A few issues on entanglement

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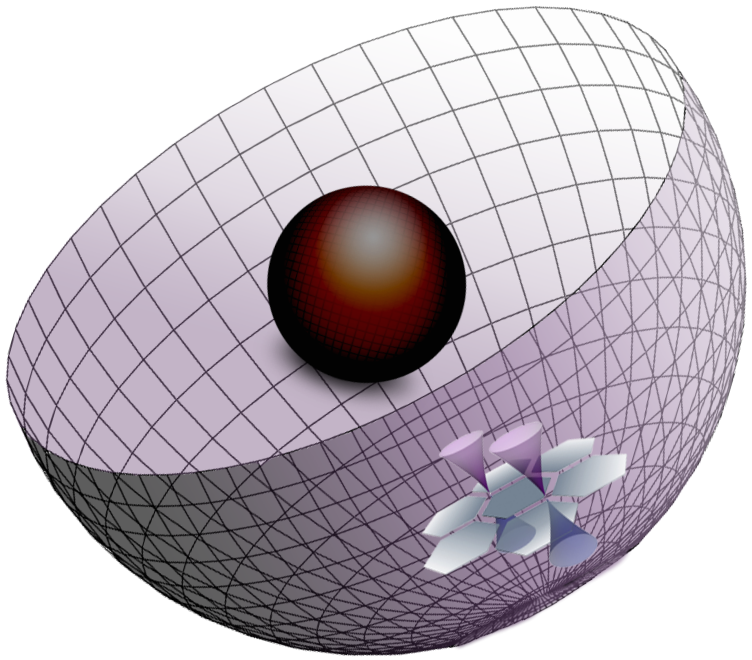


Topology in Holographic Mean-Field Theory at Zero and Finite Temperature e-Print: 2508.01767
Mean field theory for strongly coupled systems: Holographic approach *JHEP* 06 (2024) 100

A few Tales on Entanglement

I. 이별과 재회: introduction to ads/cft

II. Entanglement & locality : ads/cmt

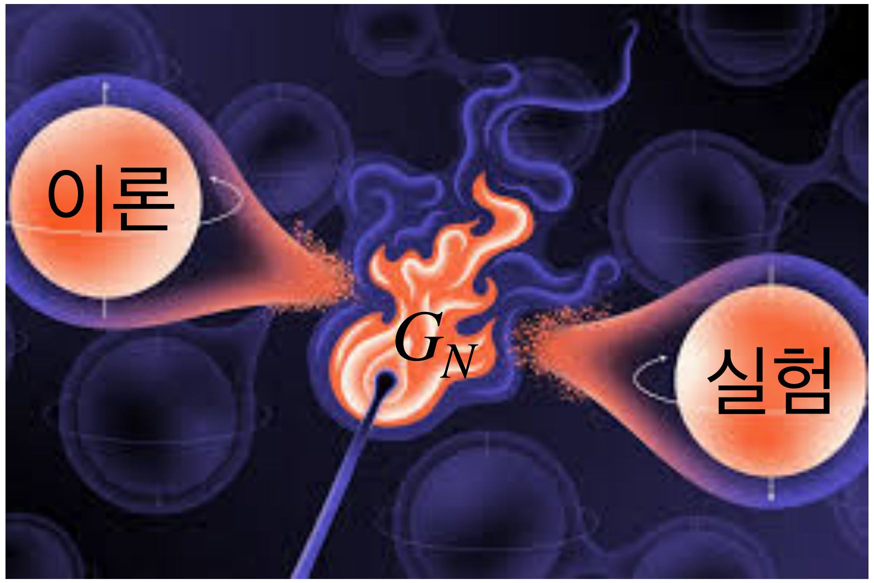
III. Entanglement and scattering.

IV. Sewing spacetime by Entanglement

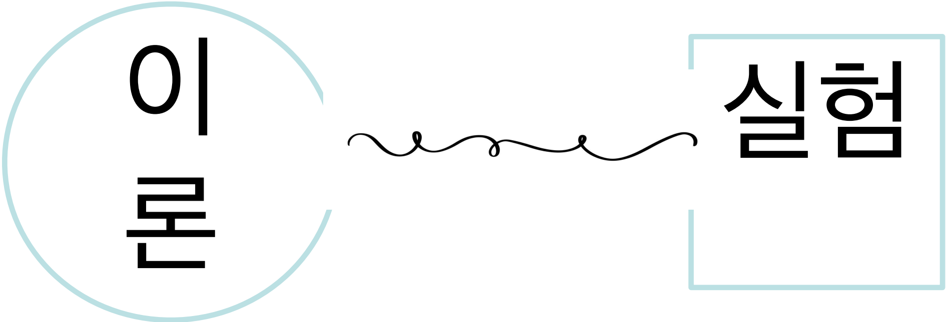
V. ER=EPR and Strange metal

I. 이별과 재회

이별:



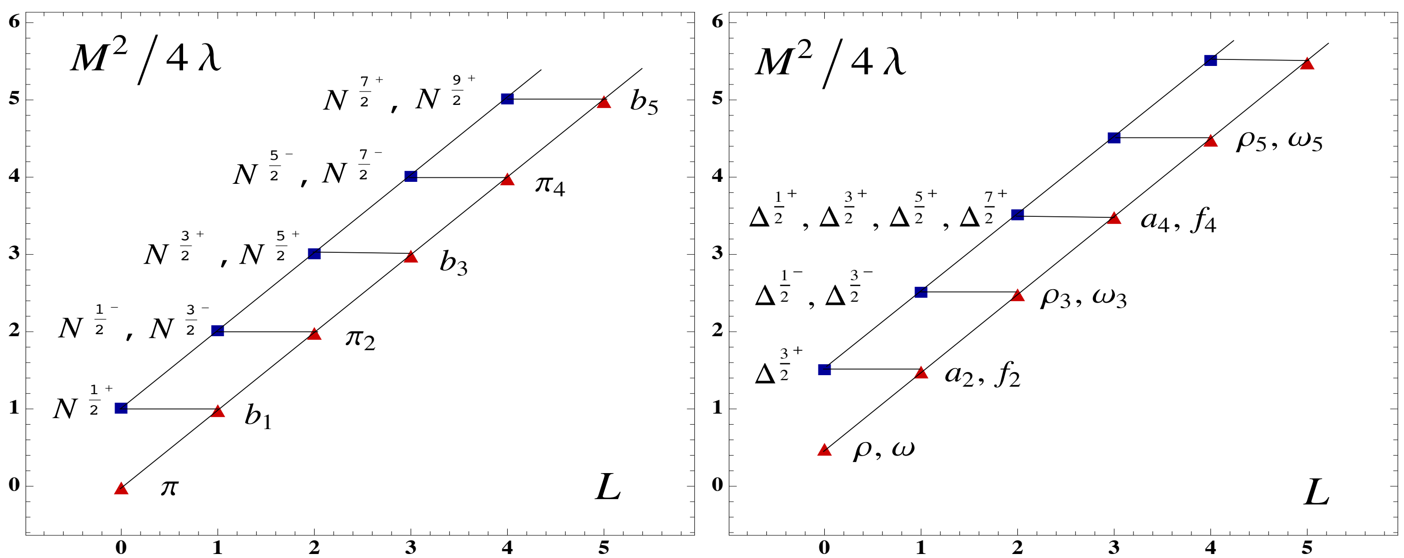
재회:



