Low-acceleration gravitational anomaly and its implications

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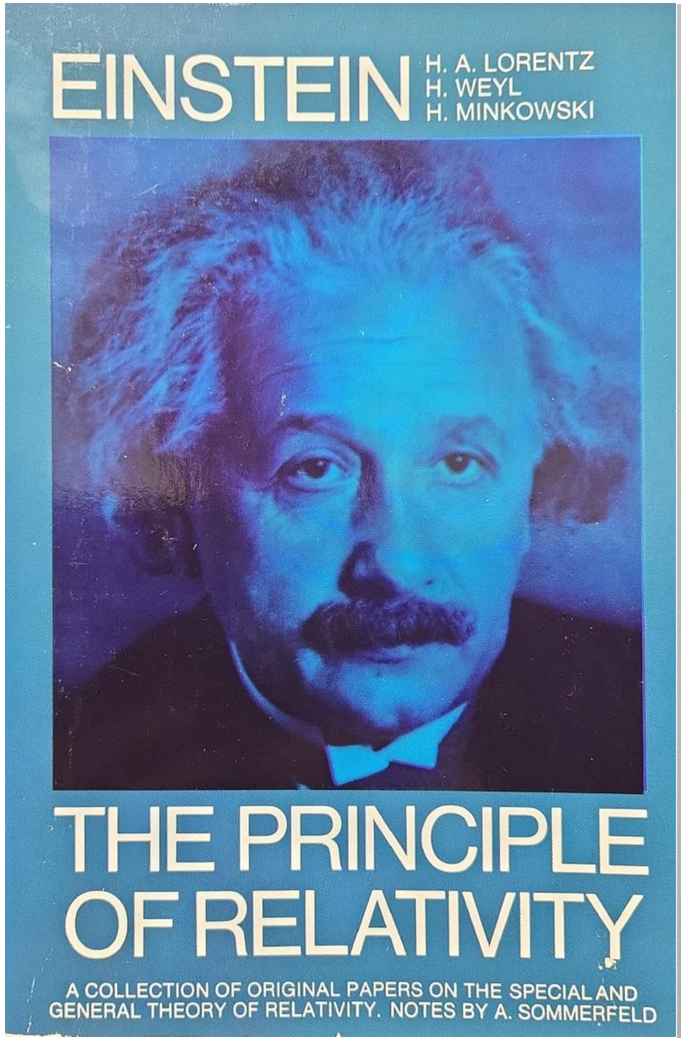
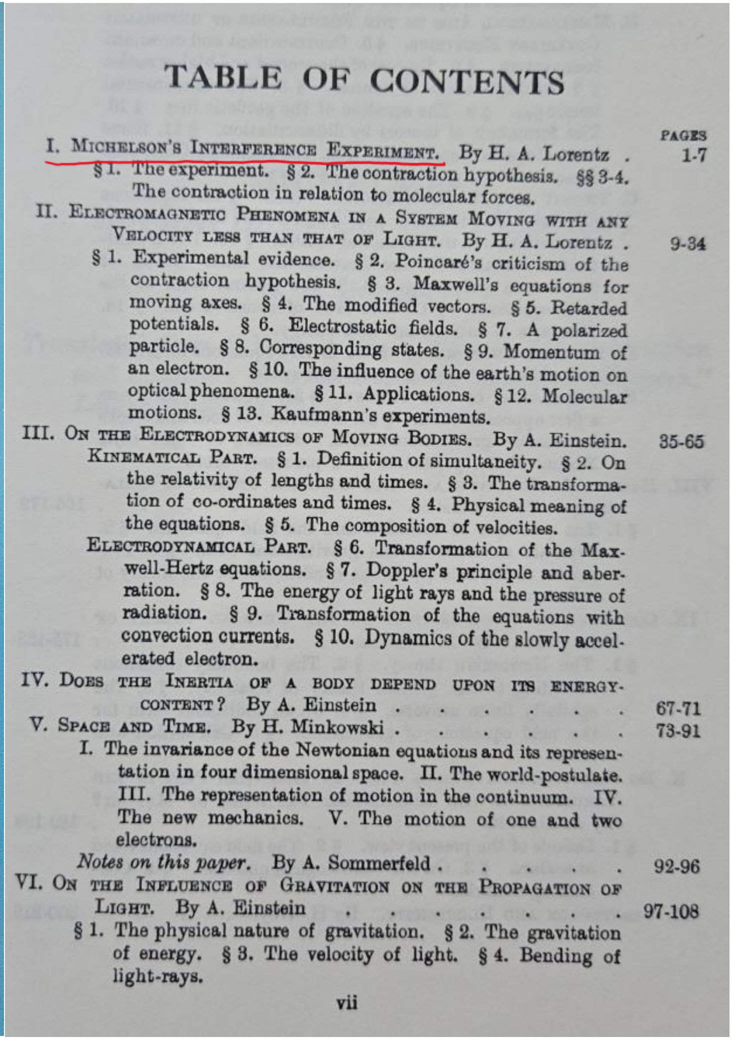
- A perspective on gravity and the dark matter problem
- A pedagogical introduction to wide binary gravity tests
- Gaia DR3 and other data
- Statistical tests with sky-projected (tangential) relative velocities between the stars
- A Bayesian orbit modeling with 3D relative velocities
- Implications
- Conclusions and prospects

A historical perspective: importance of ***one*** solid evidence

- **Revolutions** or **major developments** in physics were triggered by **solid pieces** of experimental/observational **evidence**.
- Kepler’s laws ⇒ Newton’s law of gravity
- Le Verrier’s discovery of the anomaly in the orbital motion of Mercury ⇒ Einstein’s general relativity
- **Michelson & Morley’s experiment ⇒ special relativity**
- Ultraviolet catastrophe of the blackbody radiation ⇒ Planck’s quantum theory
- Penzias & Wilson’s discovery of the cosmic microwave background radiation ⇒ Big Bang cosmology
- Galilei’s observation of Venus’s phases ⇒ The heliocentric model of the solar system
- etc





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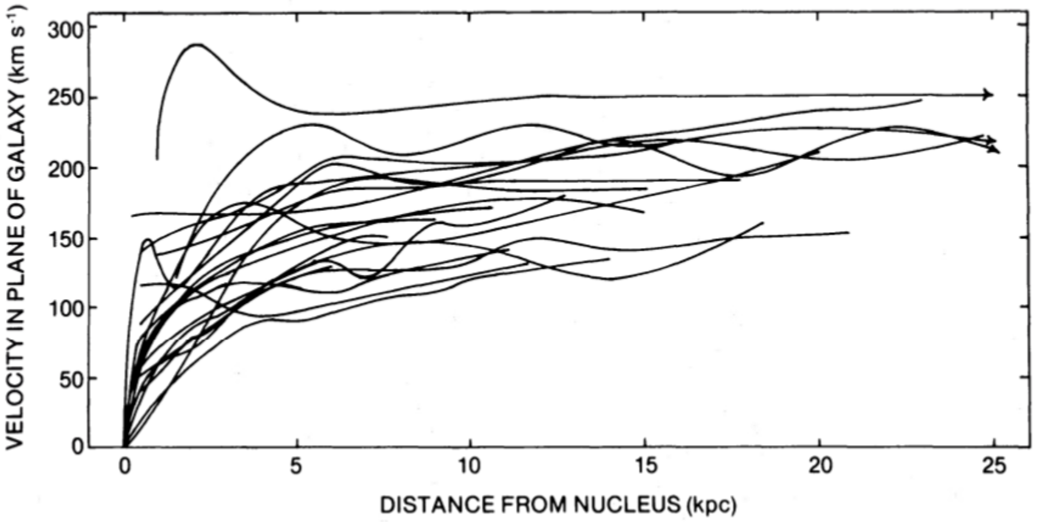
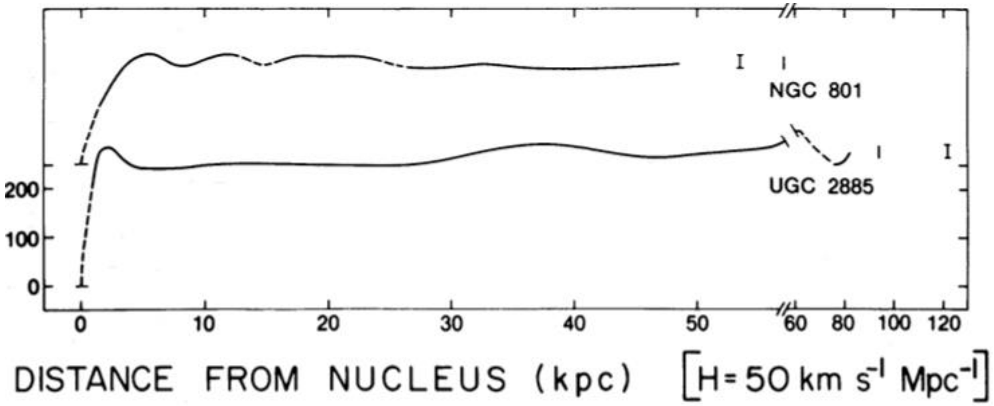
- **Real evidence** overrides (or doesn't have to respect) existing "established" theories.

- When Kepler's empirical laws were uncovered from Tycho Brahe's data, there didn't even exist physical laws (or modern physics at all).
- The observed gravitational anomaly in the motion of the perihelion of Mercury's orbit violated two-centuries respected Newton's universal law of gravity.
- The observed blackbody radiation curve violated the "perfect" Maxwell's theory of electrodynamics completed just a few decades ago at that time.
- etc

Gravitational anomalies in the 20th century after general relativity (GR) was discovered by Einstein

- For nonrelativistic gravitational phenomena, both Newtonian gravity and GR are described by Poisson's equation ("standard gravity" in the nonrelativistic regime) $\nabla^2\Phi = 4\pi G\rho$.
- In 1933, F. Zwicky discovered that motions of galaxies violated standard gravity in the Coma cluster.
- In 1970, Rubin & Ford measured a nearly flat rotation curve up to $r = 24$kpc for the Andromeda galaxy.
- In 1980, Rubin, Ford, & Thonnard reported nearly flat rotation curves for 21 Sc-type spiral galaxies.
- In 1978(Ph.D. thesis) and 1981, A. Bosma reported nearly flat rotation curves of neutral hydrogen.





