Dynamical Friction on Circular Orbit in Self-Interacting Ultra-Light Dark Matter and Fornax Globular Clusters

Hyeonmo Koo[1], Jae-Weon Lee[2]

[1]Department of Physics, University of Seoul, Seoul 02504, KOREA
[2]Department of Electrical and Electronic Engineering, Jungwon University

Abstract

Considering the self-interaction of bosonic scalar Ultra-Light Dark Matter (ULDM) relaxes the constraints on its particle mass, typically estimated to be around $\sim 10^{-22}\mathrm{eV}$. We calculate the dynamical friction (DF) coefficient for a steady-state circular orbit induced by the gravitational wake of ULDM using multipole expansion. In the strong self-coupling regime, the leading-order term for the Mach number, only arising in the dipole term ($l = 1$), closely matches the full-order calculation. Applying these findings to the Fornax dwarf spheroidal galaxy (dSph) and its globular clusters (GCs), and assuming a DM halo density profile in the Thomas-Fermi limit, we provide a plausible explanation for the extended lifetime of Fornax GCs. Furthermore, considering self-interaction can reduce the DF force, and further increases their lifetime.

Self-Interacting ULDM

$$S = S_{\mathrm{EH}} + S_\phi = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}m_a^2\phi^2 - \frac{\lambda}{4}\phi^4\right]$$

$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi)\delta_{ij}dx^i dx^j$$
$$\phi(\vec{x},t) = \frac{\hbar}{\sqrt{2m}}\left[\psi(\vec{x},t)e^{-im_ac^2t/\hbar} + \mathrm{c.c.}\right]$$

Gross-Pitaevskii-Poisson (GPP) System

$$\begin{cases} -\dfrac{\hbar^2}{2m_a}\nabla^2\psi + m_a\Phi_0\psi + \dfrac{\hbar^3\lambda}{2m_a^2c}|\psi|^2\psi = i\hbar\dfrac{\partial\psi}{\partial t} \\ \nabla^2\Phi_0 = 4\pi G m_a|\psi|^2 \end{cases}$$

- Madelung formalism $\begin{cases}\psi(\vec{x},t) = \sqrt{\rho(\vec{x},t)/m_a}\,e^{i\theta(\vec{x},t)} \\ \vec{v}(\vec{x},t) = (\hbar/m_a)\nabla\theta(\vec{x},t)\end{cases}$ introduces two pressure terms on ULDM fluid.

Self-Interacting Pressure $P = \frac{\lambda\hbar^3}{4m_a^4c}\rho^2$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0\,,\qquad \frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} + \nabla\Phi + \frac{1}{\rho}\nabla P + \nabla Q = 0,$$

Quantum Pressure $Q = -\frac{\hbar^2}{2m_a^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}$

DF Force from Linear Perturbation

- Linear density perturbation $\alpha(\vec{x},t) \equiv \rho(\vec{x},t)\bar{\rho} - 1$ by perturber's potential $\Phi_{\mathrm{P}}(\vec{x},t)$ is represented by Green function method.

$$\alpha(\vec{x},t) = \int d^3\vec{x}'dt'G(\vec{x}-\vec{x}',t-t')\nabla^2\Phi_{\mathrm{P}}(\vec{x}',t')$$

$$G(\vec{R},\tau) = \int\frac{d^3\vec{k}}{(2\pi)^3}\frac{d\omega}{2\pi}\frac{\exp\left[i\left(\vec{k}\cdot\vec{R}-\omega\tau\right)\right]}{\frac{\hbar^2k^4}{4m_a^2} + c_s^2k^2 - \omega^2}$$

