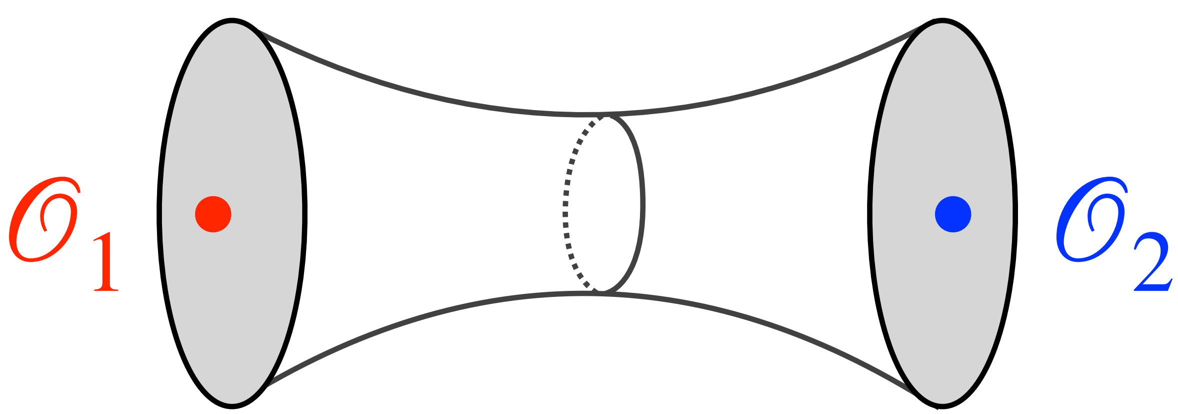
TPQ-like State still looks problematic



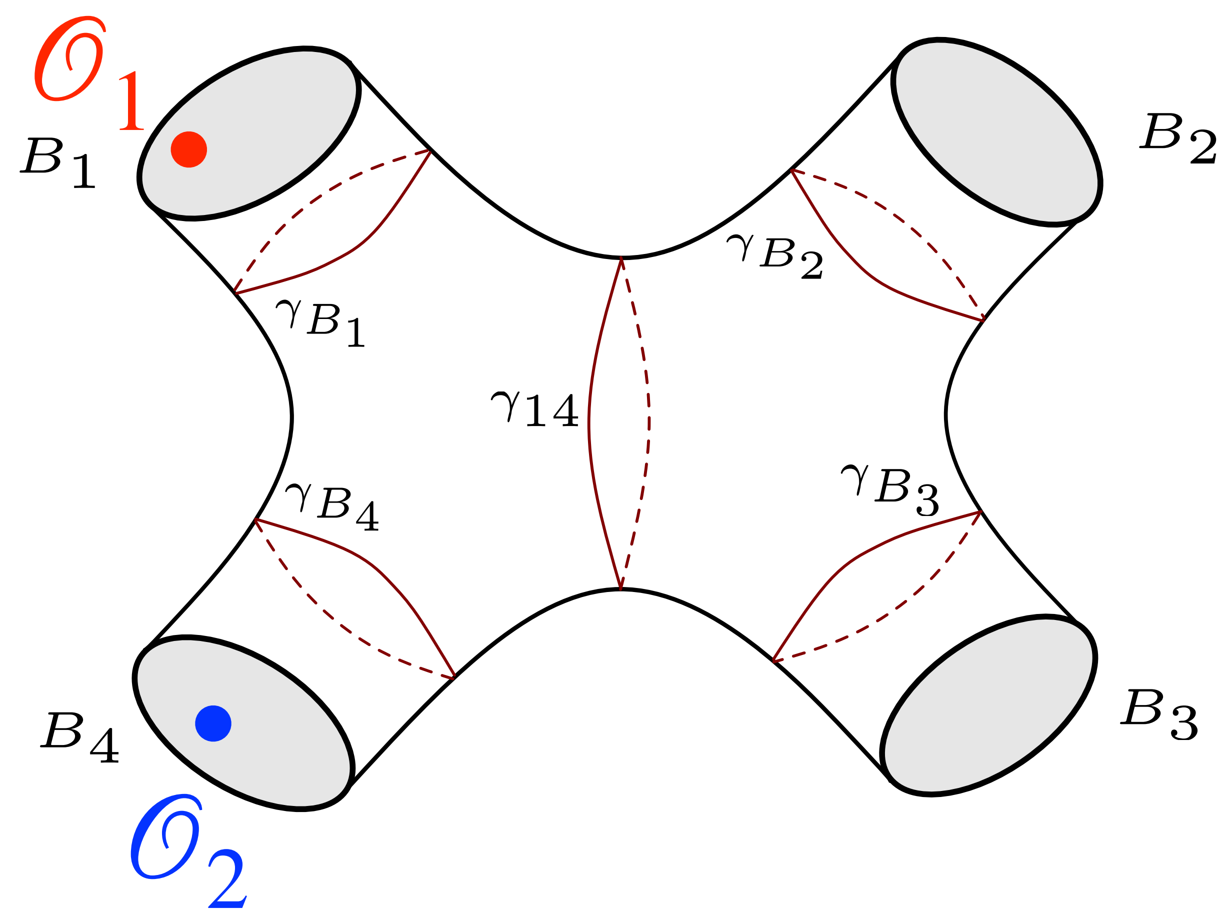
$$\frac{\overline{\langle \Psi_\beta | \mathcal{O}_1 \mathcal{O}_2 | \Psi_\beta \rangle}}{\overline{\langle \Psi_\beta | \Psi_\beta \rangle}} = \frac{1}{Z(\beta)} \mathrm{Tr}\left(\mathcal{O}_1 \mathcal{O}_2 \, e^{-\beta H} \right)$$