Rubin, Ford, Thonnard
(1980, Astrophysical Journal)

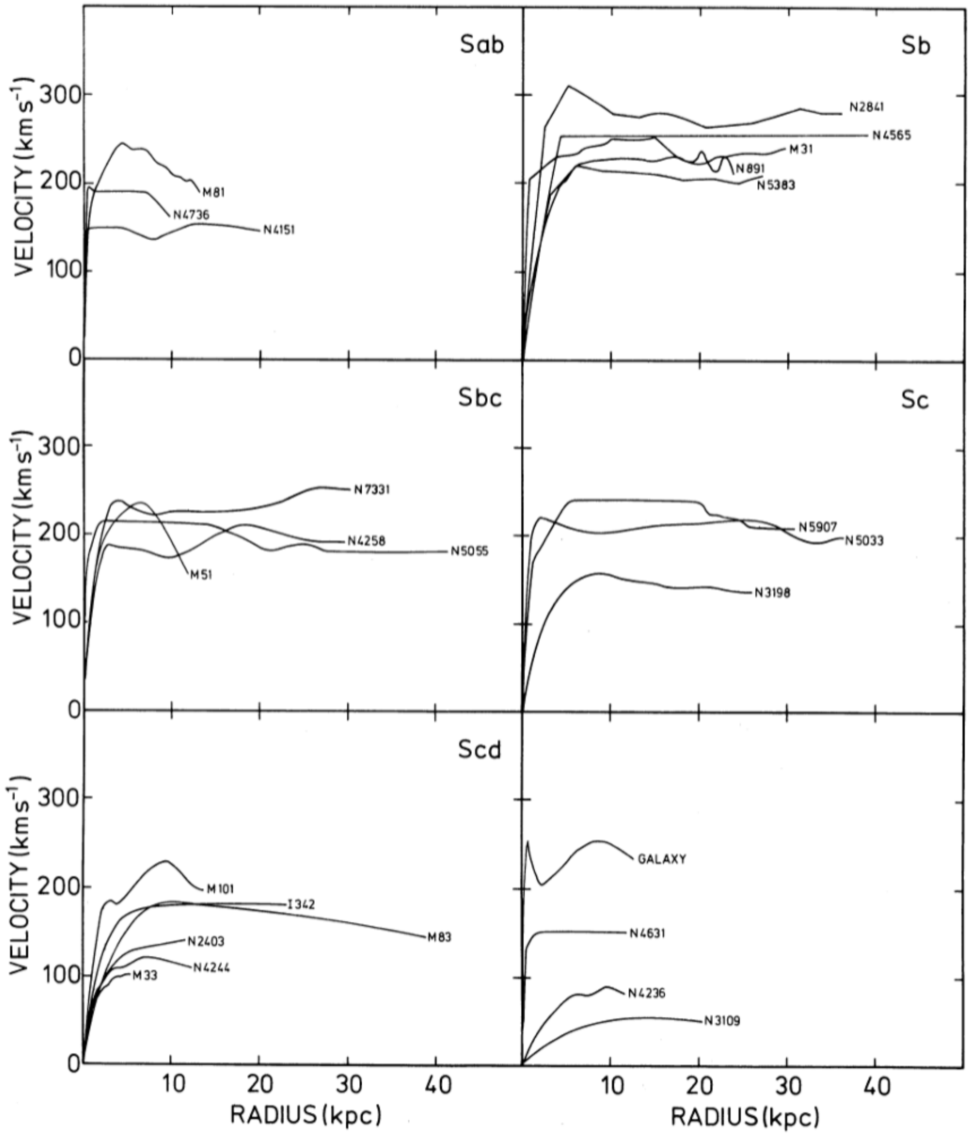
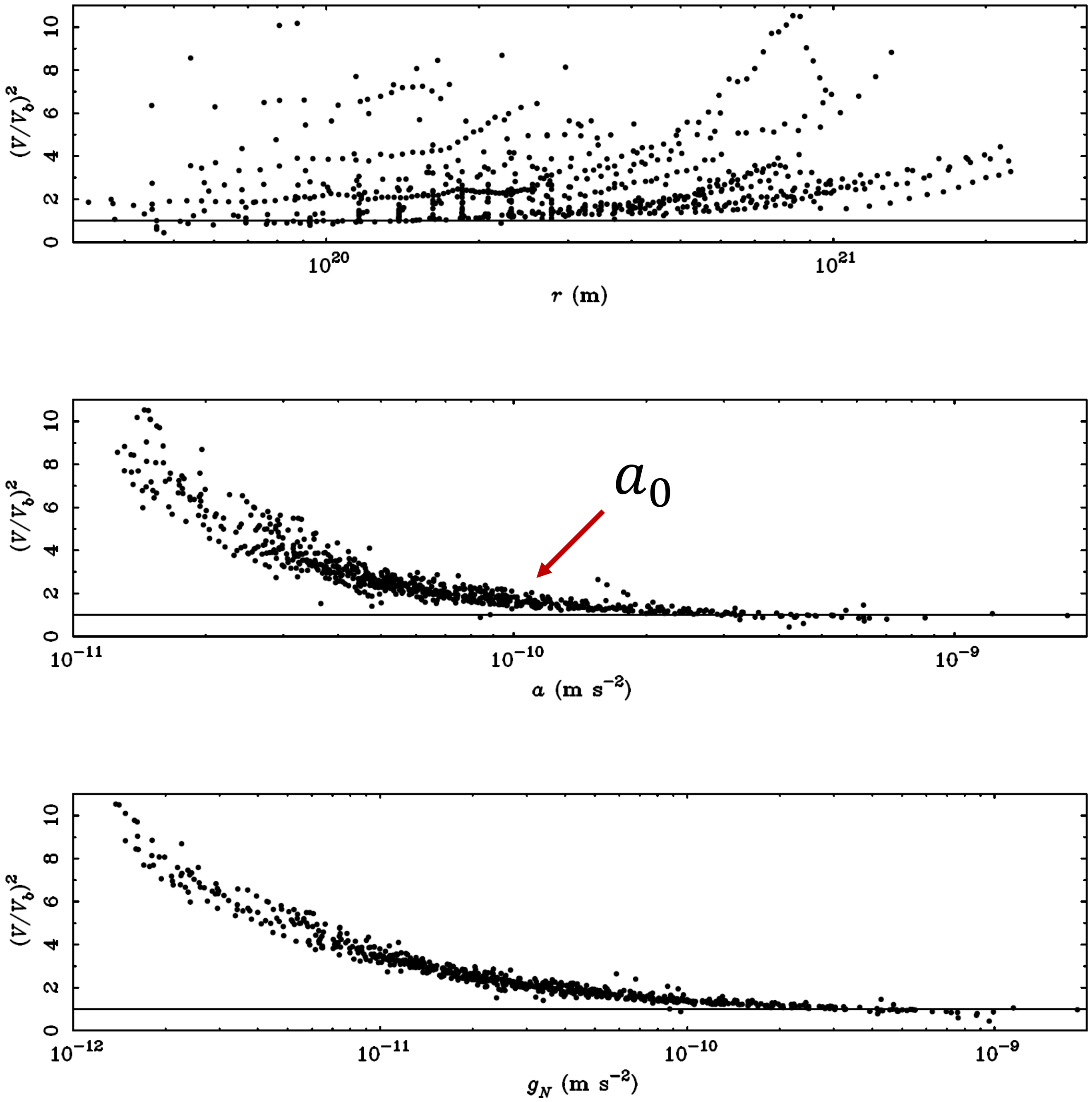


FIG. 3. Rotation curves of 25 galaxies of various Hubble types.

A. Bosma
(1981, Astronomical Journal)

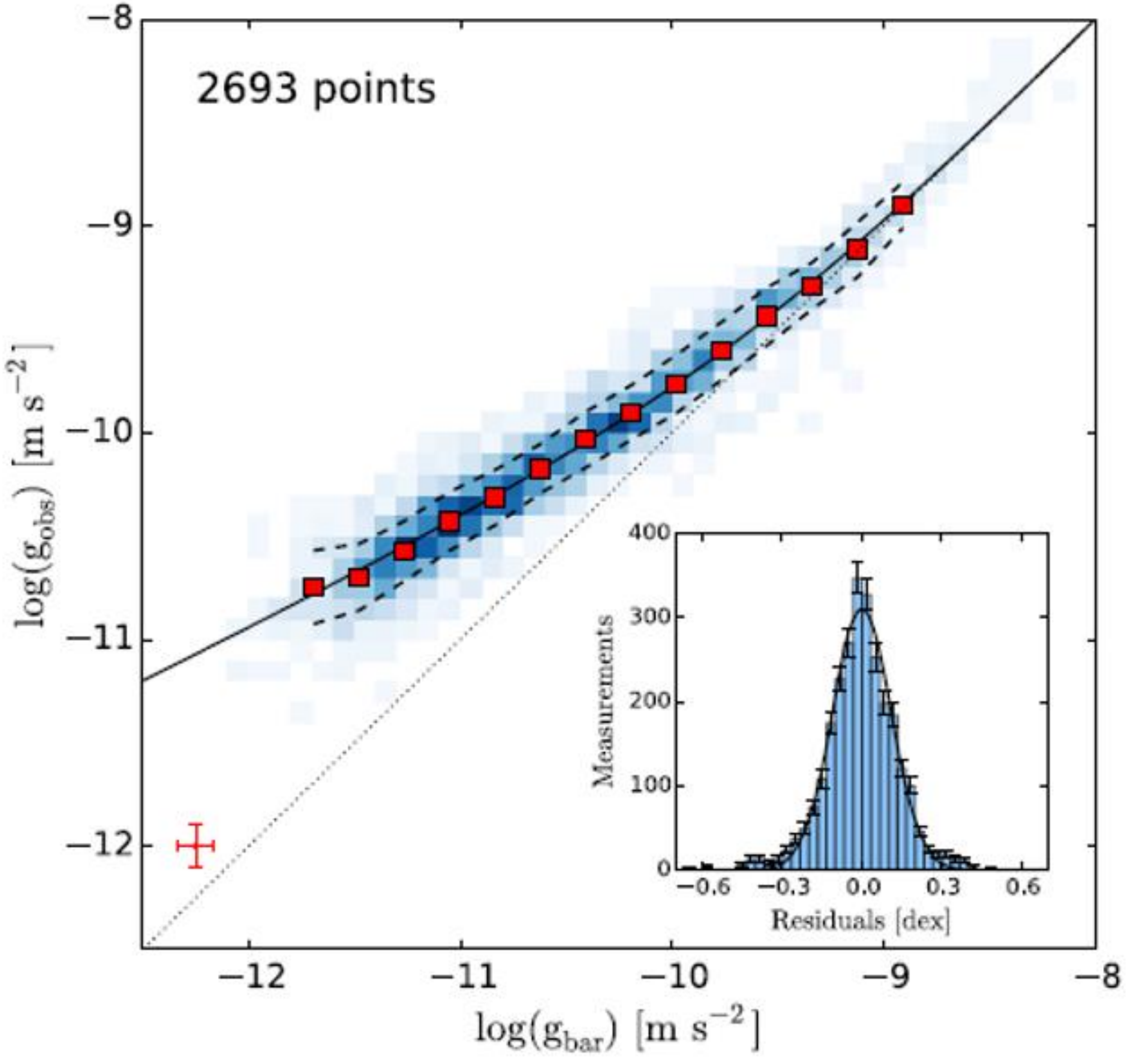
A summary of galactic rotation curves: *gravitational anomalies in galaxies occur at* ***low accelerations***

(see S. McGaugh 2004, also a review article by Famaey & McGaugh 2012).



