“Hairy Black Holes by Spontaneous Symmetry Breaking in ESGB theory"

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Content

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[3] Alexandros Papageorgiou, Chan Park, Miok Park, Phys.Rev.D 106 (2022) 8, 084024

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$$S = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2} \nabla_\alpha \varphi \nabla^\alpha \varphi + f(\varphi) \mathcal{G} \right],$$

where $\mathcal{G}$ is the GB term

$$\mathcal{G} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2,$$

and $f(\varphi)$ is a scalar field function

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- Einstein theory holds the no-hair theorems, proved by J. Bekenstein in 1972 and 1995.
- ESGB theory evades **the no-hair theorems** [1] [2] [3] : indicates the existence of hairy BH.

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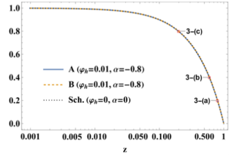
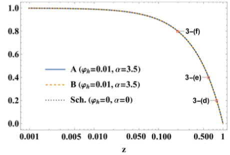
Hairy Black Hole Solutions

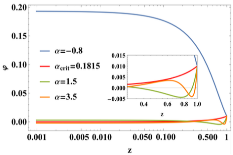
Hairy Black hole Solutions for $f = \alpha\varphi^2$

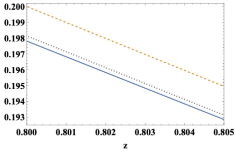
$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{B(r)}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right).$$

Hairy Black hole Solutions for $f = \alpha\varphi^2$

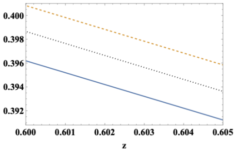








(a) Case of $\alpha = -0.8$



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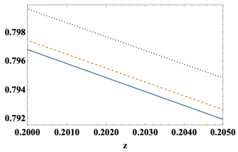

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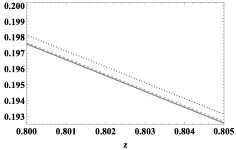
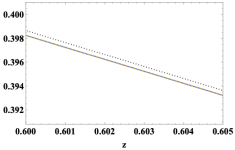


(d) Case of $\alpha = 3.5$



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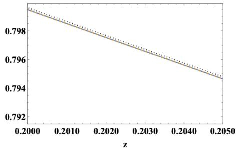

(f) Case of $\alpha = 3.5$

How these hairy black holes can be formed?

We suggest the formation of hairy black holes by spontaneous symmetry breaking [4]

[4] Boris Latosh, Miok Park, Phys.Rev.D 110 (2024) 2, 024012, “Hairy black holes by spontaneous symmetry breaking"

Our Lagrangian

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \nabla_\alpha \varphi^* \nabla^\alpha \varphi + f(\varphi)\mathcal{G} \right], \tag{1}$$

$$\mathcal{L}_\varphi = - \nabla_\alpha \varphi^* \nabla^\alpha \varphi + f(\varphi)\mathcal{G} = T - V, \qquad V = -f(\varphi)\mathcal{G} \tag{2}$$

where $\mathcal{G}$ is the Gauss-Bonnet term and the scalar field coupling function is

$$f(\varphi) = \alpha\, \varphi^*(r)\varphi(r) - \lambda \left(\varphi^*(r)\varphi(r)\right)^2, \qquad (\lambda > 0)$$

Our metric ansatz is

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{B(r)}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2\right). \tag{3}$$

- This Lagrangian respects the global $U(1)$ symmetry

$$\varphi(r) \to e^{i\chi}\varphi(r)$$

- This action allows to have Schwarzschild black holes ($\varphi = const$).

Schwarzschild BH are always stable in ESGB?