Factorization Problem

TPQ-like State still looks problematic



This should not be factorized.

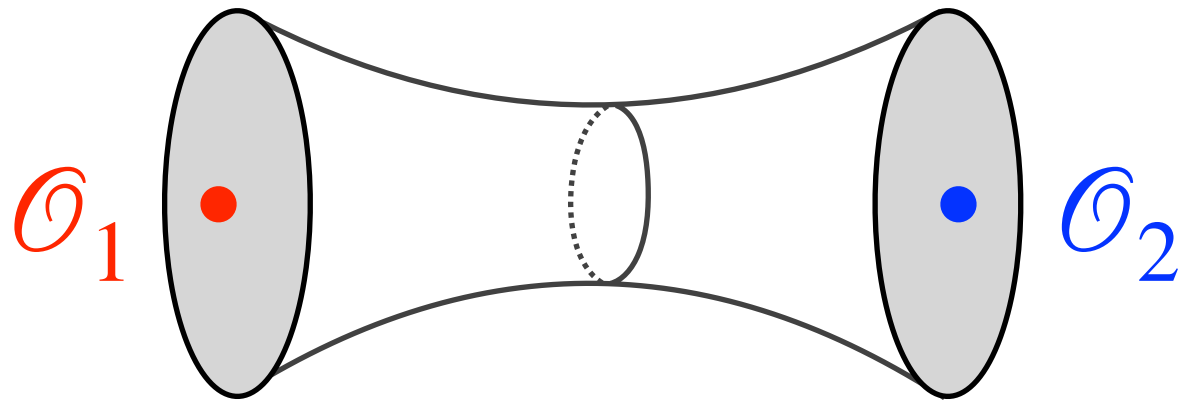


$$\frac{\overline{\langle \Psi_\beta | \mathcal{O}_1 \mathcal{O}_2 | \Psi_\beta \rangle}}{\overline{\langle \Psi_\beta | \Psi_\beta \rangle}} = \frac{1}{[Z(\beta)]^2} \mathrm{Tr}\left(\mathcal{O}_1 \, e^{-\beta H}\right) \mathrm{Tr}\left(\mathcal{O}_2 \, e^{-\beta H}\right)$$

: Factorized!!

Is it Contradiction?

Or, is TPQ-like state not holographic dual to black hole?