Sound Speed of ULDM Fluid $c_s^2 = \frac{\lambda\hbar^3}{2m_a^4c}\bar{\rho}$

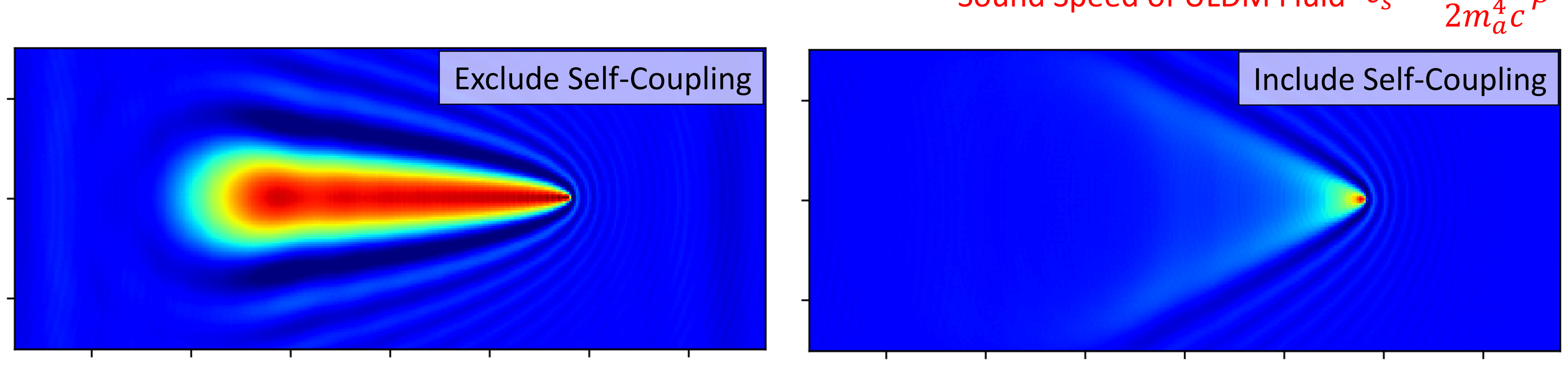
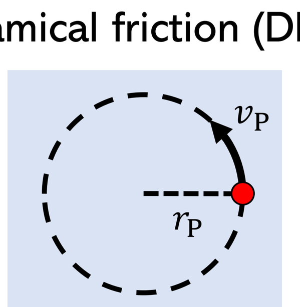


Figure 1 Density perturbation $\alpha(\vec{x},t)$ woken by linear motion of point particle $\vec{x}_{\mathrm{P}}(t) = (v_0t)\hat{x}$ is numerically simulated by using pseudo-spectral method. *[N.Glennon et al. 2312.07684]*

- For circularly moving point particle inside ULDM fluid with Mach number $\mathcal{M} = v_{\mathrm{P}}/c_s$, the dynamical friction (DF) force $\vec{F}_{\mathrm{DF}}(t) = \bar{\rho}\int d^3\vec{x}(\nabla\Phi_{\mathrm{P}})\alpha(\vec{x},t)$ is given by



$$\vec{F}_{\mathrm{DF}}(t) = -4\pi\rho\left(\frac{GM_{\mathrm{P}}}{v_{\mathrm{P}}}\right)^2\left[\Re(\mathcal{I})\hat{r}(t) + \Im(\mathcal{I})\hat{\phi}(t)\right]$$

Effective DF Coefficient C_{DF}

$$\mathcal{I} = \mathcal{M}^2\sum_{l=1}^{\infty}\sum_{m=-l}^{l-2}(-1)^{m+1}\frac{(l-m)!}{(l-m-2)!}\frac{S_{l,l-1}^m - S_{l,l-1}^{-m-1}}{\Gamma\left(\frac{1-l-m}{2}\right)\Gamma\left(\frac{3-l+m}{2}\right)\Gamma\left(\frac{2+l-m}{2}\right)\Gamma\left(\frac{2+l-m}{2}\right)}$$

$$S_{l,l-1}^m = \begin{cases} \dfrac{\pi i}{2\sqrt{1+m^2/l_q^2}}\left[j_l(l_q\mathcal{M}f_m^-)h_{l-1}^{(1)}(l_q\mathcal{M}f_m^-) - j_l(il_q\mathcal{M}f_m^+)h_{l-1}^{(1)}(il_q\mathcal{M}f_m^+)\right] & (m>0) \\ \dfrac{-\pi i}{2\sqrt{1+m^2/l_q^2}}\left[j_l(l_q\mathcal{M}f_m^-)h_{l-1}^{(2)}(l_q\mathcal{M}f_m^-) + j_l(il_q\mathcal{M}f_m^+)h_{l-1}^{(1)}(il_q\mathcal{M}f_m^+)\right] & (m<0) \\ \dfrac{\pi}{2(4l^2-1)} - \dfrac{\pi i}{2}j_l(2il_q\mathcal{M})h_{l-1}^{(1)}(2il_q\mathcal{M}) & (m=0) \end{cases}$$