Scalar field perturbation

$$\delta\varphi_1(t,r,\theta,\phi) = \sum_{l,m} \frac{\Phi(r)Y_{lm}(\theta,\phi)}{r}\, e^{-i\omega t} \tag{4}$$

$$\Phi''(r_*) - (V_{\rm eff} - \omega^2)\Phi(r_*) = 0, \qquad \mathrm{d}r_* = \frac{1}{\sqrt{AB}}\mathrm{d}r, \tag{5}$$

$$V_{\rm eff}(r) = \frac{l(l+1)A}{r^2} + \frac{1}{2\,r}\Big(A'B + AB'\Big) - \frac{1}{2} f_{\varphi_1\varphi_1} A\,\mathcal{G}, \tag{6}$$

where l is the angular momentum.

$$\int_{r_h}^{\infty} dr\, \frac{1}{\sqrt{AB}}\, V_{\rm eff}(r) < 0 \qquad \rightarrow \qquad \alpha > \alpha_{\rm Sch.} = \frac{5}{24} \approx 0.2083, \tag{7}$$

$$A(r) = B(r) = 1 - \frac{2M}{r}, \qquad l = 0 \tag{8}$$

Schwarzschild black holes in EsGB

become unstable beyond the critical value of α $(= \alpha_{\rm Sch.})$

Schwarzschild BH are always stable in ESGB? No

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Hairy Black Holes by SSB

$$V = -f(\varphi)\mathcal{G},$$
$$f(\varphi) = \alpha\,\varphi^*(r)\varphi(r) - \lambda\left(\varphi^*(r)\varphi(r)\right)^2, \qquad (\lambda > 0)$$

- Black holes in the symmetric phase :

$$\langle\varphi\rangle = 0, \qquad \varphi(r) = \frac{1}{\sqrt{2}}\left(\varphi_1(r) + i\,\varphi_2(r)\right) \tag{9}$$

- Black holes in the symmetry-broken phase : the stable minima are determined by

$$\langle\varphi\rangle = v\,e^{i\beta}, \qquad v = \sqrt{\frac{\alpha}{2\,\lambda}} \tag{10}$$

we expand a field around the ground state v by reparameterizing it as follows

$$\varphi(r) = \left(v + \frac{\sigma(r)}{\sqrt{2}}\right)e^{i\theta(r)} \tag{11}$$

Flux near the black hole horizon [5]

The conserved current is given by

$$\partial_\alpha J^\alpha = 0, \qquad J_\alpha = i\, g\, (\varphi^* \partial_\alpha \varphi - \varphi \partial_\alpha \varphi^*).$$

- In the symmetric phase :

$$\int_\Sigma J_\alpha n^\alpha \sqrt{-h}\, \mathrm{d}^3 y = \int_\Sigma \Big[g(\varphi_2 \partial_r \varphi_1 - \varphi_1 \partial_r \varphi_2) \Big] \Big[\sqrt{A(r)B(r)}\, r^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi\, \mathrm{d}t \Big] = 0 \quad (12)$$

- In the symmetry-broken phase :

$$\theta'(r) = \frac{c_2}{4r^2 \sqrt{A(r)B(r)}} \left(v + \frac{\sigma(r)}{\sqrt{2}} \right)^{-2}, \quad (13)$$

$$\int_\Sigma J_\alpha n^\alpha \sqrt{-h}\, \mathrm{d}^3 y = \int_\Sigma \left[-2\, g \left(v + \frac{\sigma(r)}{\sqrt{2}} \right)^2 \theta'(r) \right] \left[\sqrt{A(r)B(r)}\, r^2 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi\, \mathrm{d}t \right]$$
$$= -8\pi\, g\, c_2 \quad (14)$$

$c_2 = 0$ is required. The Goldstone boson is trivial.