I.1 이별

1973: Asymptotic freedom -> 강작용의 섭동론 극적인 구제.
장론을 사용한 고에너지 SU(3)xSU(2)xU(1) 동역학의 완성. 현상론의 탄생, 현상론/장론 분리

Before : open string theory: $J = \alpha' M^2, \quad 1/\alpha' = (1GeV)^2,$ (장론폐기)

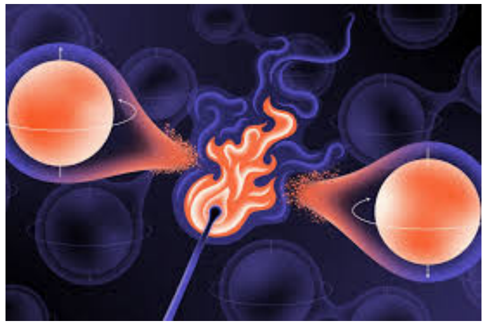


After : Closed string theory(1974): s=2 중력자 포함,

Gravity with scale $M_P = 10^{19} GeV$, $(G_N = M_P^{-2})$

practically ∞ slope/energy to excite modes.

=> 끈이론-실험의 이별 : by the presence of $G_N = M_P^{-2}$

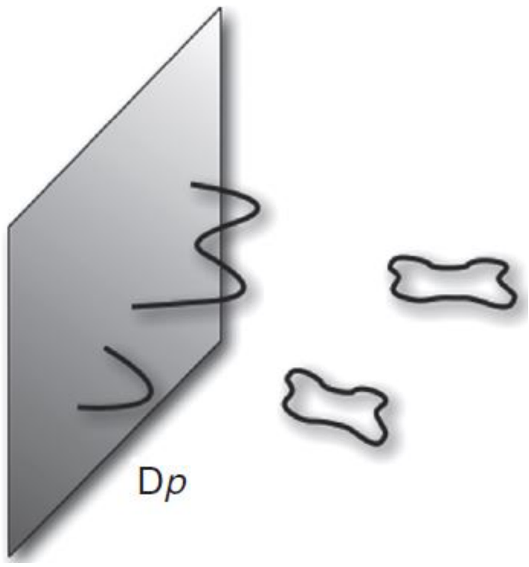


1.2 재회: AdS/CFT (1998)

D-brane(1995: Polchinski)

Near horizon limit of D-brane =AdS

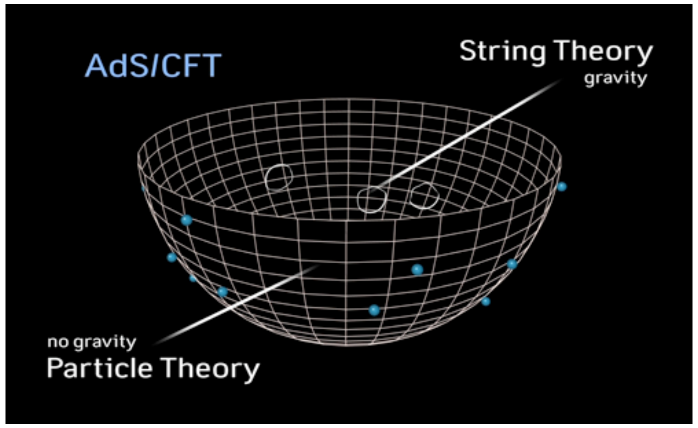
N=4 SYM <=> string th in AdS (1998: Maldacena, 2만여회 인용)



Strong-weak duality

$$\lambda = g_{YM}^2 N \qquad \lambda \sim \left(\frac{\ell_{AdS}}{\ell_{string}}\right)^4$$

$$\frac{\ell_{AdS}^{d-1}}{G_N} \sim \left(\frac{\ell_{AdS}}{\ell_P}\right)^{d-1} \sim N^2$$

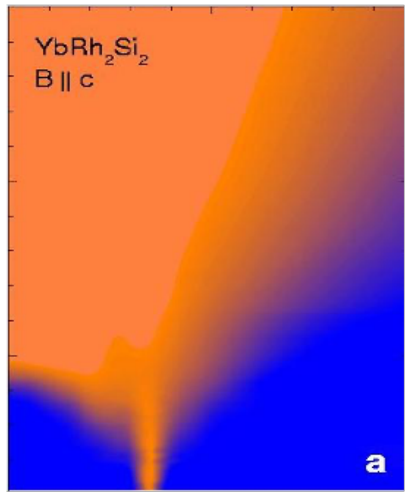


CFT has no scale => G_N decouple.

Once it decouples, we can generate any wanted scale by
a dynamical symmetry breaking

=> 재회 ? How to generate a theory for the system on the boundary?

Generalize to AdS/CMT: a calculation formalism of emergence



QCP =analogue of N=4 SYM

no scale

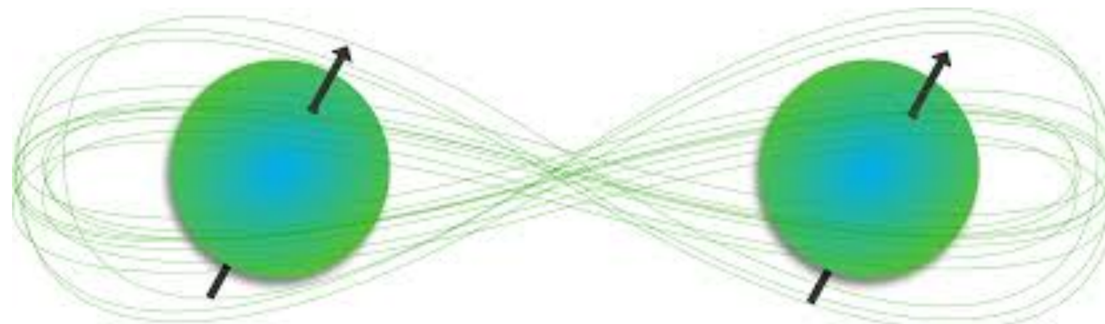
BH: no hair

- Origin of simplification at QCP :
  Democracy of scales => Information Loss => universality in SIS
- The whole system as an object indivisible.
- AdS/SYM = an soluble example.

II. Entanglement and Locality

Entanglement implies Non-locality.