where $l_q\mathcal{M} = m_ac_sr_{\mathrm{P}}/\hbar$ and $f_m^{\pm 2} = 2\sqrt{1+m^2/l_q^2} \pm 2$. *[V.M.Gorkavenko et al. 2408.00104]*

Leading-Order Approximation

- The leading-order term of DF coefficient only appears in $l = 1$.

$$\overline{\Im(\mathcal{I})} = -\frac{2}{\pi}\mathcal{M}^2\Im\left(S_{1,0}^{-1} - S_{1,0}^{0}\right) = \frac{1}{3}\mathcal{M}^3 + O(\mathcal{M}^5)$$

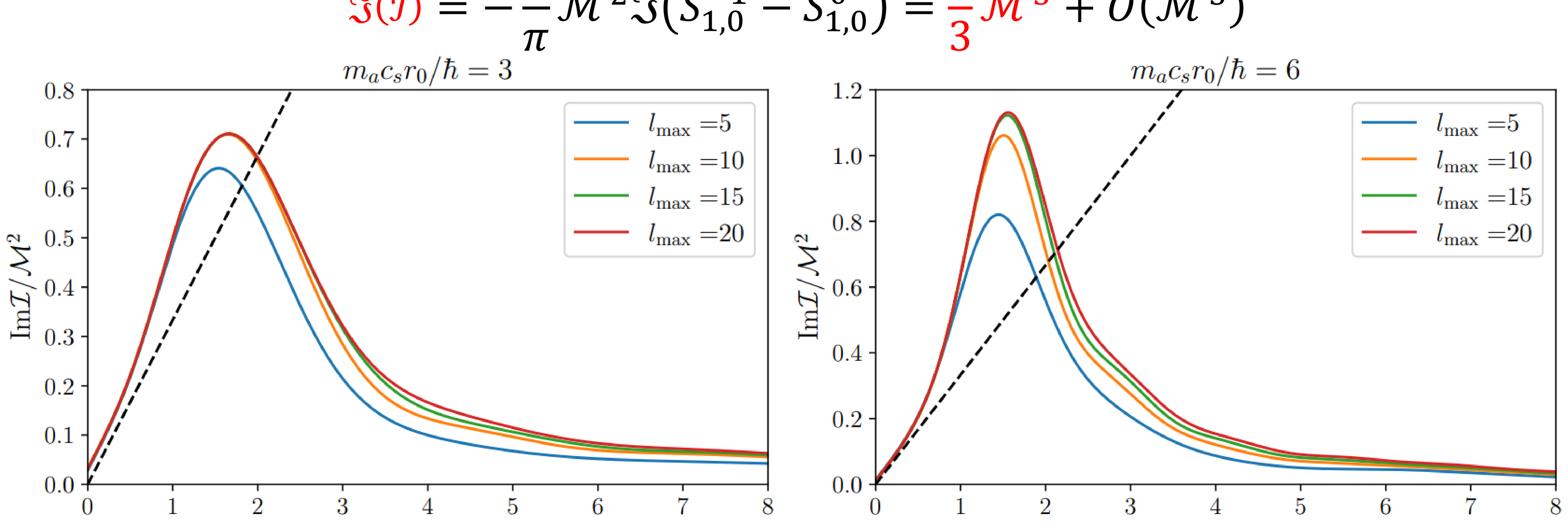


Figure 2 DF coefficient divided by square of Mach number, $\Im(\mathcal{I})/\mathcal{M}^2$ for maximum multipole $l_{\max} = 5\sim20$ and $m_ac_sr_{\mathrm{P}}/\hbar$. Dashed black line represents the leading-order approximation of $\overline{\Im(\mathcal{I})}/\mathcal{M}^2 \cong \frac{1}{3}\mathcal{M}$, which is only valid in the "subsonic" regime $\mathcal{M} < 1$.