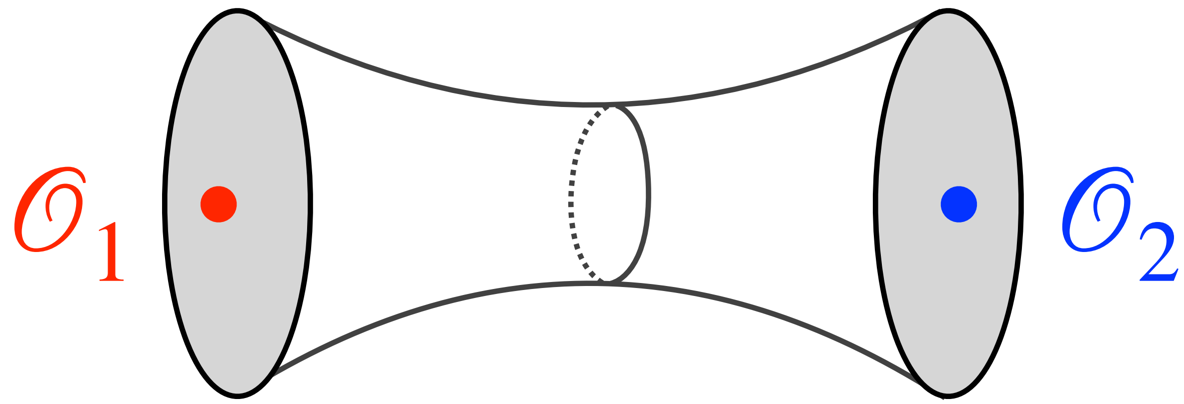


vs

$$\frac{\overline{\langle \Psi_\beta | \mathcal{O}_1 \mathcal{O}_2 | \Psi_\beta \rangle}}{\overline{\langle \Psi_\beta | \Psi_\beta \rangle}} = \frac{\mathrm{Tr}\left(\mathcal{O}_1\, e^{-\beta H}\right)\mathrm{Tr}\left(\mathcal{O}_2\, e^{-\beta H}\right)}{[Z(\beta)]^2}$$

This should not be factorized.

Factorized!!



VS

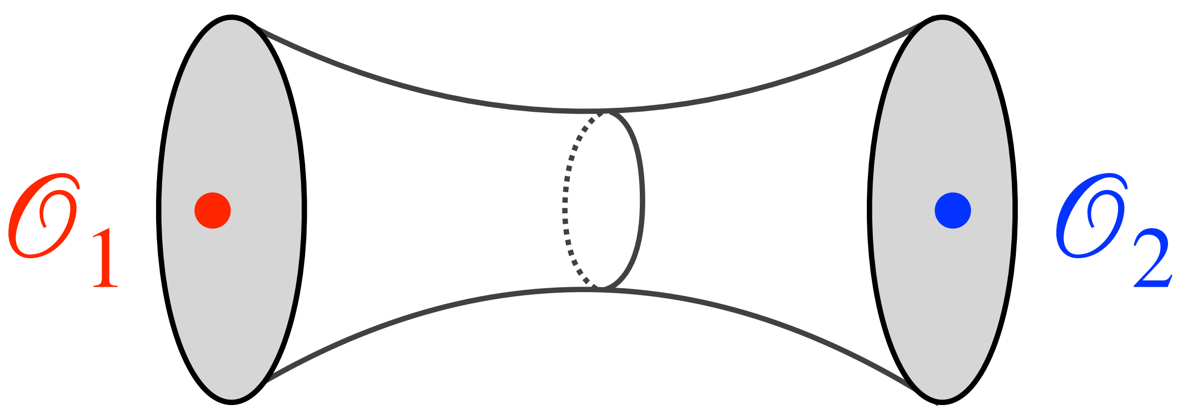
Assumption:
Operator is state-independent

$$\frac{\overline{\langle \Psi_\beta | \mathcal{O}_1 \mathcal{O}_2 | \Psi_\beta \rangle}}{\overline{\langle \Psi_\beta | \Psi_\beta \rangle}} = \frac{\mathrm{Tr}\left(\mathcal{O}_1\, e^{-\beta H}\right)\mathrm{Tr}\left(\mathcal{O}_2\, e^{-\beta H}\right)}{[Z(\beta)]^2}$$

This should not be factorized.