Measuring L can affect R, whatever is their distance=> Non-locality

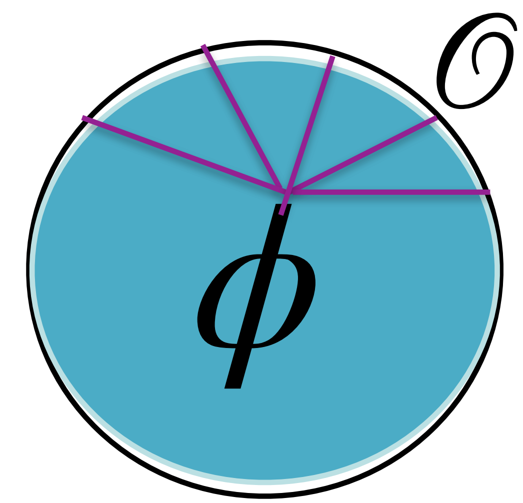


Strong Interaction => Strong Entanglement=> Non-locality => untreatable

Q: How to overcome this non-locality?

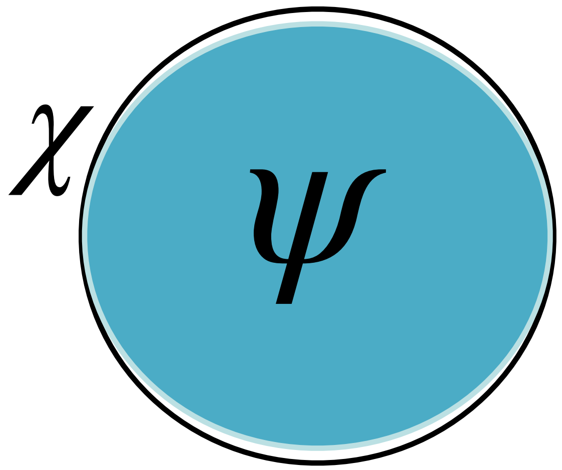
A: AdS/CFT: Bulk-Bdy correspondence is non-local.

$$\phi(x,z) = \int K(x,z;x')\mathcal{O}(x')dx'$$



Prescription of AdS/X, X=QCD, CMT, …

Finding the dual theory of known theory is meaningless.



$$S = S_0 + \int \psi_o(x)\chi(x)dx$$

$$Z_\chi := Z_{\psi(\chi)}$$

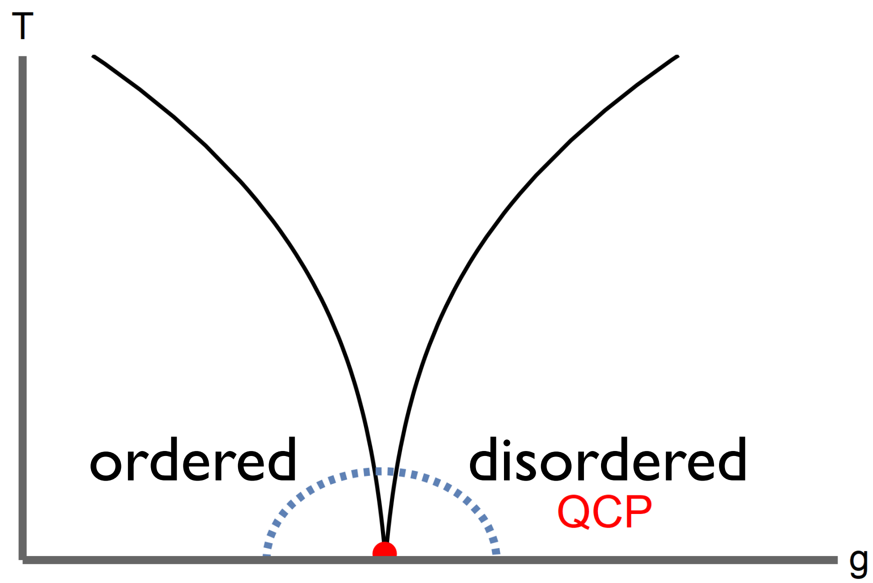
Define dof at bdy: χ
A non-local Th. of χ
Postulate the local Th. of ψ
=> calculate observable of χ

No reason to prove the equivalence

- D.o.F is defined in bdy, but its dynamics is defined by the BULK theory.
- assume the presence of basic bulk fields which is dual to the (fermion, fermion bilinear, …) and its local bulk action.
- We do not assume the locality of the boundary theory.
- bulk locality= a principle not a theorem.

Q: How to describe a realistic system using ads/CFT?

By structure & features
=> need a scale=>need a Symmetry breaking !



Need a theory off the QCP.
Mean field theory=Theory of symmetry breaking.
Theory of symmetry breaking in holographic version:
we call it as holographic mean field theory.
The first example is Holographic Superconductivity theory.

II. Holographic mean field theory (I)

Traditional MFT:

MFT= theory of condensation. => … gap generation => … order.

$\Delta \sim \langle c_k c_{-k} \rangle,\ BCS$

$\Delta \sim \langle f_k^\dagger c_{-k} \rangle,\ Kondo \quad Condensation$

$M^A \sim \langle c_k^\dagger \Gamma^A c_k \rangle,\ Charge\ density\ or\ magnetic\ ordering$

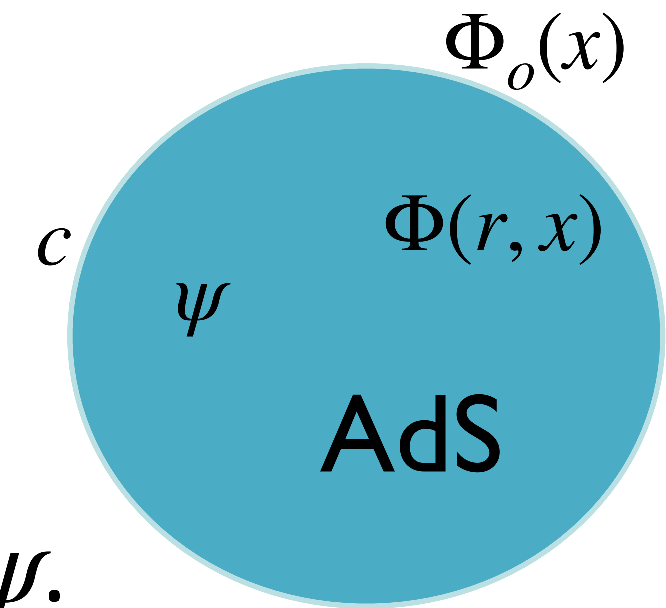
What are the condensations in particle theory?

Holographic MFT:

$$S = \int d^4x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{4} F^2 - |D_A \Phi_I|^2 \right)$$

Solution to the eq. Of M => $\Phi_I \neq 0 \rightarrow hairy\ solution$

H-MFT (ii) Effect of order in fermion spectrum



Order : $\langle \bar{c}\Gamma^A c\rangle \neq 0,$

Holographic dictionary:

Consider ψ *dual to c,* and

add $\Phi_A \cdot \bar{\psi}\Gamma^A\psi$ to $\mathscr{L}_0 = \bar{\psi}(\gamma^\mu i\partial_\mu - m)\psi.$

Find the configuration of Φ first, in the fixed BH gravity.

—> Study $\psi(z,x)$ in the fixed ($g_{\mu\nu}$, Φ)

to get spectrum of χ.