A more accurate description:
radial acceleration relation in rotationally-supported galaxies

McGaugh, Lelli, & Schombert (2016, PRL, 117, 201101)



The surge of mentions of the word "dark matter" around 1980:
Dark matter detection experiments start (to be conceived) in 1980s!

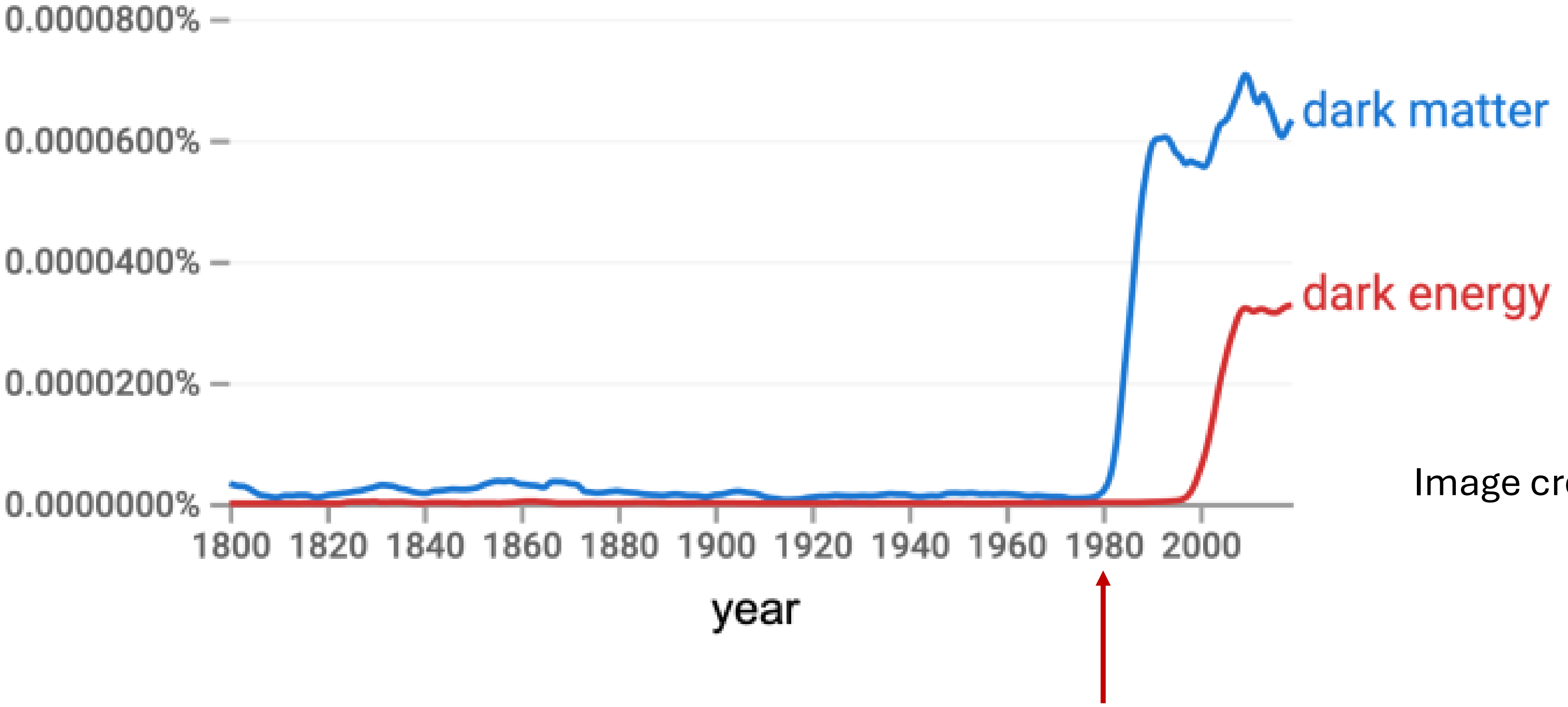


Image credit: J. M. Jee

Rotation curves by Rubin et al. and Bosma

Supporting arguments for dark matter from astronomy and astrophysics (assuming standard gravity)

- Ostriker & Peebles (1978): For the galactic disk to maintain a stable equilibrium, dark matter halo surrounding the disk is required.
- Galaxy formation and evolution under the standard Big Bang cosmology requires dark matter (note however the recent discoveries of large and massive galaxies in the early universe discovered by JWST).
- The observed distribution of the large-scale structure of galaxies requires dark matter in the standard structure formation models.
- Gravitational lensing by galaxies and galaxy clusters (with GR) requires dark matter.

Some thoughts

- Is dark matter an unavoidable logical necessity of a verified framework like top quark or the Higgs particle of the standard model of particle physics?

- *Ether was once considered a logical necessity of Maxwell's theory (plus then standard views of physics) but disproved by Michelson & Morley's experiment (Michelson was the 1st American scientist to win a Nobel prize!)*
- *Dark matter detection experiments are **valuable** whether they **detect or help to disprove** dark matter.*

- All arguments for dark matter are based on the assumption that general relativity is perfect as a classical theory of gravitational dynamics like Maxwell's theory of electrodynamics.

- ***Poisson's equation was never directly proven in the low acceleration limit** by experiments or observations.*

- In early 1980s, the same rotation curves by Rubin et al. and Bosma led **Mordehai (“Moti”) Milgrom** (a researcher at Institute for Advanced Study at that time) to conceive a **modification** of standard **gravitational dynamics**, now referred to as **modified Newtonian dynamics (MOND)** or Milgromian dynamics.

- *Milgrom posits that even non-relativistic Newtonian gravity requires modification, through either **modified Poisson’s equation** (or modified gravity) or **modified inertia**.*

- *Milgrom suggests that the **strong equivalence principle is broken** (while retaining the experimentally verified universality of free-fall), and the internal gravitational dynamics of a self-gravitating system suffers from an **external field effect** when it is falling freely under a constant external field (e.g. a binary star system freely orbiting under the gravitational field of the Milky Way).*

- Even today, we encounter conflicting news reports and “research” results supporting dark matter or alternatives.

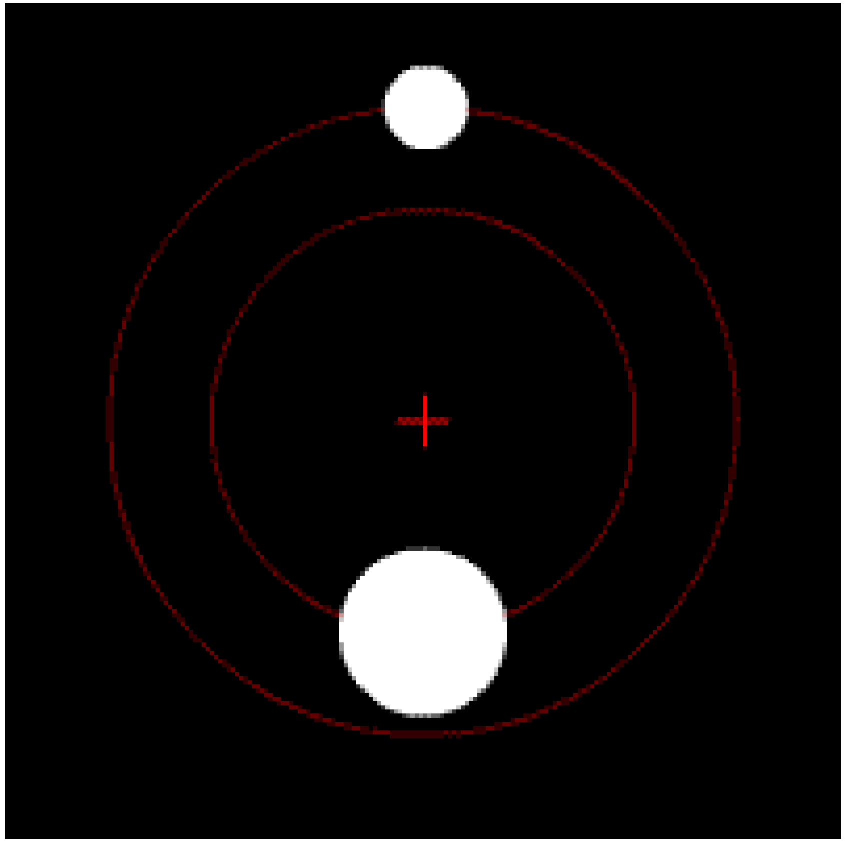
- *We are desperately in need of* ***experimental/observational facts****, not just arguments or circumstantial evidence.*
- *One route is to* ***detect/identify*** *dark matter particles or* ***exclude*** *theoretical candidates. But, how many candidates should we test? When existing candidates are excluded, one can, in principle, keep inventing more and more theoretical candidates.*
- *Another route is to* ***test gravity directly****.*