[5] special thanks to Prof. Seong Chan Park

Hairy Black Holes by SSB in EsGB with global $U(1)$

We are interested in the situation that

"Scalar fields are about to grow from black holes without scalar hair. Finally it evolves to hairy black holes."

ex. Schwarzschild black hole $\longrightarrow$ *Hairy Black holes*

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- “$V = -f(\varphi)\mathcal{G}$" as an “interacting potential" :
  - effective near the black hole horizon
  - not effective at infinity ($V \to 0$ as $r \to \infty$, since $\mathcal{G} \to 0$ as $r \to \infty$)

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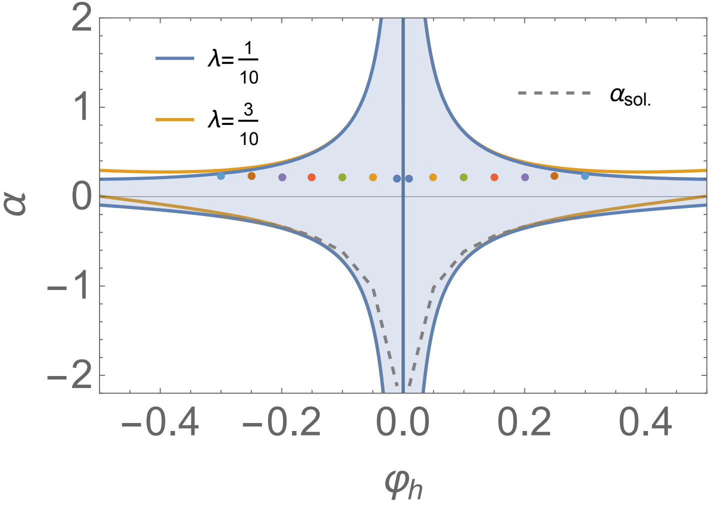
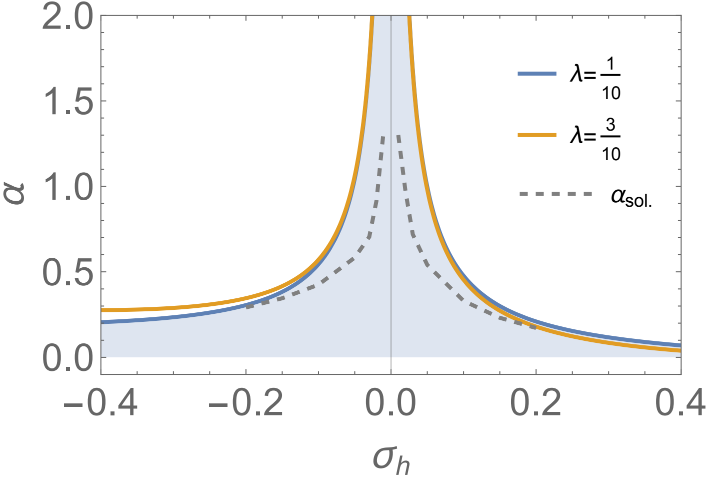
Based on these assumption,

we numerically generated the hairy black holes in symmetric and symmetry-broken phase

Hairy black holes in *symmetric phase*



Hairy black holes in *symmetry broken phase*



Hairy black holes + Scalar field perturbation [6]

[6] Young-Hwan Hyun, Boris Latosh, Miok Park, JHEP 08 (2024) 163, “Scalar field perturbation of hairy black holes in EsGB theory"

Scalar Field Perturbation of Hairy Black Holes

$$\left[\partial_x^2 - s^2 - V_{\rm eff}(r)\right]\delta\hat{\varphi}_1(s,x) = -\partial_t\delta\varphi(t,x)|_{t=0} - s\delta\varphi(t,x)|_{t=0} = \mathcal{J}(s,x) \tag{15}$$

$$V_{\rm eff}(r) = A(r)\left[\frac{l(l+1)}{r^2} + \frac{B(r)}{2\,r}\left(\frac{A'(r)}{A(r)} + \frac{B'(r)}{B(r)}\right) - \frac{1}{2}f_{\varphi_1\varphi_1}\,\mathcal{G}\right]$$