Application to Fornax dSph & GCs

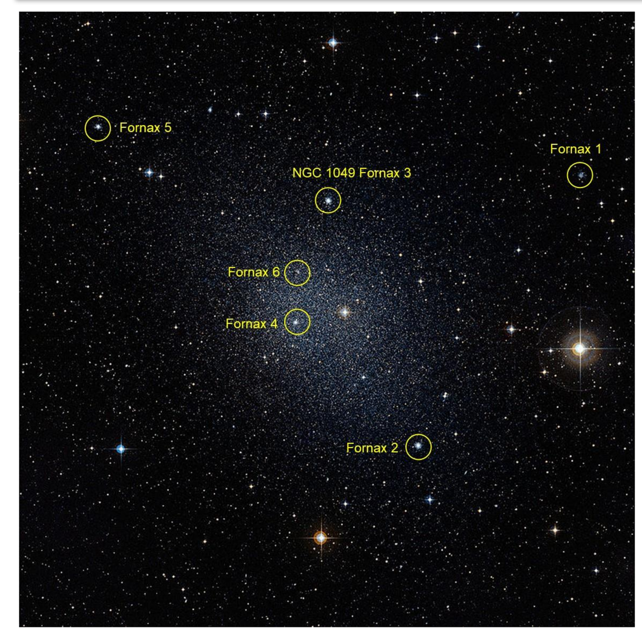


Figure 3 Digitized Sky Survey 2 image of Fornax dwarf spheroidal galaxy and its globular clusters.

- The observed globular clusters (GCs) orbiting around Fornax dwarf spheroidal (dSph) have individual "Lifetime".

$$\tau_{\mathrm{fall}} = \frac{M_{\mathrm{P}}v_{\mathrm{P}}}{F_{\mathrm{DF}}} = \frac{v_{\mathrm{P}}^3}{4\pi\rho G^2M_{\mathrm{P}}C_{\mathrm{DF}}}$$

Observational Result
$\tau_{\mathrm{fall}}\sim10\mathrm{Gyr}$ *[M.-Y.Yang et al.1809.07801]*

CDM-based Prediction
$\tau_{\mathrm{fall}}\sim1\mathrm{Gyr}$ *[K.S.Oh et al. ApJ 531 (2000) 727]*

"Timing Problem of Fornax GCs"
[S.D.Tremaine, ApJ 203 (1976) 345]

Object	dSph	GC1	GC2	GC3	GC4	GC5
$M_P\ [10^5M_\odot]$	1420	0.37	1.82	3.63	1.32	1.78
R_P [kpc]	-	1.6	1.05	0.43	0.24	1.43

Table 1 Data for the Fornax system: mass and radial distance of GCs. *[D.R.Cole et al. 1205.6327]*

- Thomas-Fermi (TF) limit, neglecting quantum pressure for fluid eq. is used for modeling DM halo of Fornax dSph.

$$\rho(r) = \frac{\pi M_{\mathrm{halo}}}{4R_{\mathrm{TF}}^3}\frac{\sin(\pi r/R_{\mathrm{TF}})}{(\pi r/R_{\mathrm{TF}})} \Rightarrow R_{\mathrm{TF}} = \sqrt{\frac{\pi\hbar^3\lambda}{8Gm_a^4c}}$$

- Scaling relation of fluid eq. $\Rightarrow$ TF-limit is valid in $R_{\mathrm{TF}} \gg R_Q \equiv \frac{\hbar^2}{GM_{\mathrm{halo}}m_a^2}$.

$$\lambda \gg 1.513\times10^{-92}\left(\frac{1.42\times10^8M_\odot}{M_{\mathrm{halo}}}\right)^2$$

[P-H.Chavanis 1103.2050]