Structure of holographic MFT

T.Yuk

- JHEP 06 (2024) 100 • e-Print: 2311.01897

S. Sukrakarn

Einstein-Maxwell-Tensor

$$S_{total} = S_\psi + S_{bdy} + S_{g,\Phi} + S_{int},$$

$$S_\psi = i\int d^dx \sum_{j=1}^{2} \sqrt{-g}\,\bar{\psi}^{(j)}\left(\not{D} - m^{(j)}\right)\psi^{(j)},$$

$$S_{bdy} = \frac{i}{2}\int_{bdy} d^{d-1}x\sqrt{-h}\left(\bar{\psi}^{(1)}\psi^{(1)} \pm \bar{\psi}^{(2)}\psi^{(2)}\right),$$

$$S_{g,\Phi} = \int d^dx\sqrt{-g}\left(R - 2\Lambda + |D_M\Phi_I|^2 - m_\Phi^2|\Phi|^2\right),$$

$$S_{int} = \int d^dx\sqrt{-g}\left(\bar{\psi}^{(1)}\Phi\cdot\Gamma\psi^{(2)} + h.c\right)$$

where Φ_I is order parameter field, $\bar{\psi}^{(1)}\Phi\cdot\Gamma\psi^{(2)}$ is constructed by considering all possible Lorentz symmetry.

$$\Phi\cdot\Gamma = \Gamma^{\underline{\mu_1\mu_2}\cdots\underline{\mu_I}}\Phi_{\underline{\mu_1\mu_2}\cdots\underline{\mu_I}}.$$

Symmetry type of the order vs pattern of spectrum

8 (half) of them have both simple pole and branch-cut types.

- $\Phi, B_i, B_{jk}, B_{tu}\ (AdS_5)$
- $\Phi, \Phi_5, B_i, B_{5i}, B_{jk}, B_{tu}\ (AdS_4)$

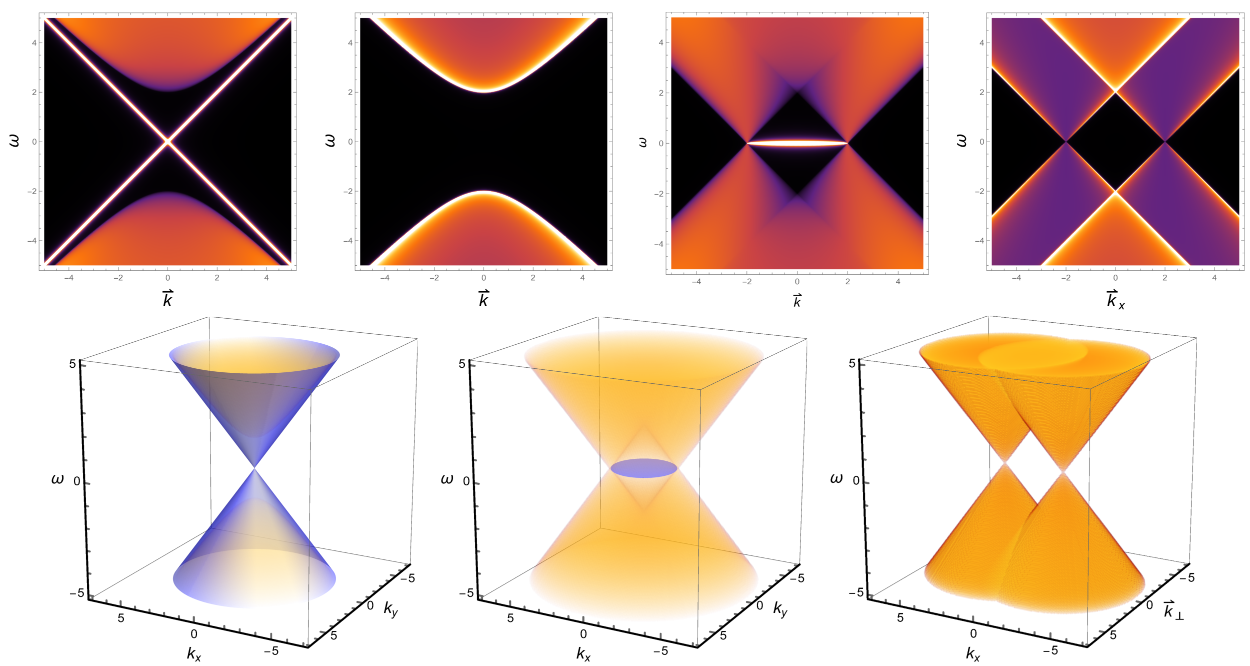
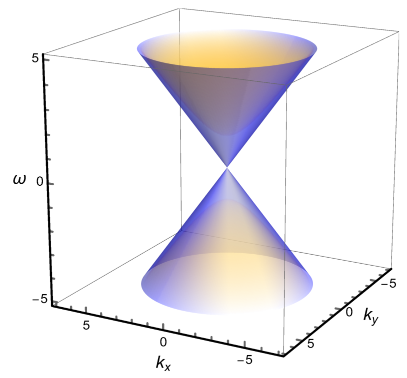
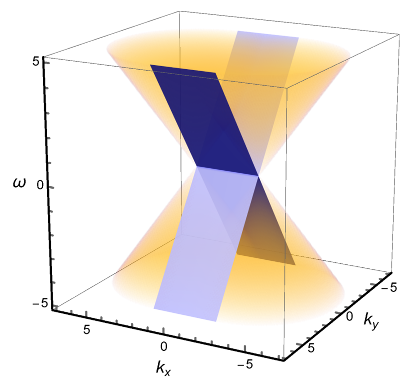
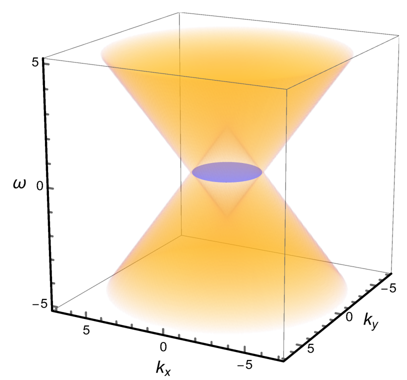


Figure: Simple pole and Branch-Cut types spectra

- 2-dimensional slice of the spectral density
- 3-dimensional spectral density

Appearing features: Gaps of s-,p-wave sym.
Flat bands of dim 1,2,3.
Nodal segment and rings

For probe: Analytic spectral functions notice: pole vs branch cut

$\Phi_{(SS)}, \Phi_{5(SA)}$		$TrG_R(k_\mu) = \dfrac{4\omega\sqrt{\vec{k}^2 - \omega^2 + M^2}}{\vec{k}^2 - \omega^2}$
$B_{x(SA)}, B_{x5(SS)}$		$TrG_R(k_\mu) = \dfrac{2\omega}{b(k_y^2 - \omega^2)}\left[(b + k_x)\epsilon_- + (b - k_x)\epsilon_+\right]$; $\epsilon_\pm = \sqrt{(b \pm k_x)^2 + k_y^2 - \omega^2}$
$B_{xy(SS)}, B_{tu(SA)}$		$TrG_R(k_\mu) = -\dfrac{2}{b\omega}\left[(b + \lvert\vec{k}\rvert)\epsilon_- + (b - \lvert\vec{k}\rvert)\epsilon +\right]$; $\epsilon_\pm = \sqrt{(b \pm \lvert\vec{k}\rvert)^2 - \omega^2}$