Green's function method provides the solution

$$\delta\hat{\varphi}_1(s,x) = \int_{-\infty}^{\infty} {\rm d}x'\;\hat{G}(s,x,x')\mathcal{J}(s,x'). \tag{16}$$

This Green's function are constructed by the following three contributions

$$G(t,x,x') = G_{\rm F}(t,x,x') + G_{\rm QNM}(t,x,x') + G_{\rm B}(t,x,x') \tag{17}$$

where

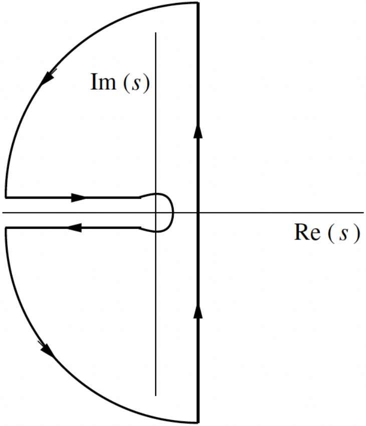
$$G_{\rm F}(t,x,x') = \lim_{R\to\infty}\left[\int_{\phi=\frac{\pi}{2}}^{\pi}\frac{{\rm d}(s=Re^{i\phi})}{2\pi i}e^{st}\hat{G}(s,x,x') + \int_{\phi=-\pi}^{-\frac{\pi}{2}}\frac{{\rm d}(s=Re^{i\phi})}{2\pi i}e^{st}\hat{G}(s,x,x')\right],$$

$$G_{\rm QNM}(t,x,x') = \lim_{R\to\infty}\oint_{\gamma_R}\frac{{\rm d}s}{2\pi i}e^{st}\hat{G}(s,x,x'),$$

$$G_{\rm B}(t,x,x') = \int_{-\infty}^{0}\frac{{\rm d}(se^{2\pi i})}{2\pi i}e^{se^{2\pi i}t}\hat{G}(se^{2\pi i},x,x') + \int_{0}^{-\infty}\frac{{\rm d}s}{2\pi i}e^{st}\hat{G}(s,x,x')$$

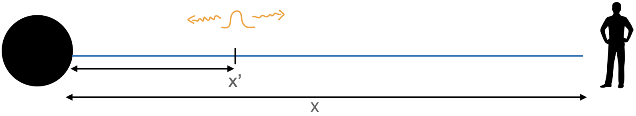
Greens function in s-space ($s = i\omega$)

$$G(t,x,x') = G_{\mathrm{F}}(t,x,x') + G_{\mathrm{QNM}}(t,x,x') + G_{\mathrm{B}}(t,x,x')$$



Solution for KG equation: Green's function method

$$G(t,x,x') = G_{\rm QNM}(t,x,x') + G_{\rm B}(t,x,x') + G_{\rm F}(t,x,x') \tag{18}$$

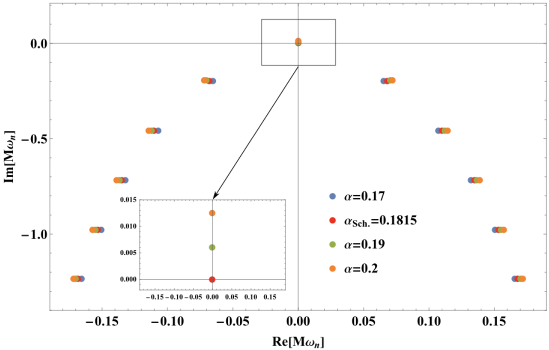


- $t \simeq x - x'$: the outward propagating wave can effectively ignore the influence of this potential and travel outward reaches the observer, $G_{\rm F}$
- $t \simeq x + x'$: (quasinormal modes) the part of the low-frequency scalar perturbation traveling towards the black hole bounces off the effective potential $V_{\rm eff}(r)$ near the horizon and propagates back to the observer, $G_{\rm QNM}$
- $t \gg x + x'$: (late-time behavior) the QNMs gradually fade away, and only the decaying modes of the signal persist. The outgoing wave that traveled to a large distance and then scattered back towards the observer due to the spacetime curvature at large distances, $G_{\rm B}$