Factorized!!

Operator is state-dependent
in the black hole



This should not be factorized.

Operator is state-dependent
in the black hole

vs

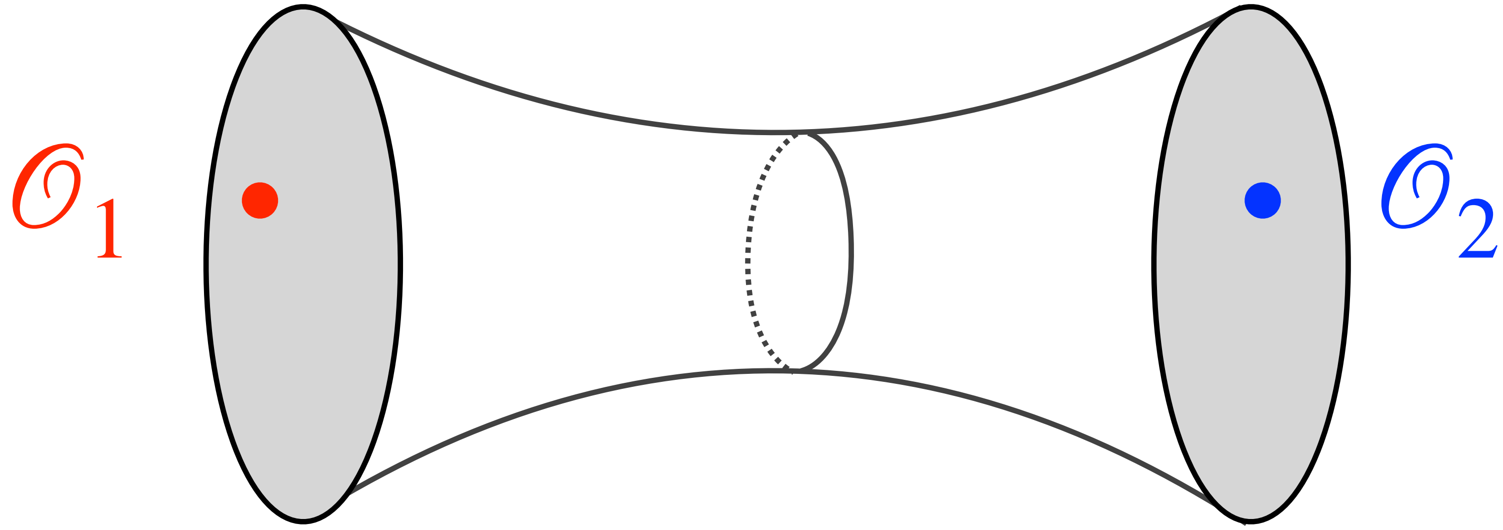
~~**Assumption**:~~
~~Operator is state-independent~~

$$\frac{\overline{\langle \Psi_\beta | \mathcal{O}_1 \mathcal{O}_2 | \Psi_\beta \rangle}}{\overline{\langle \Psi_\beta | \Psi_\beta \rangle}} = \frac{\mathrm{Tr}\left(\mathcal{O}_1\, e^{-\beta H}\right)\mathrm{Tr}\left(\mathcal{O}_2\, e^{-\beta H}\right)}{[Z(\beta)]^2}$$