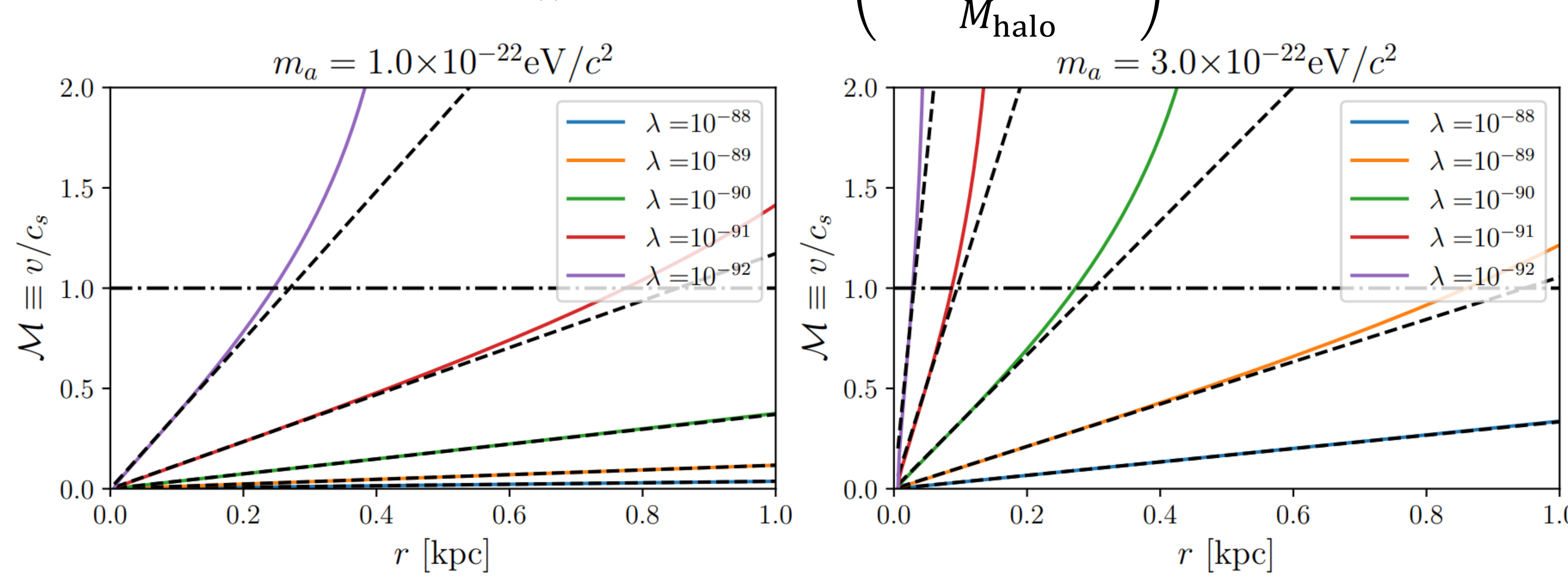


Figure 4 The Mach number $\mathcal{M}$ as a function of the radial distance r. For sufficiently large λ and small m_a, $\mathcal{M} < 1$ is satisfied at r corresponding to the radial distance of GC3/4 from dSph.