Quasinormal modes (QNM)

G_{QNM} : Schwarzschild black holes

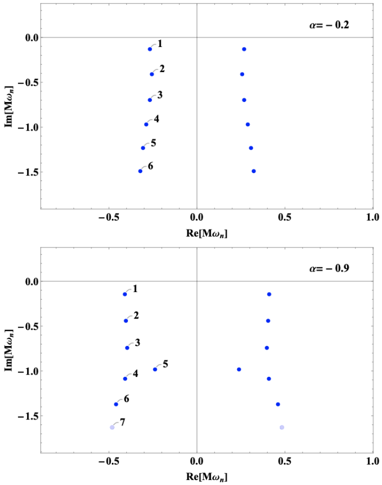
$$\delta\varphi(v,z,\theta,\phi) = \sum_{l,m} \Phi(z) Y_{lm}(\theta,\phi) e^{-i\omega v}, \quad \omega = \omega_{\rm R} + i\omega_{\rm I}$$



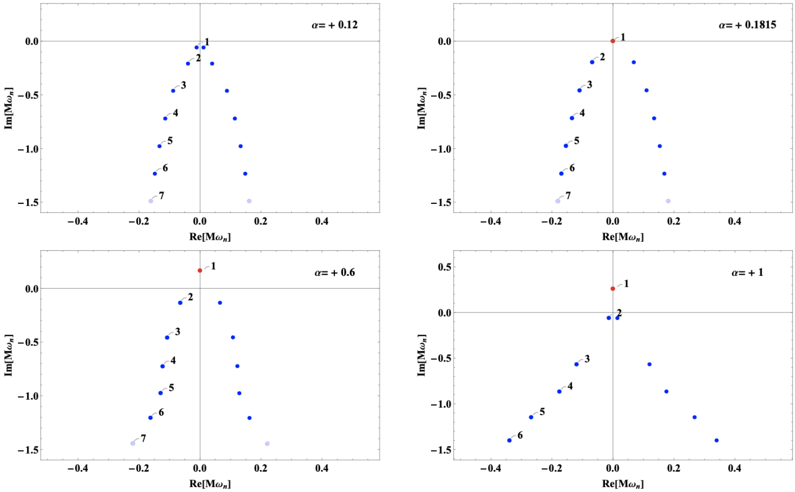
	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
α=0.17	$\pm$ 0.06528	-0.19795
	$\pm$ 0.10718	-0.45845
	$\pm$ 0.13196	-0.71851
	$\pm$ 0.15043	0.97682
$\alpha_{\rm Sch.} \approx 0.1815$	0	0.00002
	$\pm$ 0.06826	-0.19618
	$\pm$ 0.11012	-0.45776
	$\pm$ 0.13483	-0.71820
	$\pm$ 0.15324	-0.97669
α=0.19	0	0.00600
	$\pm$ 0.07013	-0.19489
	$\pm$ 0.11208	-0.45728
	$\pm$ 0.13677	-0.71798
	$\pm$ 0.15513	-0.97662
α=0.20	0	0.01254
	$\pm$ 0.07198	-0.19340
	$\pm$ 0.11415	-0.45675
	$\pm$ 0.13881	-0.71775
	$\pm$ 0.15712	-0.97654