~~**Factorized!!**~~

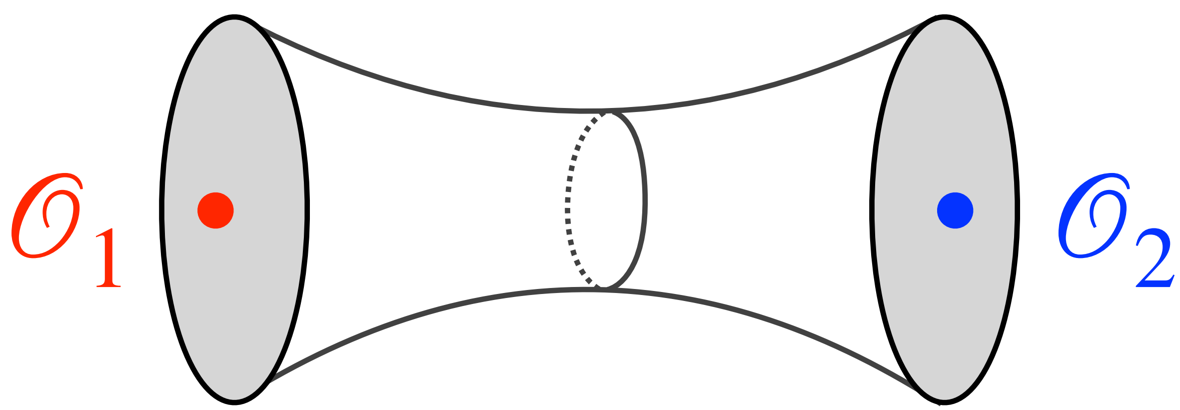
Question:

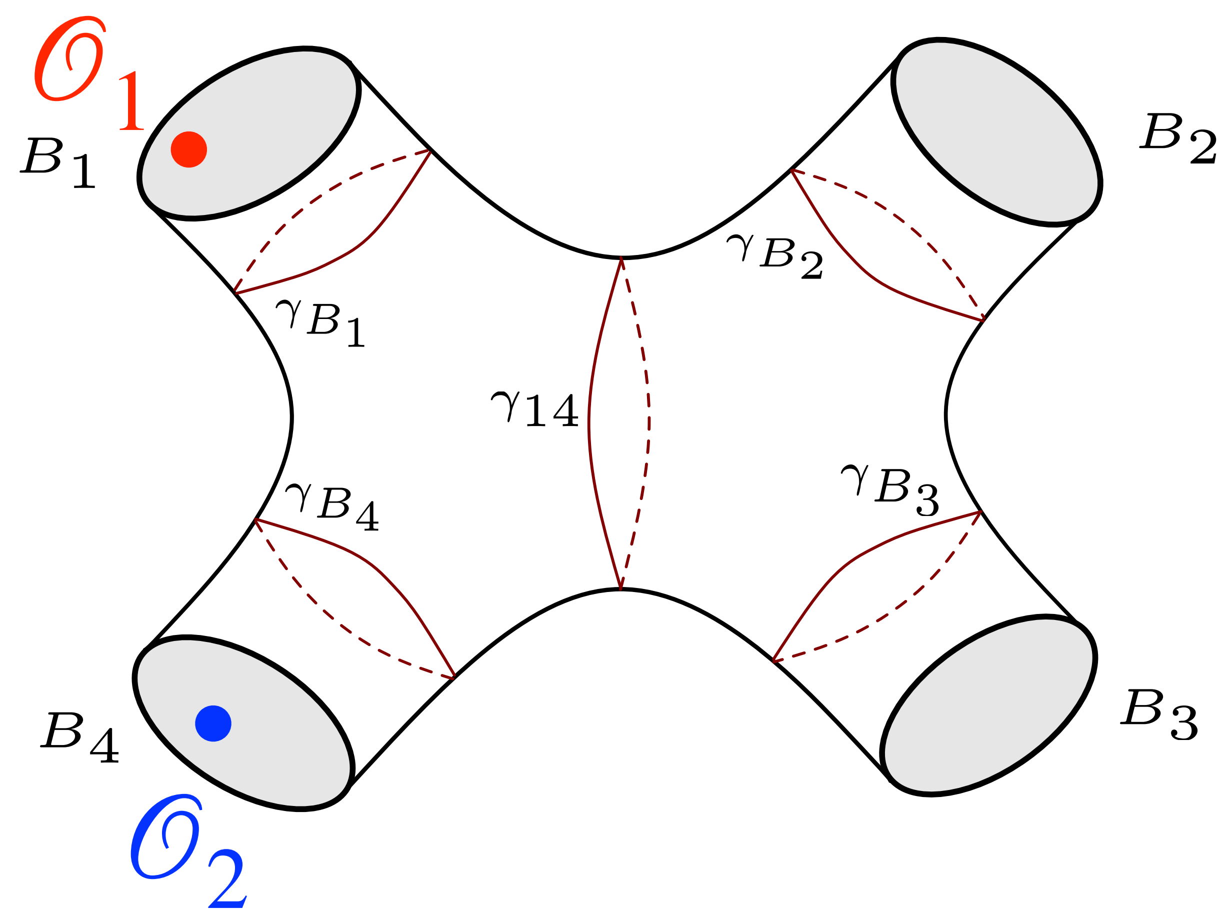
How can we define the state-dependent operator?