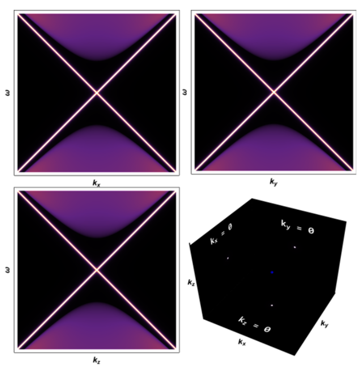
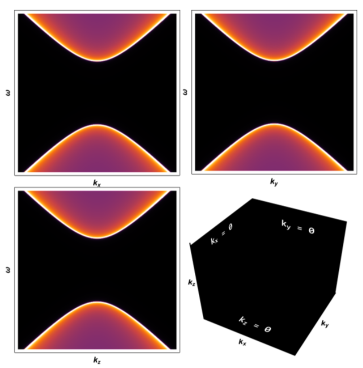
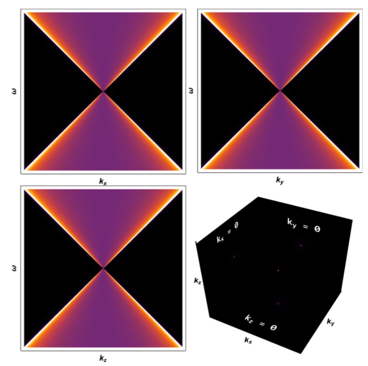
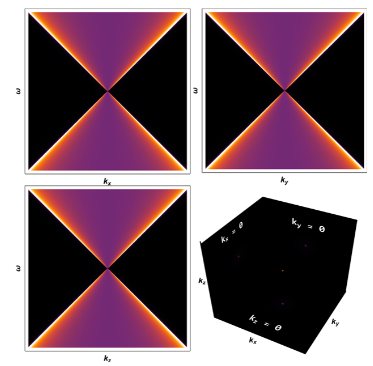
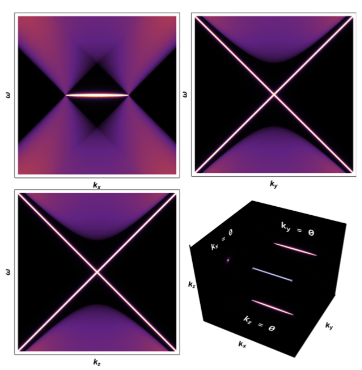
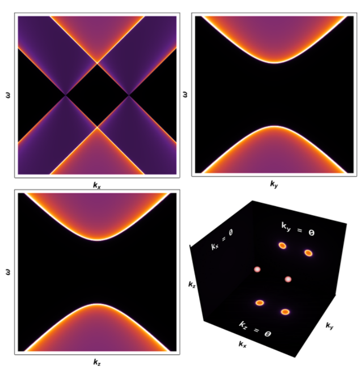
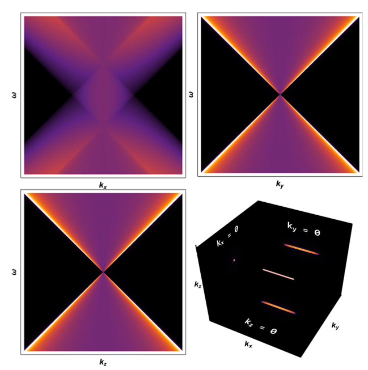
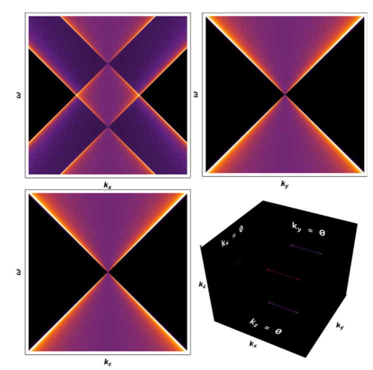
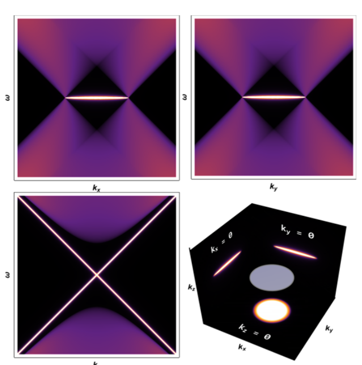
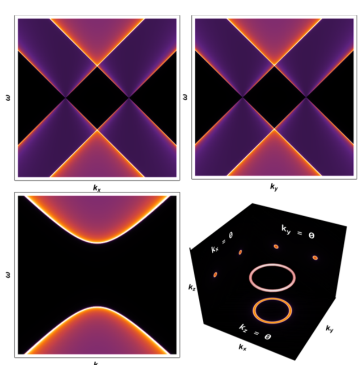
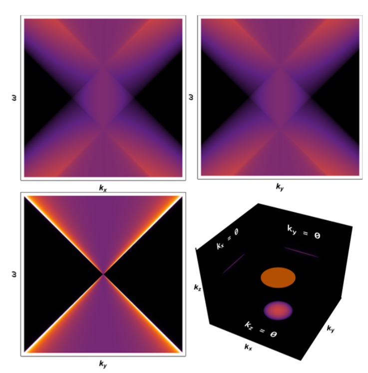
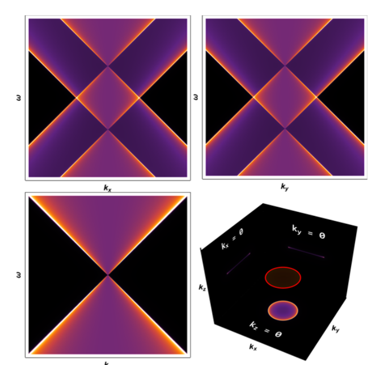
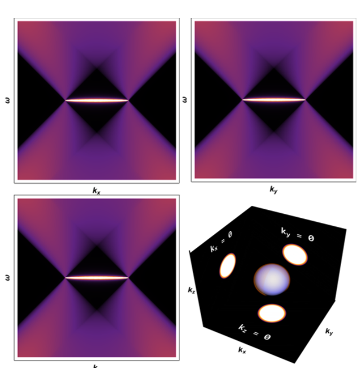
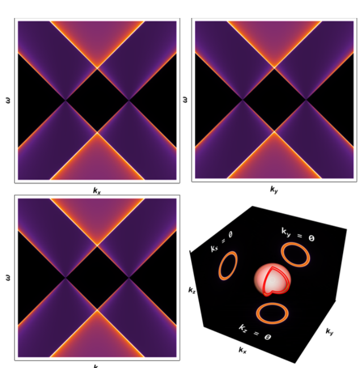
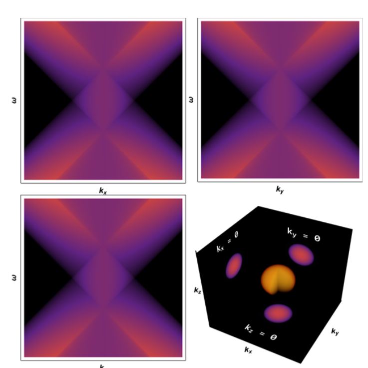
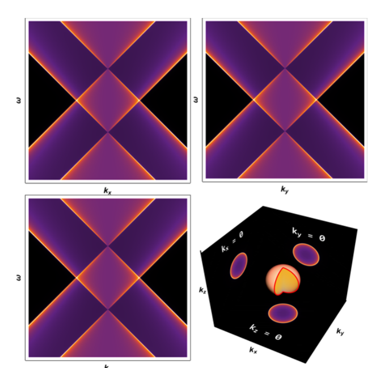
- Usual many body theory assume: $G \sim \dfrac{Z}{\omega - \epsilon - \Sigma}$:
- Poles=> FL. // Branch cut also appears in ours => New class of Non-FL