Wide binaries as a natural laboratory to test weak gravity directly

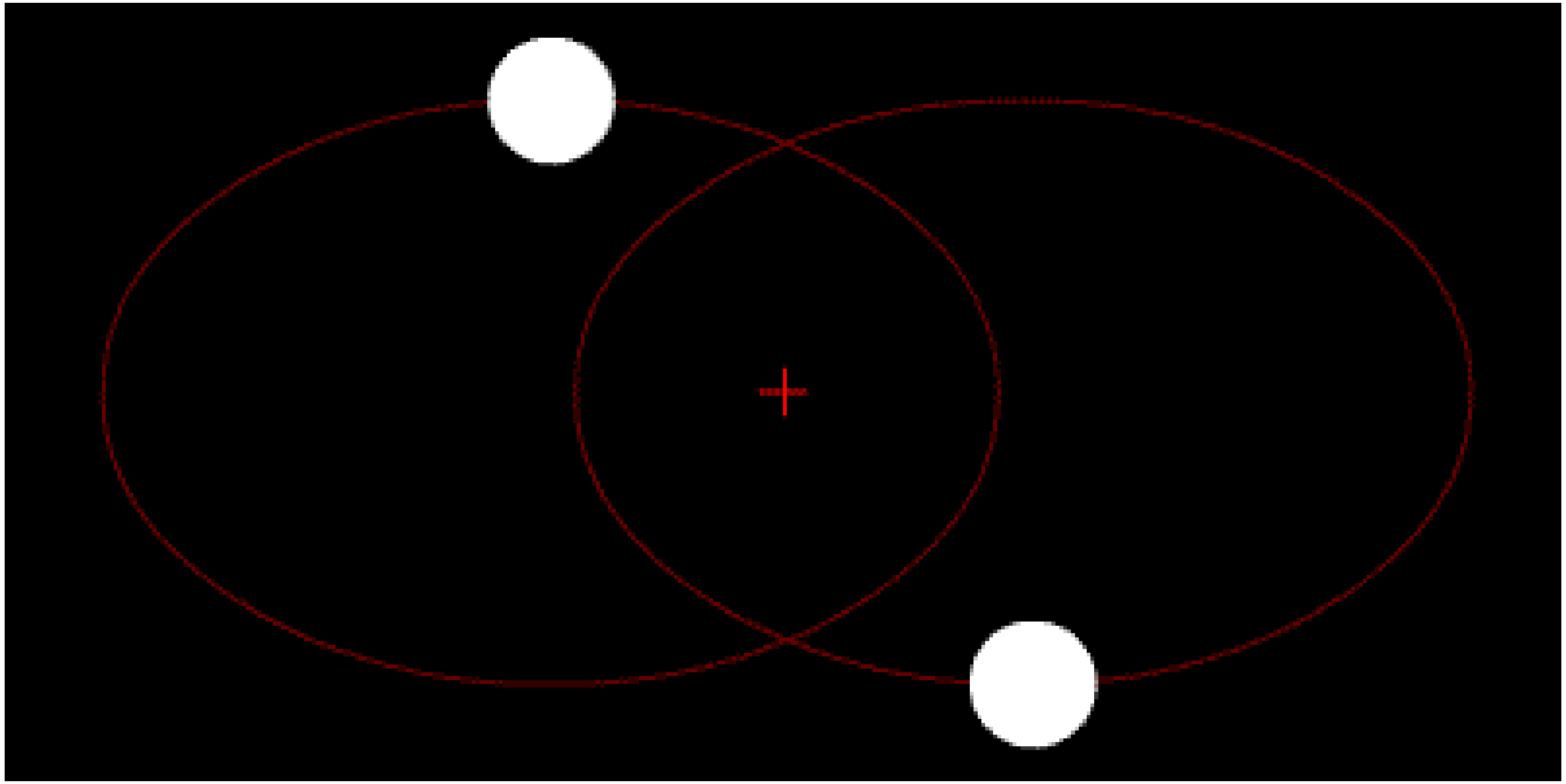
- Isolated binary systems of stars can be used to probe gravity as a function of separation.
- The dark matter mass within the space between the pair would be negligibly small even if it existed based on the Milky Way observed properties.
- When the separation between the pair is $\gtrsim 2$ kau (kilo astronomical units) for total masses in the range 1 - 2 $M_{\odot}$, the internal Newtonian gravitational acceleration gets weaker than ~ 1 nm per second squared $(10^{-9}\ \mathrm{m}\ s^{-2})$.
- First attempts were made by Hernandez et al. (2012) based on rather imprecise Hipparcos data.
- The release of Gaia DR3 allows precision tests, but more to expect in DR4 and DR5 (the final release).

Orbital motions of binary stars

(image credit: wikipedia)

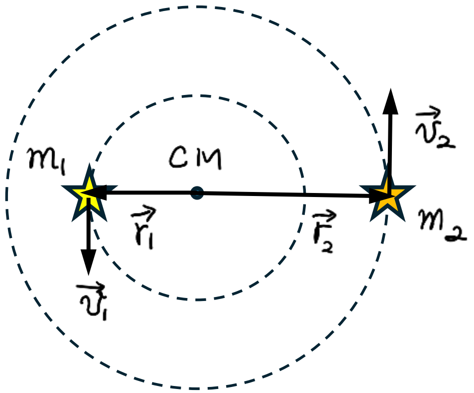


circular orbits: different masses



elliptical orbits: equal masses

A pedagogical analysis: a binary with circular orbits



In the CM frame

$$m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0$$

$$\vec{v}_2 = -\frac{m_1}{m_2} \vec{v}_1$$

relative displacement & velocity

$$\vec{r} \equiv \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\vec{v} \equiv \vec{v}_2 - \vec{v}_1$$

Equations of motion :
$$\begin{cases} \dfrac{m_1 v_1^2}{r_1} = G\dfrac{m_2 m_1}{r_{21}^2} = F_{21} \\ \dfrac{m_2 v_2^2}{r_2} = G\dfrac{m_1 m_2}{r_{12}^2} = F_{12} = F_{21} \end{cases}$$

$$m_1 v_1^2 = F_{12} r_1 \quad \& \quad m_2 v_2^2 = F_{12} r_2$$

$$m_1 v_1^2 + m_2 v_2^2 = F_{12}(r_1 + r_2) = F_{12} r_{12} = G\frac{m_1 m_2}{r_{12}}$$

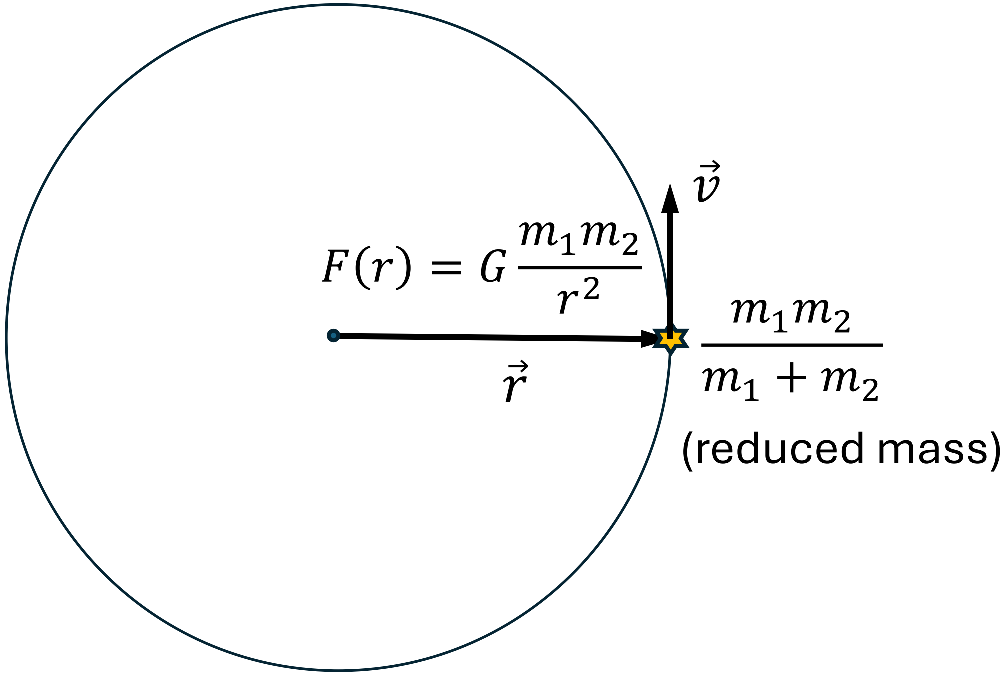
$$\frac{m_1 m_2}{m_1 + m_2} v^2 = G\frac{m_1 m_2}{r} \Rightarrow v^2 = G\frac{m_1 + m_2}{r}$$

$$\Rightarrow \frac{v^2}{r} = G\frac{M_{tot}}{r^2}$$

(centripetal acceleration) (total gravitational acceleration)

Measurements of the relative displacement (r) and velocity (v) between the pair can directly test Newtonian gravity!

One-particle equivalent description: circular, face-on

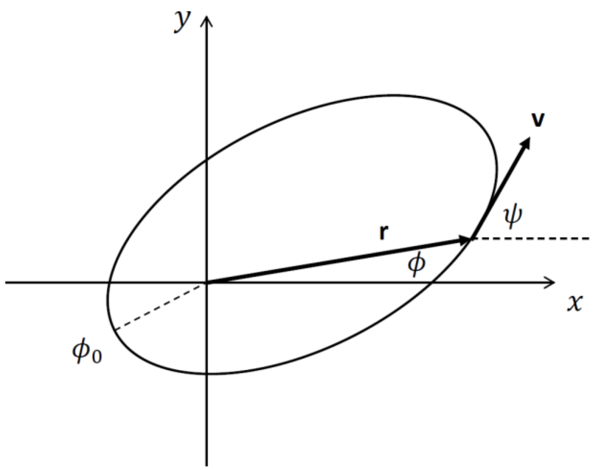


$\vec{r} = \vec{r}_2 - \vec{r}_1$

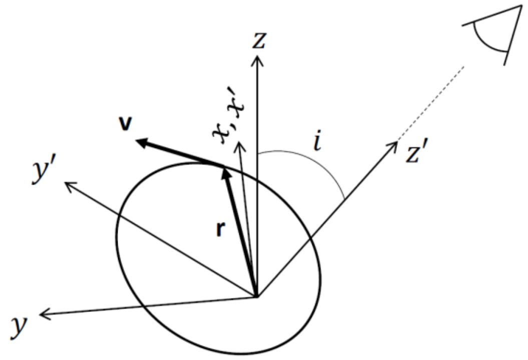
$\vec{v} = \vec{v}_2 - \vec{v}_1$

In the ideal case of **circular orbits** observed **face-on**, measurements of the **positions and proper motions** on the sky and inferences of the two **stellar masses** can be used to **test gravity** as a function of r.