Multi-partite Thermal Pure Quantum State

Incorporate the state-dependent



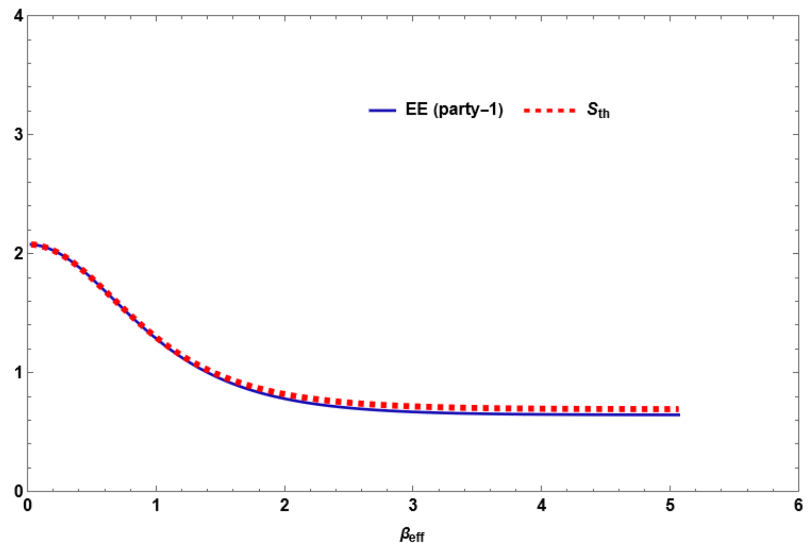


$$|\Psi_\alpha\rangle \equiv \prod_{i=1}^{n} e^{-\frac{\alpha}{2}H^{(k)}}|\psi\rangle$$

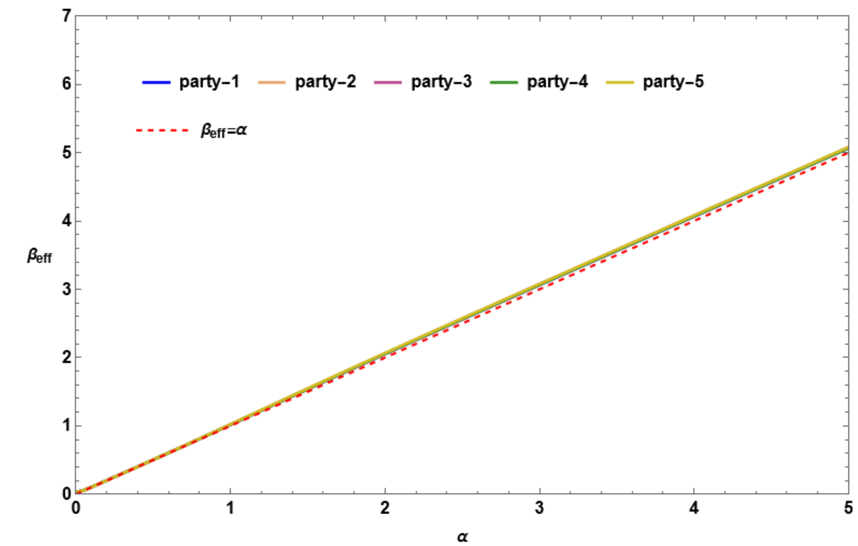
For details, see our papers, [2411.11961], [2510.19922]