Analytic spectral functions for AdS4

Interactions	**Trace of analytic Green's functions** $(\mathbf{AdS}_4)$	**Features/Classification**
M_0/M_{05}	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{M_0} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{M_{50}} = \dfrac{4\omega}{\sqrt{\boldsymbol{k}^2 - \omega^2 + M_0^2}}$	Gapful/s-wave gap
	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{M_0} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{M_{50}} = 4\omega \dfrac{\sqrt{\boldsymbol{k}^2 - \omega^2 + M_0^2}}{\boldsymbol{k}^2 - \omega^2 - i\epsilon}$	Topological liquid
B_x/B_{5x}	$\mathrm{Tr}\,G^{(SS)}_{B_x^{(0)}} \equiv \mathrm{Tr}\,G^{(SA)}_{B_{5x}^{(0)}} = \dfrac{2\omega}{\sqrt{(b-k_x)^2 + k_y^2 - \omega^2}} + \dfrac{2\omega}{\sqrt{(b+k_x)^2 + k_y^2 - \omega^2}}$	Shifting cones/p-wave gap
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_x^{(0)}} \equiv \mathrm{Tr}\,G^{(SS)}_{B_{5x}^{(0)}} = \dfrac{2\omega}{b}\left[\dfrac{(b+k_x)\sqrt{(b-k_x)^2 + k_y^2 - \omega^2} + (b-k_x)\sqrt{(b+k_x)^2 + k_y^2 - \omega^2}}{k_y^2 - \omega^2 - i\epsilon}\right]$	1D flat band
B_{xy}/B_{tu}	$\mathrm{Tr}\,G^{(SA)}_{B_{xy}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{tu}^{(-1)}} = \dfrac{2\omega}{\sqrt{(b-\boldsymbol{k})^2 - \omega^2}} + \dfrac{2\omega}{\sqrt{(b+\boldsymbol{k})^2 - \omega^2}}$	Nodal ring
(anti-symmetric)	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{xy}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{tu}^{(-1)}} = -\dfrac{2}{b}\left[\dfrac{(b+\lvert\boldsymbol{k}\rvert)\sqrt{(b-\boldsymbol{k})^2 - \omega^2} + (b-\lvert\boldsymbol{k}\rvert)\sqrt{(b+\boldsymbol{k})^2 - \omega^2}}{\omega + i\epsilon}\right]$	2D flat band
B_u	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_u^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_u^{(0)}} = \dfrac{4\omega}{\sqrt{\boldsymbol{k}^2 - \omega^2}}$	QCP
B_{ux}/B_{5u}	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{ux}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{5u}^{(-1)}} = 4\omega \dfrac{b^2 + \boldsymbol{k}^2 - \omega^2 + f_+ f_-}{f_+ f_- (f_+ + f_-)}\ ;\ f_\pm = \sqrt{k_x^2 - \left(b \pm \sqrt{\omega^2 - k_y^2}\right)^2}$	Filled nodal line
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{ux}^{(-1)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{5u}^{(-1)}} = 4\omega \dfrac{(f_+ + f_-)\sqrt{\omega^2 - k_y^2} - b(f_+ - f_-)}{\sqrt{\omega^2 - k_y^2}(b^2 + \boldsymbol{k}^2 - \omega^2 + f_+ f_-)}\ ;\ f_\pm = \sqrt{k_x^2 - \left(b \pm \sqrt{\omega^2 - k_y^2}\right)^2}$	Non-singular segment
B_t/B_{5t}	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_t^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{5t}^{(0)}} = 2\left(\dfrac{b+\omega}{\sqrt{\boldsymbol{k}^2 - (b+\omega)^2}} - \dfrac{b-\omega}{\sqrt{\boldsymbol{k}^2 - (b-\omega)^2}}\right)$	Filled nodal ring
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_t^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{5t}^{(0)}} = \dfrac{2}{b}\left[\sqrt{\boldsymbol{k}^2 - (b-\omega)^2} - \sqrt{\boldsymbol{k}^2 - (b+\omega)^2}\right]$	Non-singular disk