One-particle equivalent description: elliptical, inclined



orbital plane
(face-on view)

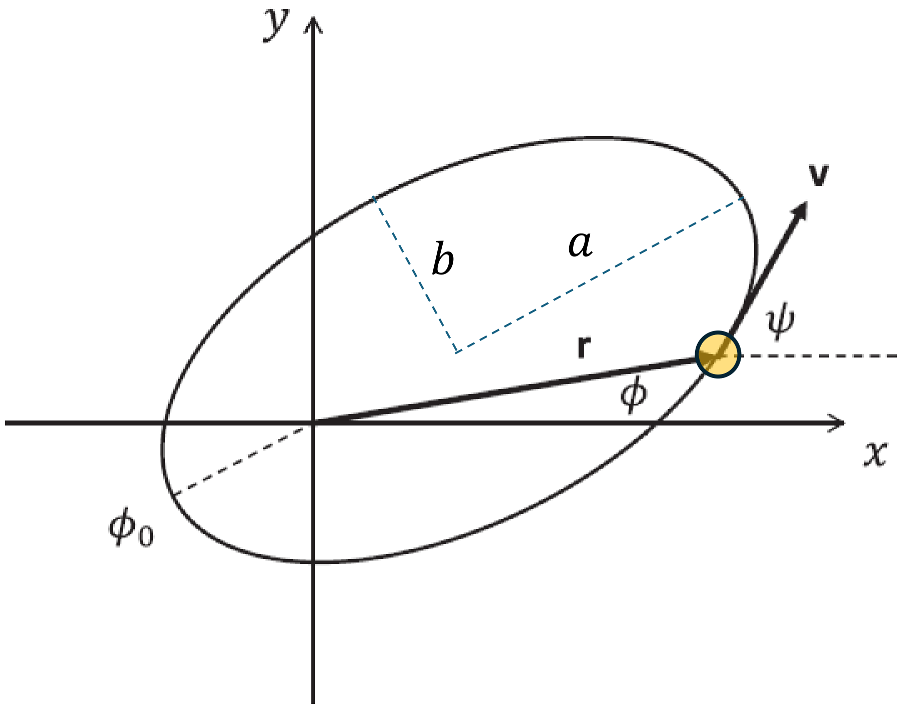


observer's view
(3D geometry)

Unknowns in testing gravity
- Eccentricity e
- Phase ϕ and periastron ϕ_0
- Inclination i

*Due to **long periods** ($\gtrsim 10^5\ \mathrm{yr}$ for separation $\gtrsim 3\ \mathrm{kau}$), observations over a few years correspond to **snapshots** and thus **cannot determine** the above **unknowns**.*

Newtonian dynamics: **elliptical orbits**



orbital plane

orbit equation: $r = \frac{a(1-e^2)}{1+e\cos(\phi-\phi_0)}$

semi-major axis: a

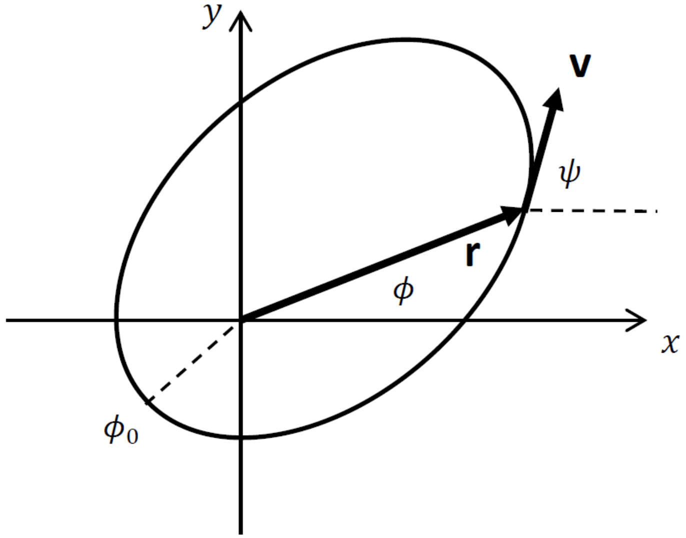
eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$

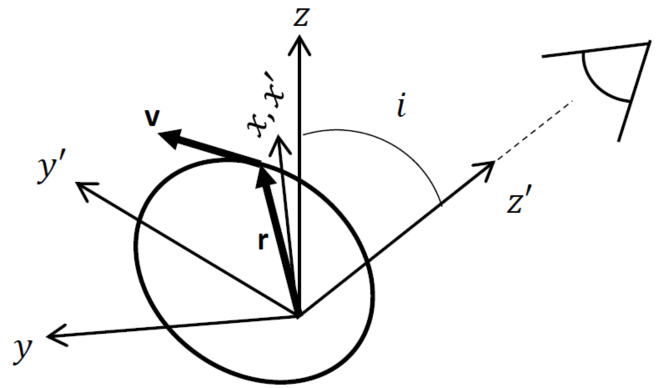
Newtonian prediction: $v(r) = \sqrt{\frac{GM_{\text{tot}}}{r}\left(2 - \frac{r}{a}\right)}$

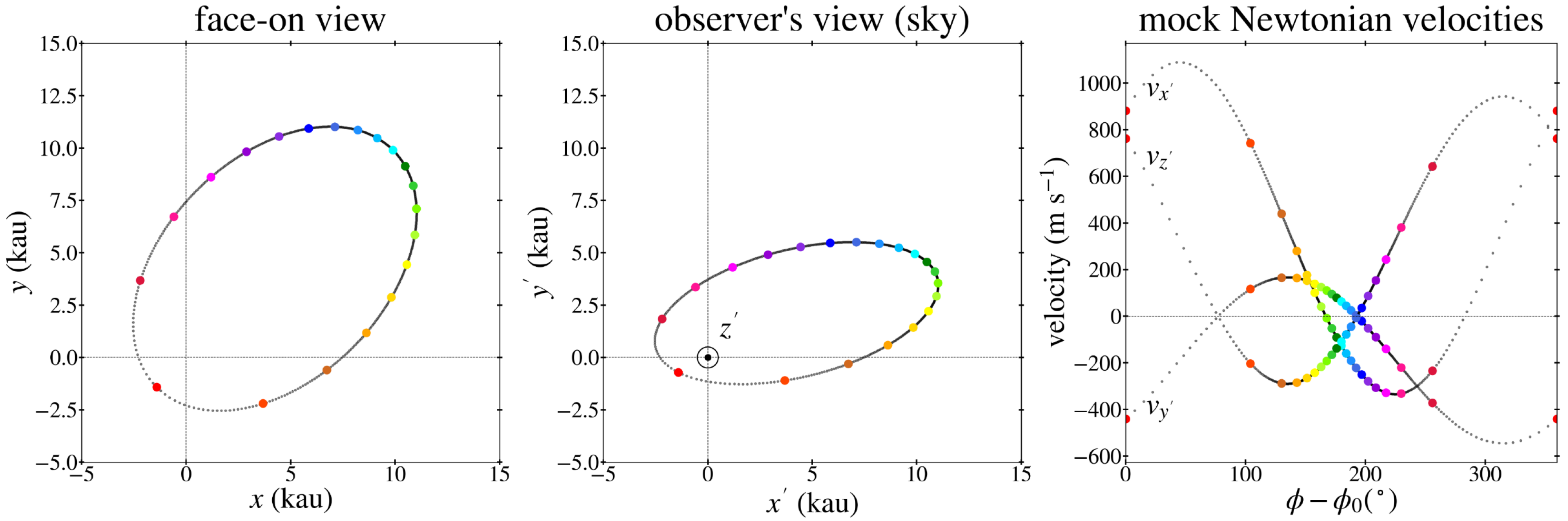
One-particle equivalent description
of the relative motion between the pair

A mock Newtonian wide binary (to be used later for Bayesian modeling)

$M_{\mathrm{tot}} = 2\mathrm{M}_{\odot}$
$a = 8\mathrm{kau}$
$e = 0.75$
$i = 60°$







Properties of the current data (Gaia DR3)

- The **sky positions** $(\boldsymbol{x'}, \boldsymbol{y'})$ are very **accurate and precise**.
- **Tangential velocities** $(\boldsymbol{v_{x'}}, \boldsymbol{v_{y'}})$ as measured by proper motions on the sky are very **accurate and precise**.
- The line-of-sight displacements ($\boldsymbol{\Delta z'}$) between the stars in the pairs are **not precise** (i.e. two distances are not precise enough to measure $\Delta z'$).
- Gaia measured **velocities** in the longitudinal (i.e. **radial** from the observer) direction $(\boldsymbol{v_{z'}})$ are **not as precise** as tangential velocities.

<u>**Important**</u>: ***"Observations and observation proposals are in progress to measure $v_{z'}$ as accurately and precisely as $v_{x'}, v_{y'}$."***

Statistical analyses based on v_p only

- The key is to **compare** some **measured/inferred** parameter Y_{obs} with the **Newton-predicted** (through a Monte Carlo method) parameter Y_{pred} at a variable X.
- Because individual values of Y_{obs} and Y_{pred} are drawn probabilistically from the observed quantities and possible ranges of the unobserved quantities, their **medians and/or distributions are compared**.

method	Y (gravity-sensitive parameter)	X (independent variable)	comments
stacked velocity profile	v_p	s	$v_p =$ projected relative velocity $s =$ projected separation
normalized velocity profile	$\tilde{v} \equiv \frac{v_p}{v_c(s)}$	$\frac{s}{r_{\mathrm{M}}}$	$v_c(s) =$ Newtonian circular velocity at s $r_{\mathrm{M}} = \sqrt{\frac{GM_{\mathrm{tot}}}{a_0}}$: MOND radius for Milgrom's constant $a_0 = 1.2 \times 10^{-10}\ \mathrm{m\ s^{-2}}$
acceleration plane	$g = \frac{v^2}{r}$ ("kinematic acceleration")	$g_{\mathrm{N}} = \frac{GM_{\mathrm{tot}}}{r^2} = \frac{v_c^2(r)}{r}$ ("Newtonian gravity")	$v =$ relative velocity in 3D $r =$ separation in 3D *Monte Carlo deprojection is required.*

From observed proper motions (PMs) to sky-projected velocities:

$$\Delta\mu = \left[(\mu^*_{\alpha,A} - \mu^*_{\alpha,B})^2 + (\mu_{\delta,A} - \mu_{\delta,B})^2\right]^{1/2}$$