Example of MTPQ State

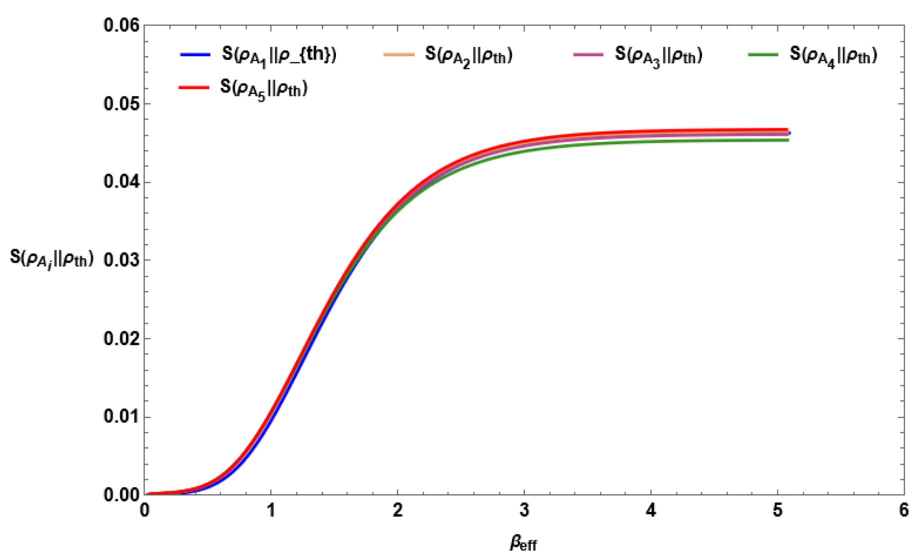
5-party 3 qubit SYK Model (N=6)



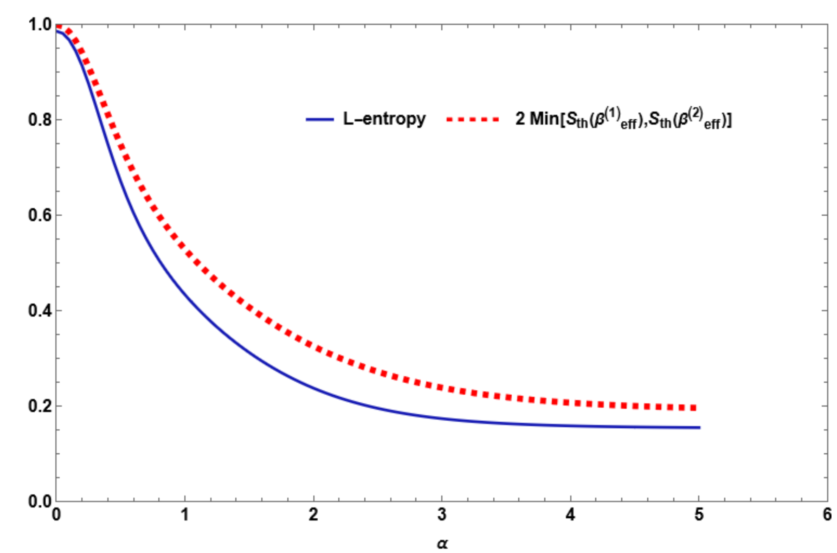
EE of one party vs thermal entropy



parameter α vs effective (inverse) temperature



relative entropy between reduced density matrix and thermal density matrix



L-entropy vs $2\min(S_{th}^{(1)}, S_{th}^{(2)})$

Future Works

Applications and Beyond

- MTPQ state: Holography of multi-boundary wormhole
- K-uniform state: quantum secret sharing and quantum cryptography
- TPQ state: study of black hole micro state
- TPQ state: quantum simulations

Thank You