Figure 4 & Table 1: QNMs of Schwarzschild black holes for serval values α

G_{QNM} : Symmetric Phase with $\alpha < 0$



G_{QNM} : Symmetric Phase with $\alpha > 0$



G_{QNM} : Symmetric Phase with $\alpha > 0$

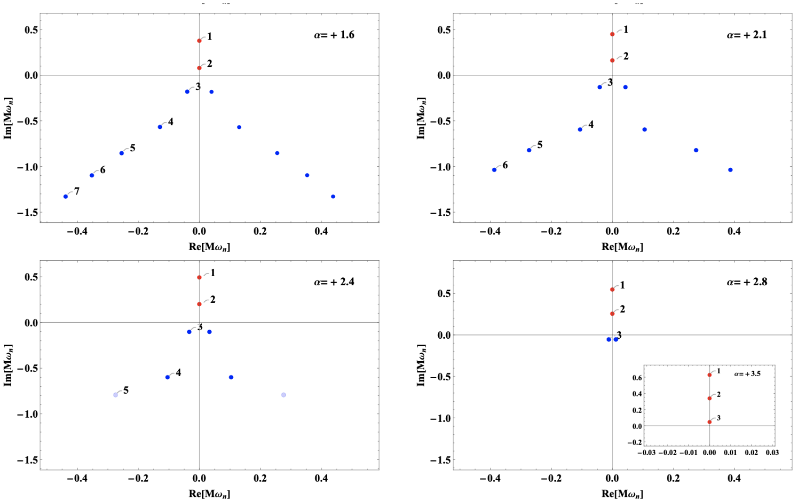
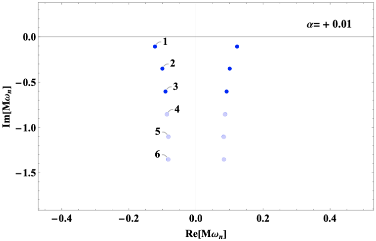
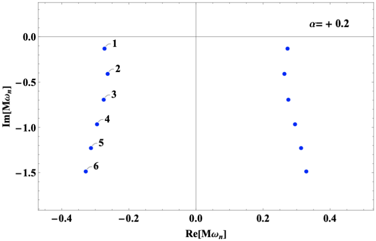


Figure 6: QNMs with positive values of α for $\varphi_h = 0.01$. Blue dots are tolerable less than 10^{-3}, whereas lighter blue dots have a tolerance of 10^{-2}.

G_{QNM} : Symmetry-Broken Phase

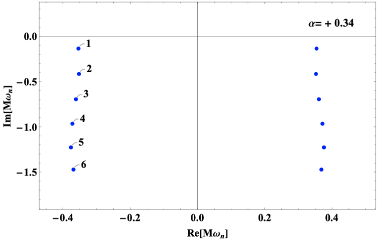


	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.12229	-0.10701
2	± 0.10025	-0.35077
3	± 0.09104	-0.60312
4	± 0.08662	-0.85448
5	± 0.08226	-1.10205
6	± 0.08295	-1.35246

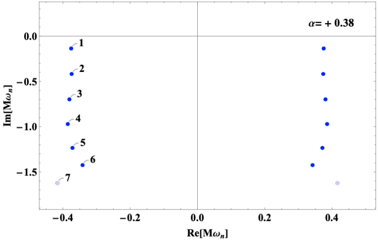


	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.27261	-0.13084
2	± 0.26325	-0.40969
3	± 0.27500	-0.69474
4	± 0.29492	-0.96587
5	± 0.31297	-1.22847
6	± 0.32848	-1.4865

G_{QNM} : Symmetry-Broken Phase



	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.35393	-0.13610
2	± 0.35217	-0.41623
3	± 0.36116	-0.69570
4	± 0.37179	-0.96582
5	± 0.37610	-1.22716
6	± 0.36866	-1.47193



	$\mathrm{Re}[M\omega_n]$	$\mathrm{Im}[M\omega_n]$
1	± 0.37538	-0.13731
2	± 0.37399	-0.41842
3	± 0.38069	-0.69878
4	± 0.38549	-0.97137
5	± 0.37156	-1.23482
6	± 0.34159	-1.42465
7	± 0.41624	-1.62298

Figure 11 & Table 3: QNMs with various values of α for $\sigma_h = 0.01$. Blue dots are tolerable less than 10^{-3}, whereas lighter blue dots have a tolerance of 10^{-2}.