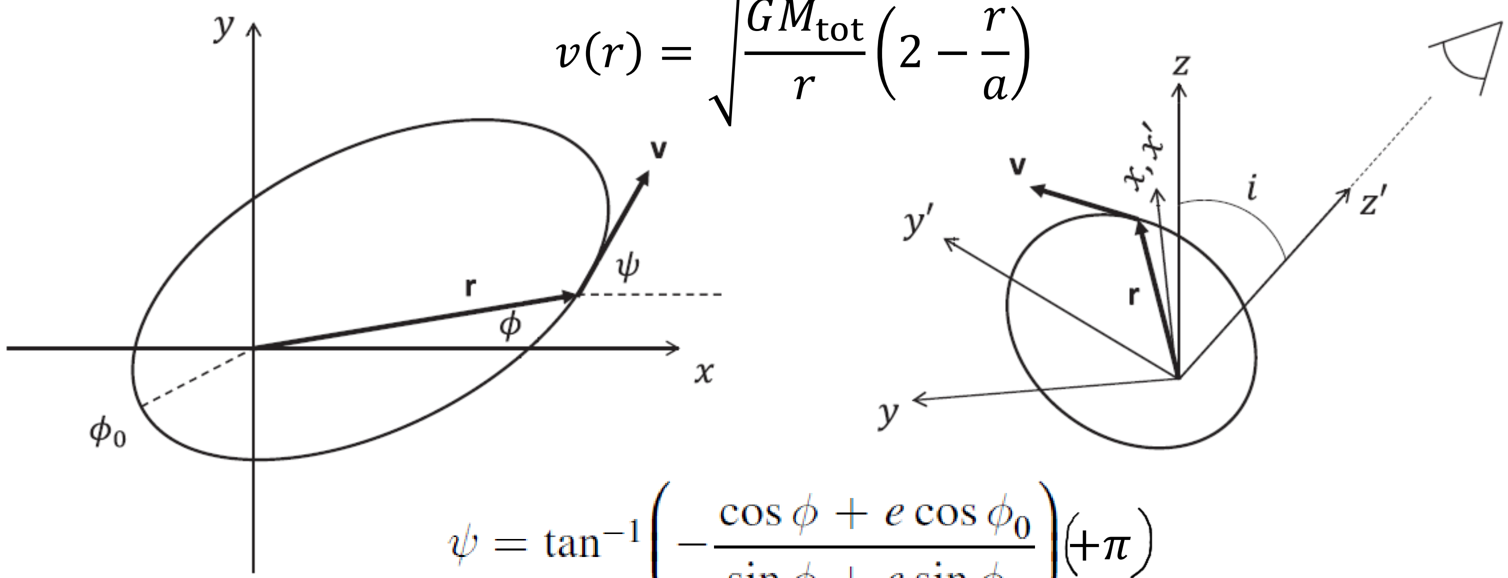
$$\Rightarrow \quad v_p = 4.7404 \times 10^{-3} \text{ km s}^{-1} \times \Delta\mu \times d$$

Here $\Delta\mu$ in $\mathrm{mas\ yr^{-1}}$, d in parsec (pc).

Since $s \ll d$, the two stars are assumed to be at the same distance once they are determined to be a gravitationally bound system.

Newtonian prediction of v_p

$$v(r) = \sqrt{\frac{GM_{\text{tot}}}{r}\left(2 - \frac{r}{a}\right)}$$



$$\psi = \tan^{-1}\left(-\frac{\cos\phi + e\cos\phi_0}{\sin\phi + e\sin\phi_0}\right)(+\pi)$$

orbital plane
(face-on view)

observer's view
(3D geometry)

Observer's sky plane: $x'y'$

Inclination: i

Sky-projected separation:
$s = r\sqrt{1 - \sin^2 i \sin^2 \phi}$

Sky-projected velocity:
$v_{p,x'} = v(r)\cos\psi$
$v_{p,y'} = v(r)\cos i \sin\psi$
$v_p(s) = v(s)\sqrt{1 - \sin^2 i \sin^2 \psi}$

Newton predicted numerical value for a binary:

$$v(s) = 0.9419\ \mathrm{km\ s^{-1}} \sqrt{\frac{M_{\mathrm{tot}}/\mathrm{M}_\odot}{s/\mathrm{kau}} \sqrt{1 - \sin^2 i \sin^2 \phi} \left(2 - \frac{1-e^2}{1+e\cos(\phi-\phi_0)}\right)}$$

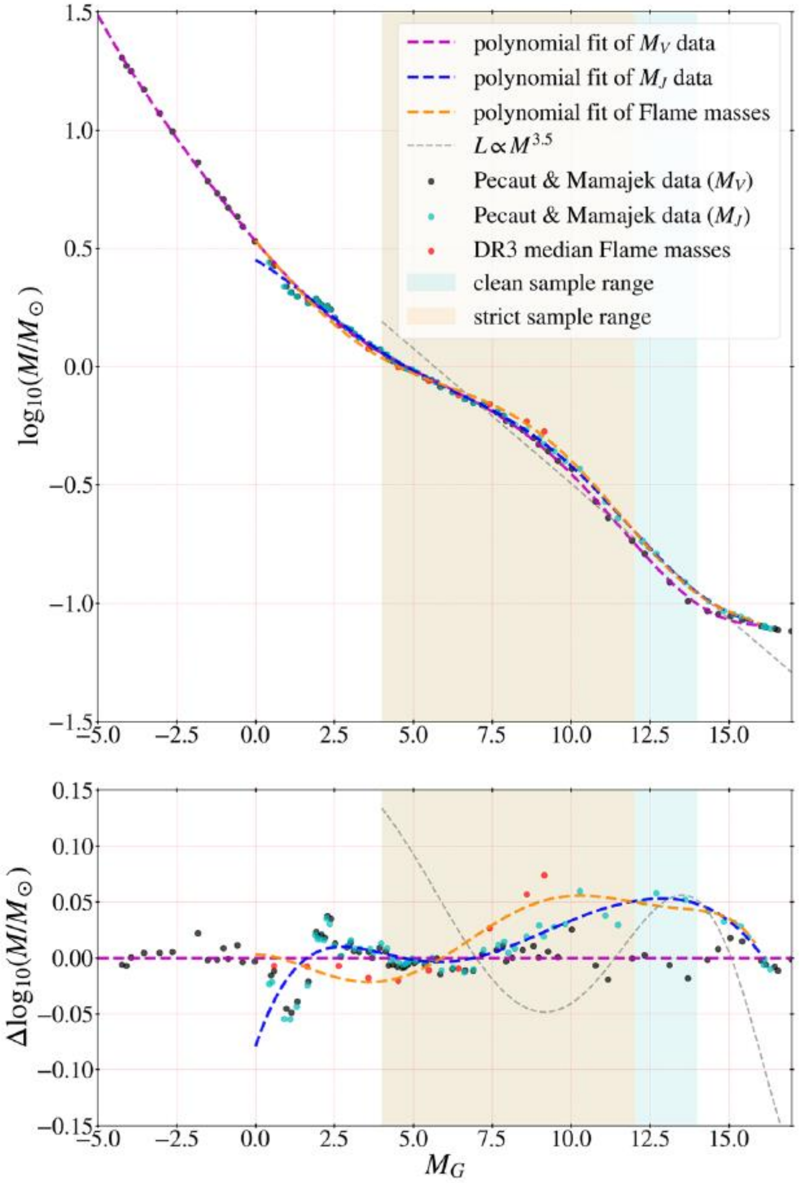
where $\boldsymbol{s}$ and $\boldsymbol{M}_{\mathrm{tot}}$ are **measured** quantities, and $\boldsymbol{i}, \boldsymbol{e}, \boldsymbol{\phi},$ and $\boldsymbol{\phi_0}$ are **MC drawn** values.

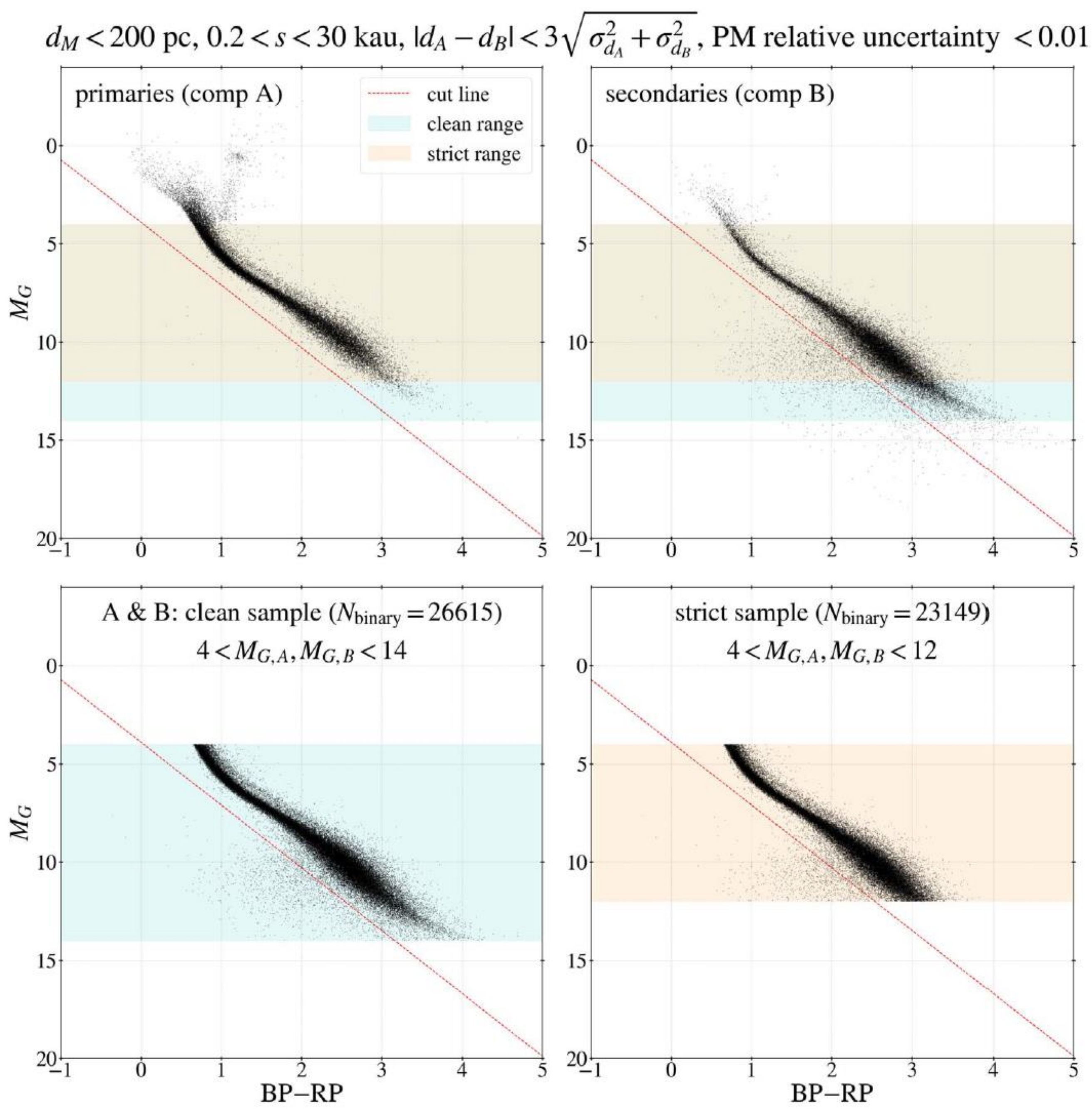
How to determine the masses of the observed stars?