Analytic spectral functions for AdS5

M_0	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{M_0} = \dfrac{4\omega}{\sqrt{\boldsymbol{k}^2-\omega^2+M_0^2}}$	(4.3)	Gapful/s-wave gap	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{M_0} = 4\omega\dfrac{\sqrt{\boldsymbol{k}^2-\omega^2+M_0^2}}{\boldsymbol{k}^2-\omega^2-i\epsilon}$	(4.2)	Topological liquid	Pole
B_x	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_x^{(0)}} = \dfrac{2\omega}{\sqrt{(b-k_x)^2+\boldsymbol{k}_\perp^2-\omega^2}} + \dfrac{2\omega}{\sqrt{(b+k_x)^2+\boldsymbol{k}_\perp^2-\omega^2}}$	(4.10)	Shifting cones/p-wave gap	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_x^{(0)}} = \dfrac{2\omega}{b}\Big[\dfrac{(b+k_x)\sqrt{(b-k_x)^2+\boldsymbol{k}_\perp^2-\omega^2}+(b-k_x)\sqrt{(b+k_x)^2+\boldsymbol{k}_\perp^2-\omega^2}}{\boldsymbol{k}_\perp^2-\omega^2-i\epsilon}\Big]$	(4.11)	1D flat band	Pole
B_{xy}	$\mathrm{Tr}\,G^{(SA)}_{B_{xy}^{(-1)}} = \dfrac{2\omega}{\sqrt{(b-\lvert\boldsymbol{k}_\perp\rvert)^2+k_z^2-\omega^2}} + \dfrac{2\omega}{\sqrt{(b+\lvert\boldsymbol{k}_\perp\rvert)^2+k_z^2-\omega^2}}$	(4.15)	Nodal ring	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{xy}^{(-1)}} = \dfrac{2\omega}{b}\Big[\dfrac{(b+\lvert\boldsymbol{k}_\perp\rvert)\sqrt{(b-\lvert\boldsymbol{k}_\perp\rvert)^2+k_z^2-\omega^2}+(b-\lvert\boldsymbol{k}_\perp\rvert)\sqrt{(b+\lvert\boldsymbol{k}_\perp\rvert)^2+k_z^2-\omega^2}}{k_z^2-\omega^2-i\epsilon}\Big]$	(4.14)	2D flat band	Pole
B_{tu}	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{tu}^{(-1)}} = \dfrac{2\omega}{\sqrt{(b-\lvert\boldsymbol{k}\rvert)^2-\omega^2}} + \dfrac{2\omega}{\sqrt{(b+\lvert\boldsymbol{k}\rvert)^2-\omega^2}}$	(4.8)	Nodal shell	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{tu}^{(-1)}} = -\dfrac{2}{b}\Big[\dfrac{(b+\lvert\boldsymbol{k}\rvert)\sqrt{(b-\lvert\boldsymbol{k}\rvert)^2-\omega^2}+(b-\lvert\boldsymbol{k}\rvert)\sqrt{(b+\lvert\boldsymbol{k}\rvert)^2-\omega^2}}{\omega+i\epsilon}\Big]$	(4.9)	3D flat band	Pole
B_u	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_u^{(0)}} \equiv \mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_u^{(0)}} = \dfrac{4\omega}{\sqrt{\boldsymbol{k}^2-\omega^2}}$	(4.5)	QCP	Branch-cut
B_{ux}	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{ux}^{(-1)}} = 4\omega\dfrac{b^2+\boldsymbol{k}^2-\omega^2+f_+f_-}{f_+f_-(f_++f_-)}\ ;\ f_\pm = \sqrt{k_x^2-\Big(b\pm\sqrt{\omega^2-\boldsymbol{k}_\perp^2}\Big)^2}$	(4.12)	Filled nodal segment	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{ux}^{(-1)}} = 4\omega\dfrac{(f_++f_-)\sqrt{\omega^2-\boldsymbol{k}_\perp^2}-b(f_+-f_-)}{\sqrt{\omega^2-\boldsymbol{k}_\perp^2}\,(b^2+\boldsymbol{k}^2-\omega^2+f_+f_-)}\ ;\ f_\pm = \sqrt{k_x^2-\Big(b\pm\sqrt{\omega^2-\boldsymbol{k}_\perp^2}\Big)^2}$	(4.13)	Non-singular segment	Branch-cut & nonsingular
B_{tz}	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_{tz}^{(-1)}} = 4\omega\dfrac{b^2+\boldsymbol{k}^2-\omega^2+h_+h_-}{h_+h_-(h_++h_-)}\ ;\ h_\pm = \sqrt{\boldsymbol{k}_\perp^2-\Big(b\pm\sqrt{\omega^2-k_z^2}\Big)^2}$	(4.16)	Filled nodal ring	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_{tz}^{(-1)}} = 4\omega\dfrac{(h_++h_-)\sqrt{\omega^2-\boldsymbol{k}_\perp^2}-b(h_+-h_-)}{\sqrt{\omega^2-\boldsymbol{k}_\perp^2}\,(b^2+\boldsymbol{k}^2-\omega^2+h_+h_-)}\ ;\ h_\pm = \sqrt{\boldsymbol{k}_\perp^2-\Big(b\pm\sqrt{\omega^2-k_z^2}\Big)^2}$	(4.17)	Non-singular disk	Branch-cut & nonsingular
B_t	$\mathrm{Tr}\,\mathbb{G}^{(SS)}_{B_t^{(0)}} = 2\Big(\dfrac{b+\omega}{\sqrt{\boldsymbol{k}^2-(b+\omega)^2}} - \dfrac{b-\omega}{\sqrt{\boldsymbol{k}^2-(b-\omega)^2}}\Big)$	(4.6)	Filled nodal shell	Branch-cut
	$\mathrm{Tr}\,\mathbb{G}^{(SA)}_{B_t^{(0)}} = \dfrac{2}{b}\Big[\sqrt{\boldsymbol{k}^2-(b-\omega)^2}-\sqrt{\boldsymbol{k}^2-(b+\omega)^2}\Big]$	(4.7)	Non-singular bowl	Branch-cut & nonsingular

Order p. & Dims	*Flat bands*	*Gaps*	*Order p. & Dims*	*Nonsingular/Gapless*	*ω-shiftings/Gapless*
Φ $\mathbf{d_{eff}=0}$	SS, (figure 2) 	SA, (figure 2) 	$\mathbf{B_u}$ $\mathbf{d_{eff}=0}$	SS,SA 	SS,SA 
$\mathbf{B_x}$ $\mathbf{d_{eff}=1}$	SA, (figure 5) 	SS, (figure 5) 	$\mathbf{B_{ux}}$ $\mathbf{d_{eff}=1}$	SA, (figure 6) 	SS, (figure 6) 
$\mathbf{B_{xy}}$ $\mathbf{d_{eff}=2}$	SS, (figure 7) 	SA, (figure 7) 	$\mathbf{B_{tz}}$ $\mathbf{d_{eff}=2}$	SS, (figure 8) 	SA, (figure 8) 
$\mathbf{B_{tu}}$ $\mathbf{d_{eff}=3}$	SA, (figure 4) 	SS, (figure 4) 	$\mathbf{B_t}$ $\mathbf{d_{eff}=3}$	SA, (figure 3) 	SS, (figure 3) 

III. Entanglement and Scattering

1. 문제
2. 엔트로피 정의 및 산란 공식
3. 섭동론적 결과 및 케이스별 예시
4. 구체적 장론/양자역학 모델과 엔트로피 변화
5. 결론

1. EPR = ER, scattering amplitude and entanglement entropy change
Shigenori Seki(Hanyang U.), Sang-Jin Sin(Hanyang U.) (Apr 3, 2014)
Published in: *Phys.Lett.B* 735 (2014) 272-276 • e-Print: 1404.0794 [hep-th] 15 citations

2. Variation of Entanglement Entropy in Scattering Process
Shigenori Seki(Hanyang U.), I.Y. Park(Philander Smith Coll.), Sang-Jin Sin(Hanyang U.) (Dec 26, 2014)
Published in: *Phys.Lett.B* 743 (2015) 147-153 • e-Print: 1412.7894 [hep-th] 51 citations

문제와 기본적 정의

문제: 얽히지 않은 두 입자도 산란후엔 얽힌다.

$$|\psi\rangle = \lim_{T\to\infty} U|i\rangle = \sum_f |f\rangle\langle f|S|i\rangle = \sum_f |f\rangle S_{fi}$$

Q: 산란에 의해 생성된 얽힘의 양은 산란행렬과 어떤관계에 있는가?