Late time tails : branch cut

G_B : branch cut

If considering the low-frequency limit ($s = i\omega$) such as $A_1 s \to 0$ and $\varphi_{1,1} s \to 0$

$$G_{\mathrm{B}}(t,x,x') = \int_{-\infty}^{0} \frac{\mathrm{d}(se^{2\pi i})}{2\pi i} e^{se^{2\pi i}t} \hat{G}(se^{2\pi i},x,x') + \int_{0}^{-\infty} \frac{\mathrm{d}s}{2\pi i} e^{st} \hat{G}(s,x,x'),$$

$$G_B(t,r,r') \simeq (-1)^{-l} \frac{A_1 2^{2l+1} \Gamma(l+1)^2 r^{l+1} (r')^{l+1}}{\Gamma(2l+2)^2} \int_{-\infty}^{0} \mathrm{d}s\, e^{s(t-r-r')} s^{2(l+1)} {}_1F_1(2rs)\, {}_1F_1(2r's),$$

where ${}_1F_1(2rs) = {}_1F_1(l+1; 2l+2; 2rs)$ is the confluent hypergeometric ft. of the first kind.

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where ${}_1F_1(2rs) = {}_1F_1(l+1; 2l+2; 2rs)$ is the confluent hypergeometric ft. of the first kind.

- In the limit $t \to \infty$ with r fixed, we obtain

$$G_B(t,r,r') \simeq (-1)^{l+1} \frac{2(-A_1)(2l+2)!}{((2l+1)!!)^2} \frac{r^{l+1}(r')^{l+1}}{t^{2l+3}}.$$

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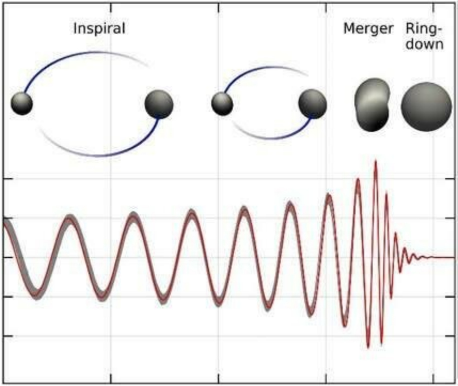
- In the limit $v \to \infty$ ($t \to \infty$ with t/r fixed), we can take $r \to \infty$ and $r's \ll 1$, then

$$\begin{aligned} G_B(t,r,r') &\simeq -\frac{A_1(r')^{l+1}}{(2l+1)!!} \int_{0}^{-\infty} {\rm d}s\; e^{s(t-r)} s^{l+1} + \frac{A_1(-1)^l (r')^{l+1}}{(2l+1)!} \int_{0}^{-\infty} {\rm d}s\; e^{s(r+t)} s^{l+1}, \\ &= \frac{(l+1)!}{(2l+1)!!} \left[\frac{(-A_1)(-1)^{l+1}}{u^{l+2}} - \frac{A_1}{v^{l+2}} \right] (r')^{l+1} \\ &\simeq (-1)^{l+1}(-A_1) \frac{(l+1)!}{(2l+1)!!} \frac{(r')^{l+1}}{u^{l+2}} \end{aligned}$$

Ring-down phase



The newly formed, perturbed black hole relaxes to a stable state (usually a Kerr black hole characterized by its mass M and spin J) by emitting gravitational waves. These waves are described by the black hole's quasinormal modes (QNMs)

Ringdown phase with QNMs for gravitational wave

However, as we have seen that *QNMs generally do not form a complete set in the usual mathematical sense due to their non-Hermitian nature* and the boundary conditions imposed. They can be part of a generalized basis when combined with other modes, such as:

$$\psi(t.x) = \sum_n c_n \psi_n(x) e^{-i\omega_n t} + \text{branch cut contribution}$$