Use the magnitude(M_G)-mass(M) relation for main-sequence stars:

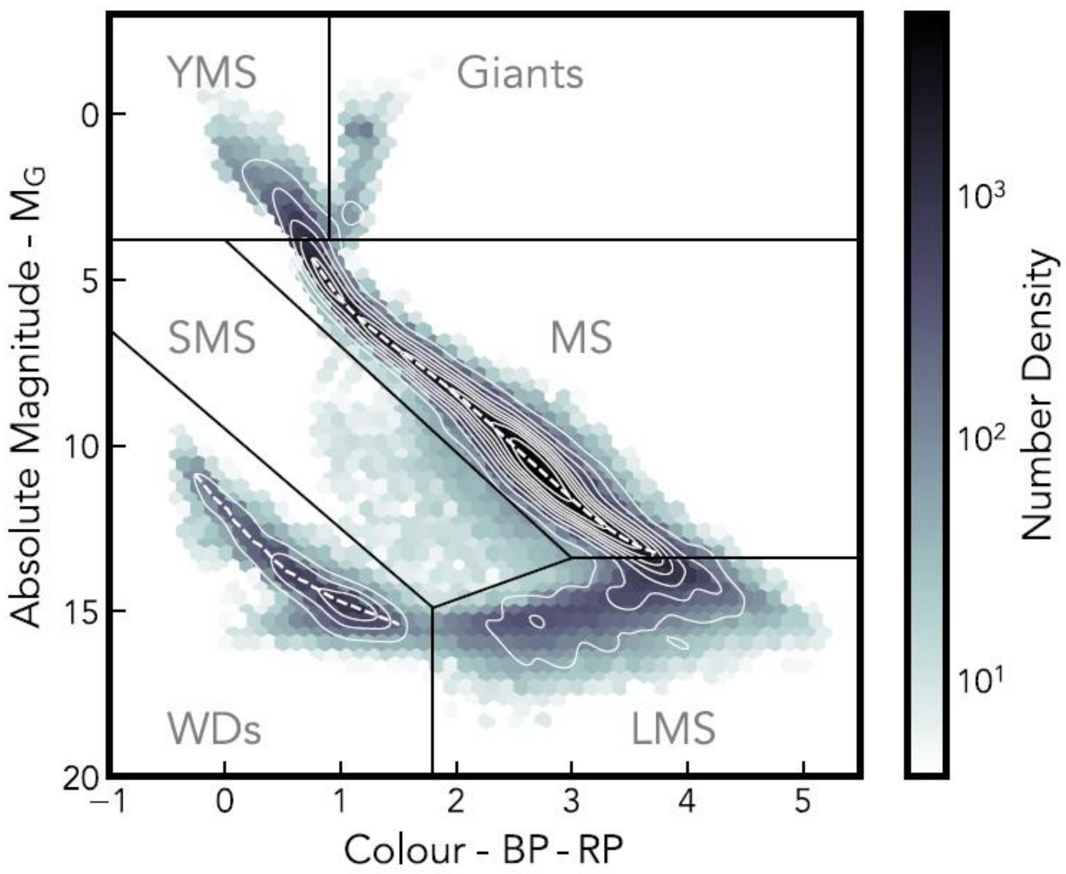
Use the Pecaut & Mamajek (2013) relations that are consistent with short-period binary data of Mann et al. (2019).

The Gaia DR3 ` FLAME' masses are also considered for checking the relations in the corresponding magnitude range.





Selection of main sequence (MS) stars



Penoyre et al. (2022)
MNRAS, 513, 5270

e (eccentricity), i (inclination), and ϕ (orbital phase) are drawn from the following distributions.

(1) eccentricity: empirical ranges or
power-law distribution $p(e) = (1 + \alpha)p^{\alpha}$

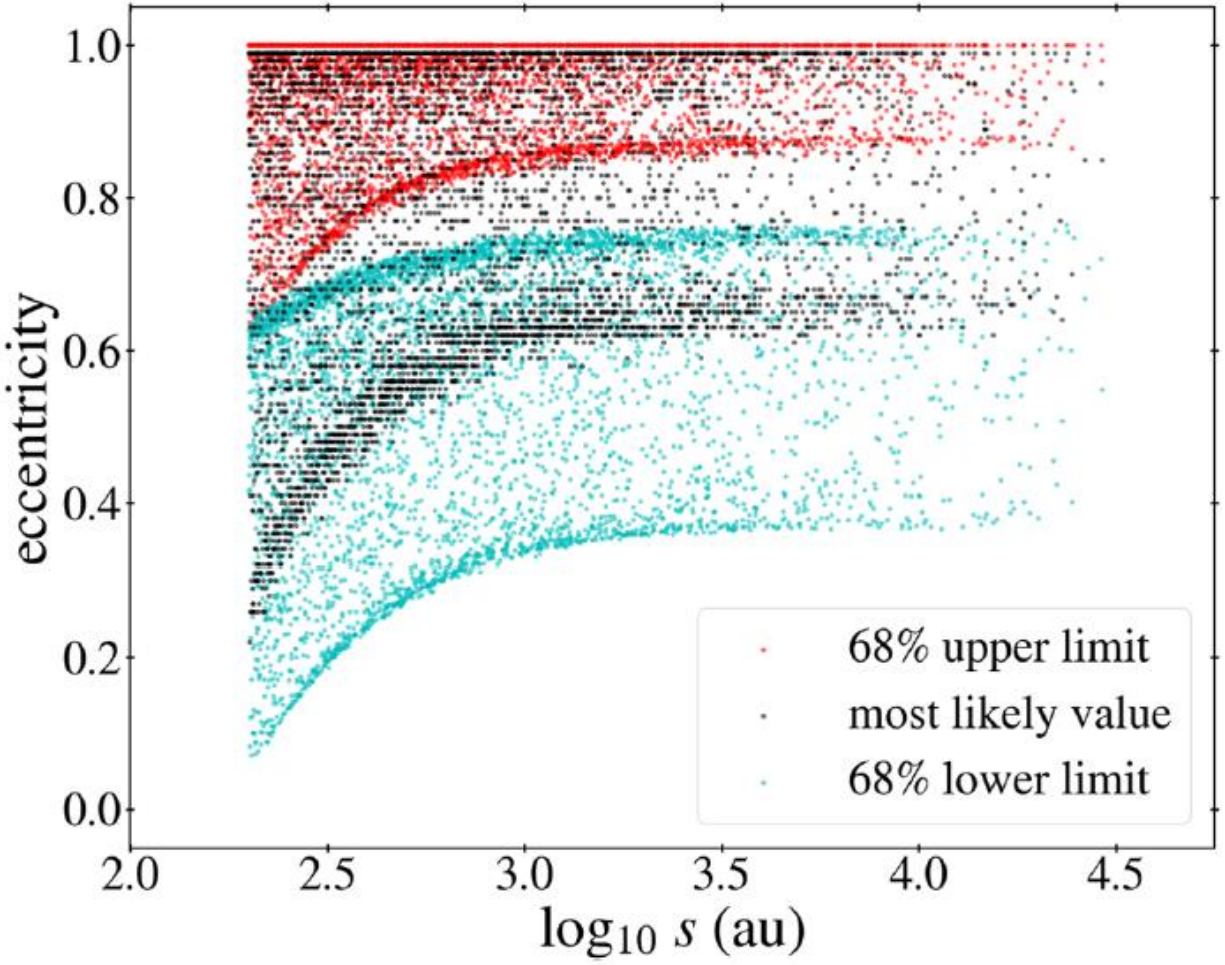
(2) inclination distribution: $p(i) = \sin i$

(3) ϕ distrbution from the time distribution along the orbit :

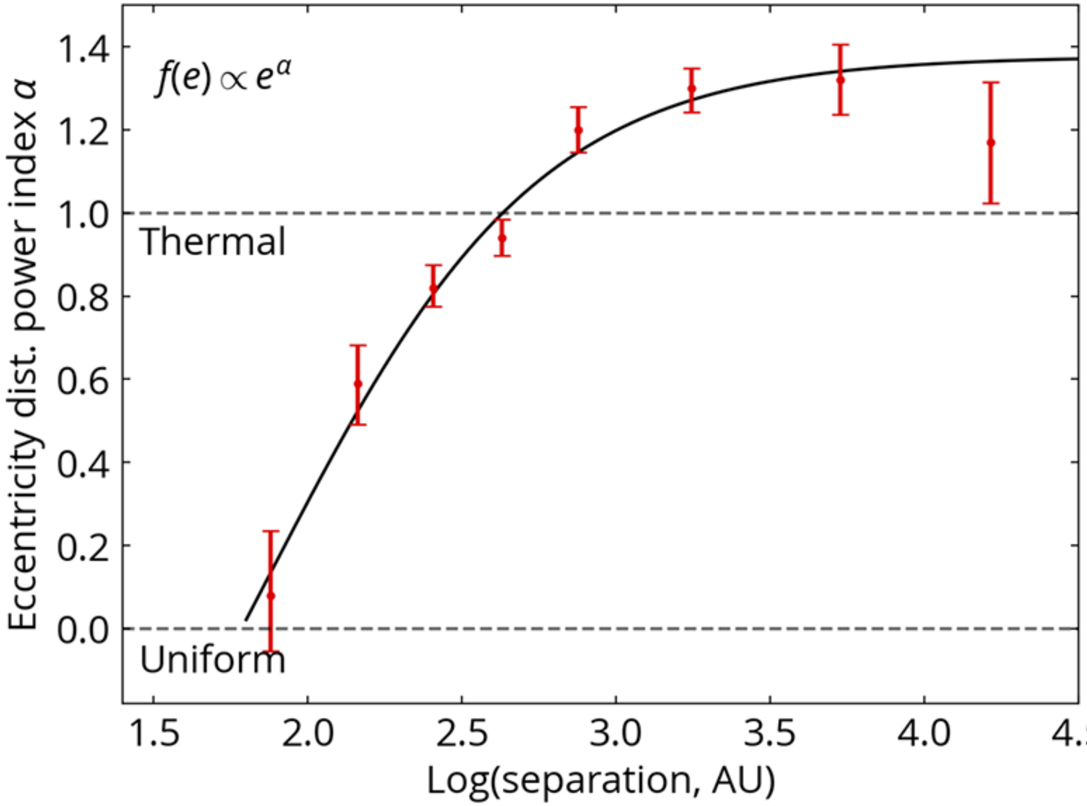
$$t \propto \int_{\phi_0}^{\phi} d\phi' \frac{1}{(1+e\cos(\phi'-\phi_0))^2}$$