Parameter Space

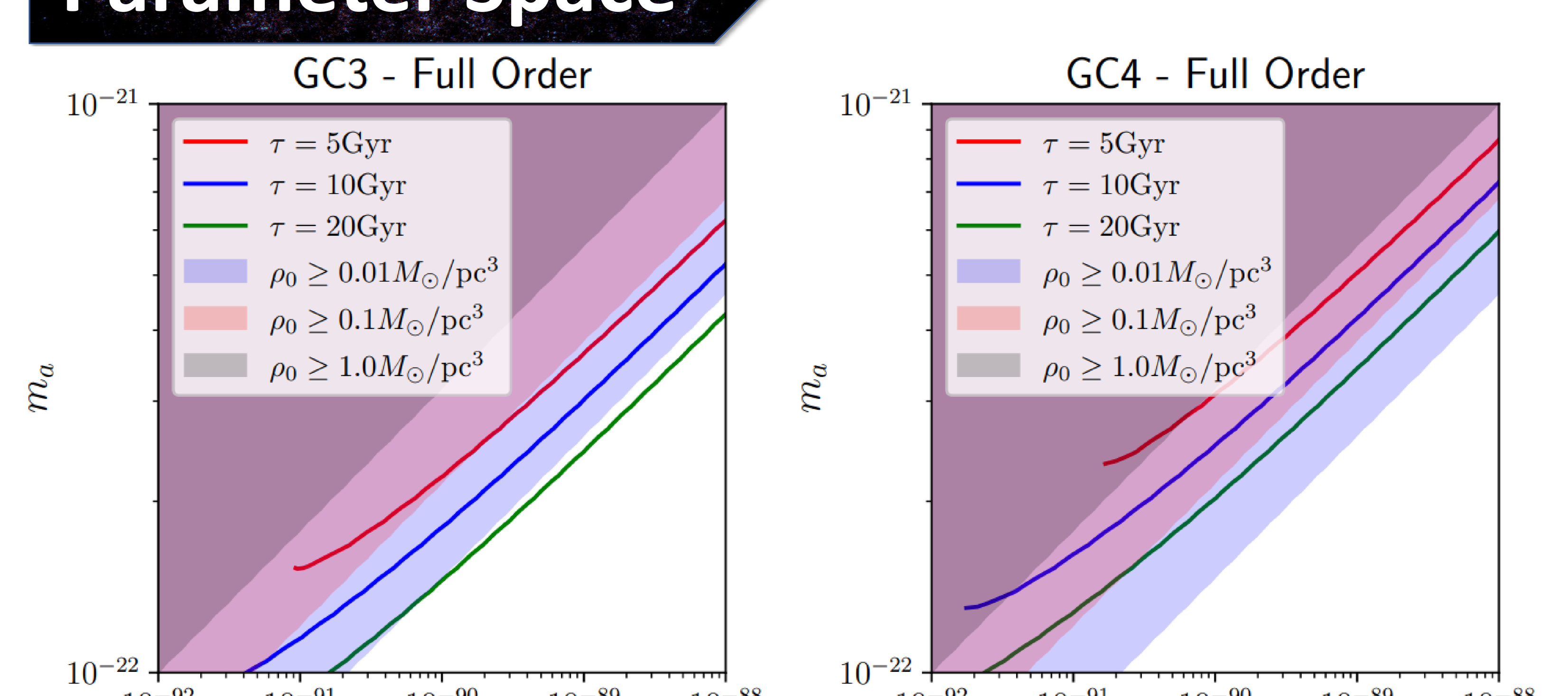


Figure 5 Parameter space in ULDM particle mass m_a vs self-coupling constant λ for various lifetimes τ_{fall} of GC3/4 and DM halo central density ρ_0 of TF-limit-profile Fornax dSph. Maximum multipole $l_{\max} = 20$ is used.

- Instead of $\Im(\mathcal{I})$, leading-order DF coefficient $\overline{\Im(\mathcal{I})}$ is useful for $\mathcal{M} < 1$ to simply calculate the lifetime.