기본적 정의:

(A) 얽힘 엔트로피의 정의

- 분할된 두 입자계 의 엔트로피:

$$S_E = -\operatorname{Tr}_A[\rho_A \log \rho_A]$$

(B) 산란 과정과 S-행렬

- $|\text{fin}\rangle = S|\text{ini}\rangle, \quad S = 1 + iT$

- 산란 전/후의 엔트로피 변화량: $\Delta S_E = S_E^{\text{fin}} - S_E^{\text{ini}}$

케이스별 엔트로피 변화(섭동론적 결과)

(A) 얽힘 없는 초기 상태

- 밀도행렬의 퍼터베이션 전개:
- 엔트로피 변화 leading term: $\Delta S_E \approx g^2 \log g^2 \int \mathrm{d}k \,|\, T_{kl,p_1 q_1} |^2$

(: 상호작용 결합상수)

(B) 초기 상태가 얽힘인 경우 : $|\,\mathrm{ini}\rangle = u_1 | p_1, q_1\rangle + u_2 | p_2, q_2\rangle$

- 엔트로피 변화: 상호 전이항 포함, 새로운 항이 order g에서 등장
- 최대 얽힘($u_1 = u_2$)의 경우 1차 항 소거됨

Explicit examples

(A) 스칼라 장론:

$$S = \int \mathrm{d}^{d+1}x \left[\frac{1}{2}(\partial\phi_A)^2 + \frac{1}{2}(\partial\phi_B)^2 + \frac{1}{2}m^2(\phi_A^2 + \phi_B^2) + \lambda\phi_A^2\phi_B^2\right]$$

- 산란진폭 $\mathscr{T}$과 엔트로피 증가율

  $\Delta S_E \propto |\mathscr{T}|^2 \propto \sigma_{\text{scattering}}$

- center of mass frame: $\Delta S_E \sim p_{\text{cm}}^{d-2} E_{\text{cm}}^3$

(B) 양자역학

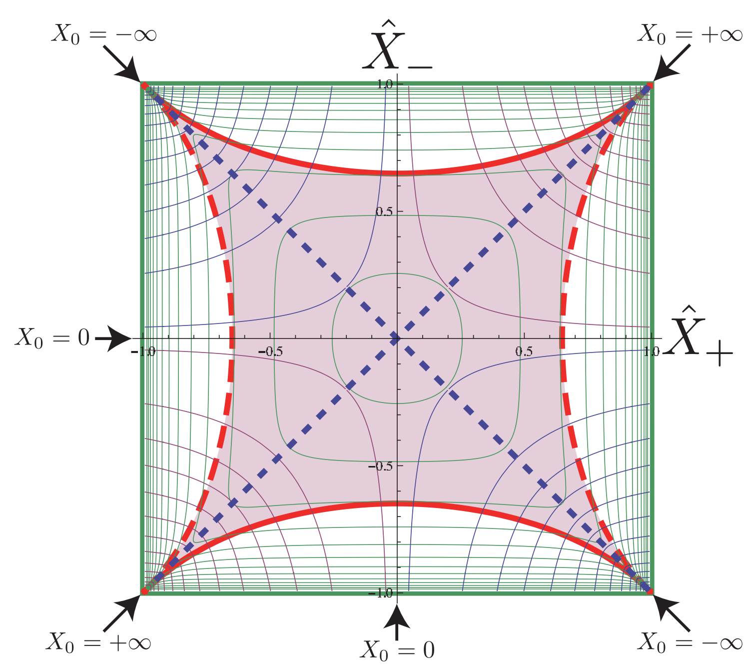
- with time-dependent Hamiltonian $H_{int}(t)$,
- $\Delta S_E \propto |\mathscr{T}|^2 \propto \sigma_{\text{scattering}}$: the same!

ads/cft 와 scattering

world sheet wormhole

- AdS/CFT 대응에서는 산란진폭 및 엔트로피가 모두 bulk minimal surface의 영역에 대응:
- world sheet has wormhole structure.

Actually this was the motivation



(a)

IV. Entanglement and spacetime sewing (Raamsdonk)

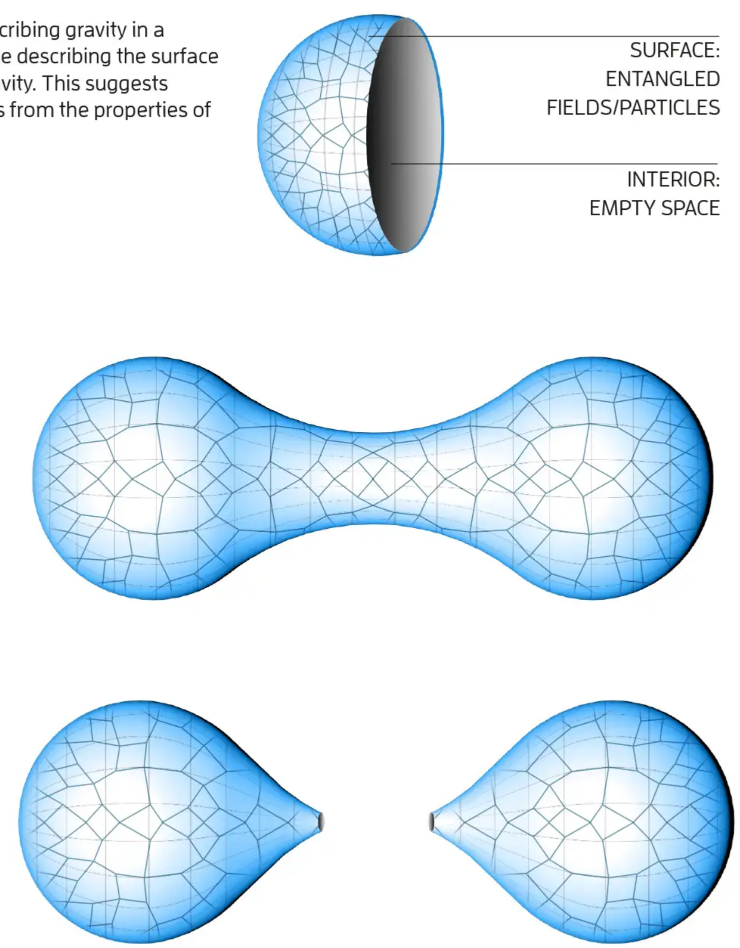
Quantum gum

Space-time might be woven from quantum entanglement

In a model universe, the equations describing gravity in a volume of space are equivalent to those describing the surface of that volume, which don't include gravity. This suggests space on the inside somehow emerges from the properties of the outside, namely entanglement

Sure enough, when you reduce the entanglement connecting two regions of the outside surface, the space inside pulls apart as if pulling at two ends of a piece of chewing gum

If you eliminate all entanglement, the space inside splits in two, suggesting that entanglement is the thread that binds space-time



V. ER=EPR
Relation to the strange metal

Conclusion

- Locality is essential for the simplification, which is lost by the entanglement.
- This difficulty can be overcome using the holographic duality.
- $\Delta S_E = k\sigma$

Conclusion

- Entanglement is still poorly understood.
- Recently, many idea has been much development.
- Entanglement is the herb of new idea.