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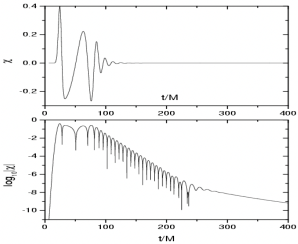
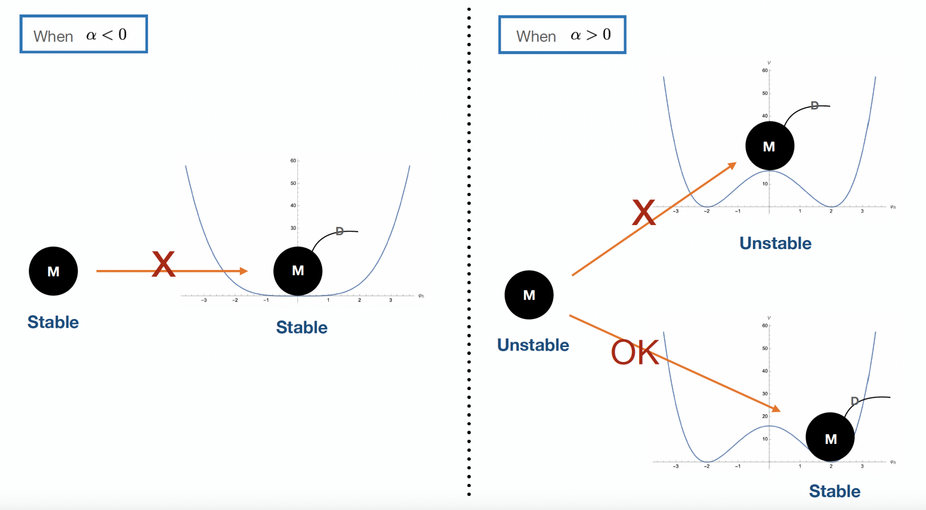


Figure 4: *The response of a Schwarzschild black hole as a Gaussian wave packet impinges upon it. The QNM signal dominates the signal after $t \approx 70M$ while at later times (after $t \approx 300M$) the signal is dominated by a power-law fall-off with time.*

Ref : Kostas D. Kokkotas, Bernd G. Schmidt, "Quasi-Normal Modes of Stars and Black Holes",Volume 2, article number 2, (1999)

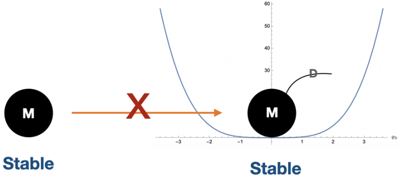
Summary



When $\alpha < 0$
M
X
M
D
Stable
Stable
When $\alpha > 0$
M
D
X
Unstable
M
Unstable
OK
M
D
Stable

Summary

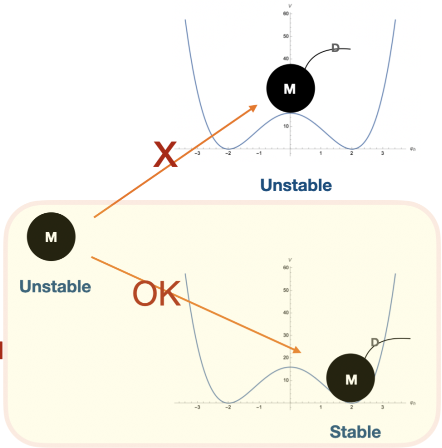
When $\alpha < 0$



Our Claim :

Phase transition will occur and form black hole's hairs in symmetry-broken phase!

When $\alpha > 0$



Future works

1. quasinormal modes with metric perturbations
2. dynamical evolution from non-hairy black holes to hairy black holes
3. hairy black holes by SSB with $U(1)$
4. thermodynamic stability
5. relation with Love number

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Thank you!