$$\overline{\tau_{\mathrm{fall}}} = \frac{3c_s^3}{4\pi\rho G^2M_{\mathrm{P}}}$$

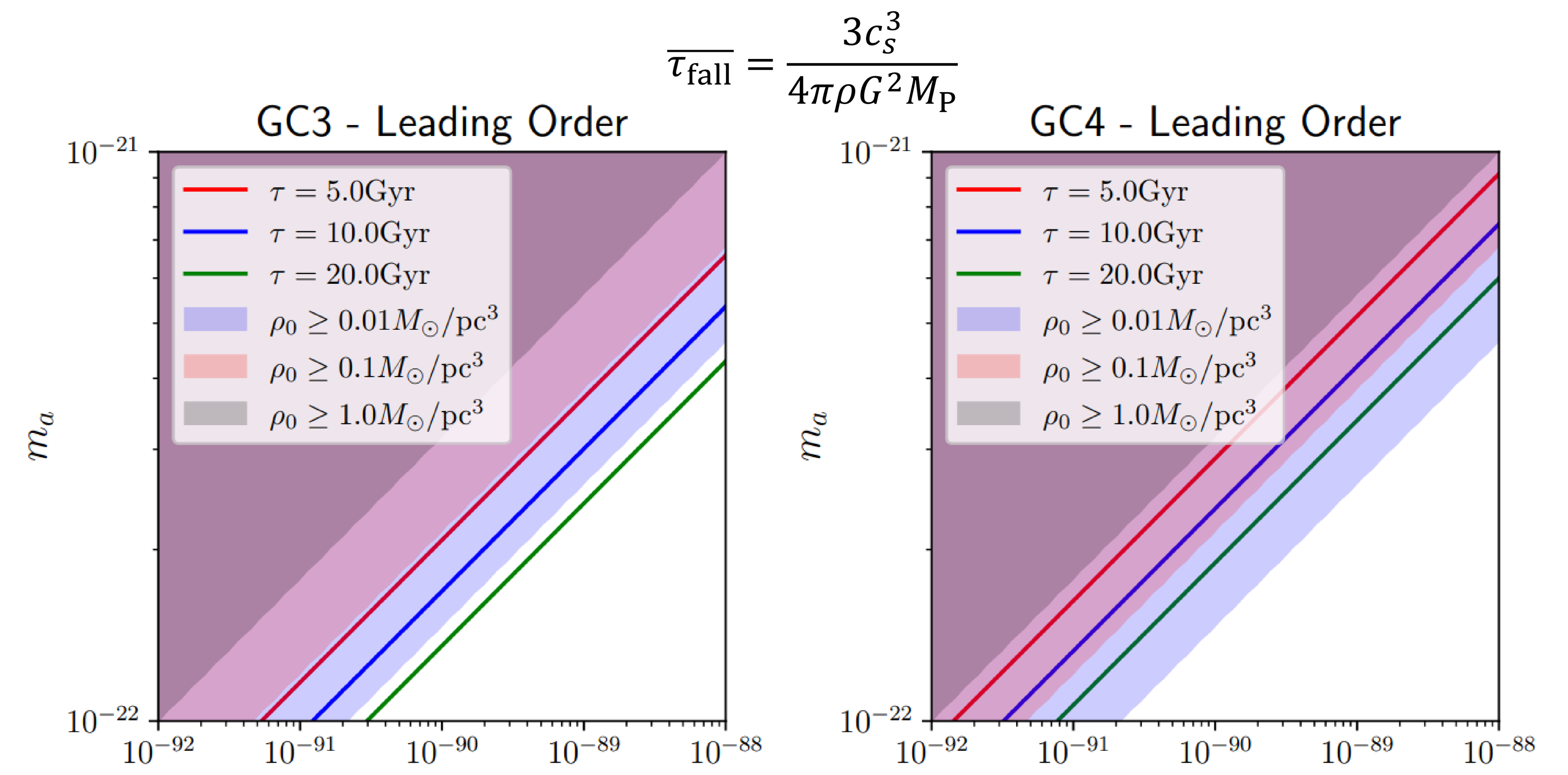


Figure 6 Parameter space of leading-order approximated lifetimes $\overline{\tau_{\mathrm{fall}}}$. These results match well with the calculation of $\Im(\mathcal{I})$ with $l_{\max} = 20$ for $\lambda \geq 10^{-90}$.

- Assuming the Fornax dSph central density has a range of $10^{-2} \leq \rho_0[M_\odot/\mathrm{pc}^3] \leq 10^{-1}$, m_a and λ are simultaneously constrained as:

$$4.680\mathrm{eV} < \frac{m_a}{\lambda^{1/4}} < 6.869\mathrm{eV}$$

Object	CDM	ULDM, SI X	ULDM, SI O
GC3	0.62	2.2	4.27~15.23
GC4	0.37	10	13.06~42.79

Table 2 Lifetimes[Gyr] of GC3/4 for CDM, ULDM without/with self-interaction. Left two columns are taken from *[L.Hui et al. 1610.08297]* with $m_a = 3\times10^{-22}\mathrm{eV}$. From above constrain, λ has a range of $[3.638\times10^{-90}, 1.689\times10^{-89}]$, so that determines $\overline{\tau_{\mathrm{fall}}}$ at the last column.

❖ Considering self-interaction in ULDM could extend lifetimes of Fornax GCs predicted by CDM, much more than ULDM without self-interaction.