Example: eccentricity distribution for Gaia binaries from Hwang et al. (2022)

Individual ranges



power-law distribution
("statistical eccentricities")



Calculating $(g_{\mathrm{obs}}, g_{\mathrm{N}})$ from observed quantities

Monte Carlo deprojection of v_p to physical velocities v:

$$v = v_p/\sqrt{1-\sin^2 i \sin^2 \psi}$$

$$\Rightarrow\ g_{\mathrm{obs}} = v^2/r \text{ with } r = s/\sqrt{1-\sin^2 i \sin^2 \phi}$$

Newtonian gravity is

$$g_{\mathrm{N}} = GM_{\mathrm{tot}}/r^2 \text{ with } r = s/\sqrt{1-\sin^2 i \sin^2 \phi}$$

How to obtain g_{pred} from observed separation and magnitudes

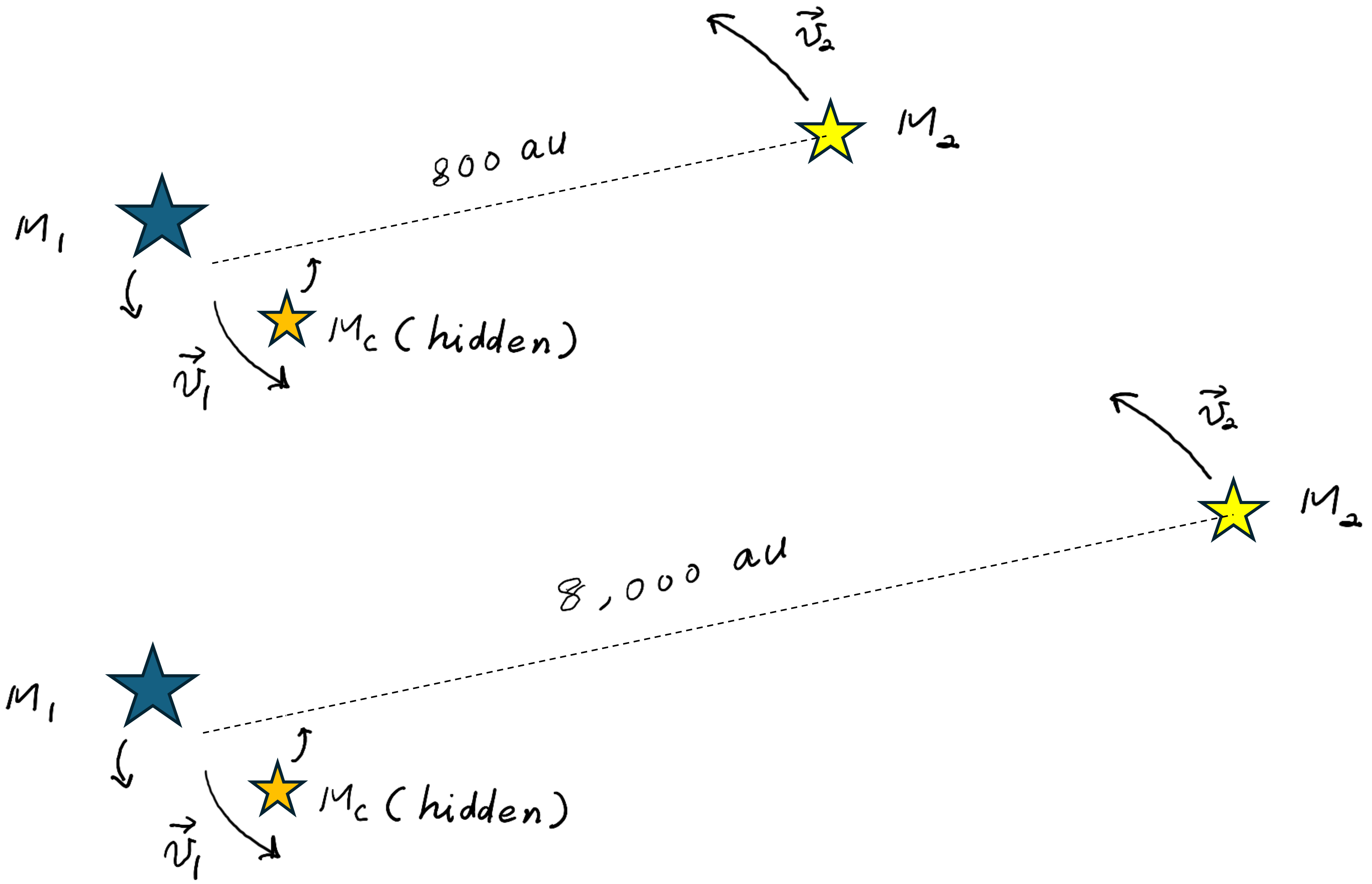
Calculate $v(r) = \sqrt{\frac{GM_{\text{tot}}}{r}\left(2 - \frac{r}{a}\right)}$ with $\frac{a}{r} = (1 + e\cos(\phi - \phi_0))/(1 - e^2)$ and $r = s/\sqrt{1 - \sin^2 i \sin^2 \phi}$.

Sky-projected velocity components: $v_{p,x'} = v(r)\cos\psi,\ v_{p,y'} = v(r)\cos i \sin\psi$

From sky-projected velocity components obtain **mock proper motions** and **replace the observed proper motions with them** to derive g_{pred}.

$$\mu^*_{\alpha,A} = \mu^*_{\alpha,M} + (M_B/M_{\text{tot}})v_{p,x'}/d_A,$$
$$\mu^*_{\alpha,B} = \mu^*_{\alpha,M} - (M_A/M_{\text{tot}})v_{p,x'}/d_B,$$
$$\mu_{\delta,A} = \mu_{\delta,M} + (M_B/M_{\text{tot}})v_{p,y'}/d_A,$$
$$\mu_{\delta,B} = \mu_{\delta,M} - (M_A/M_{\text{tot}})v_{p,y'}/d_B,$$

binaries with hidden close companions (“hierarchical systems”)



How to take into account unresolved hierarchical systems?

- Their gravitational effects must be included: statistical properties from various surveys can be used.
- Their occurrence rate must be properly calibrated.

- The self-calibration can be done by requiring Newtonian regime data $s \lesssim 1$ kau to agree with the Newtonian prediction. Then, use the self-calibrated value of $f_{\rm multi}$ assuming that it does not vary from $s \lesssim 1$ kau to the low-acceleration regime $s \gtrsim 5$ kau.
- **If all stars are selected with the same photometric, astrometric, and kinematic criteria, the occurrence rate of unresolved companions should not depend on s.**

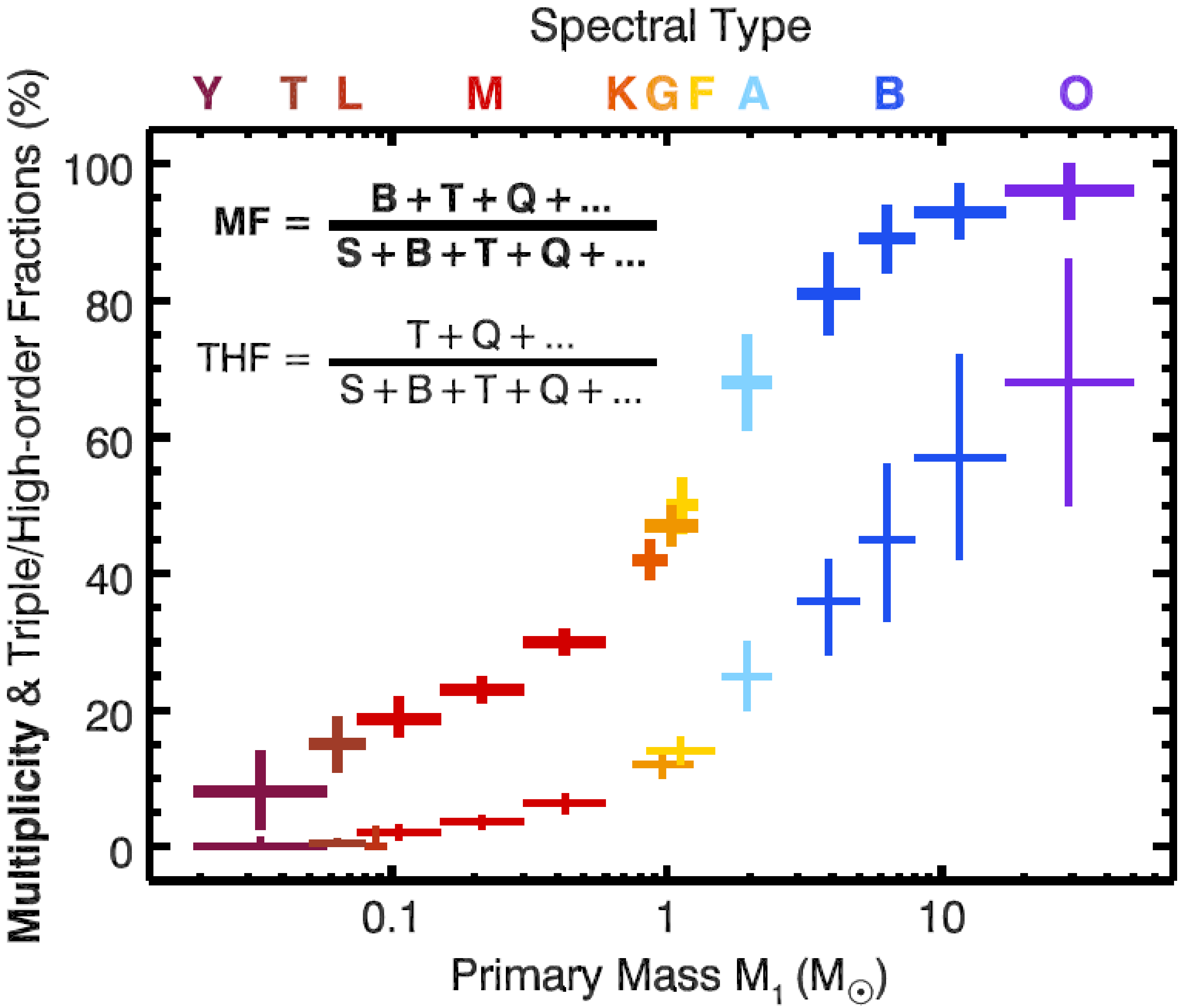
Occurrence rate of multiples (triples or higher-order) among binaries:

$$f_{\rm multi} \equiv \frac{\text{number of apprent binaries with additional hidden component(s)}}{\text{all apparent binaries}}$$

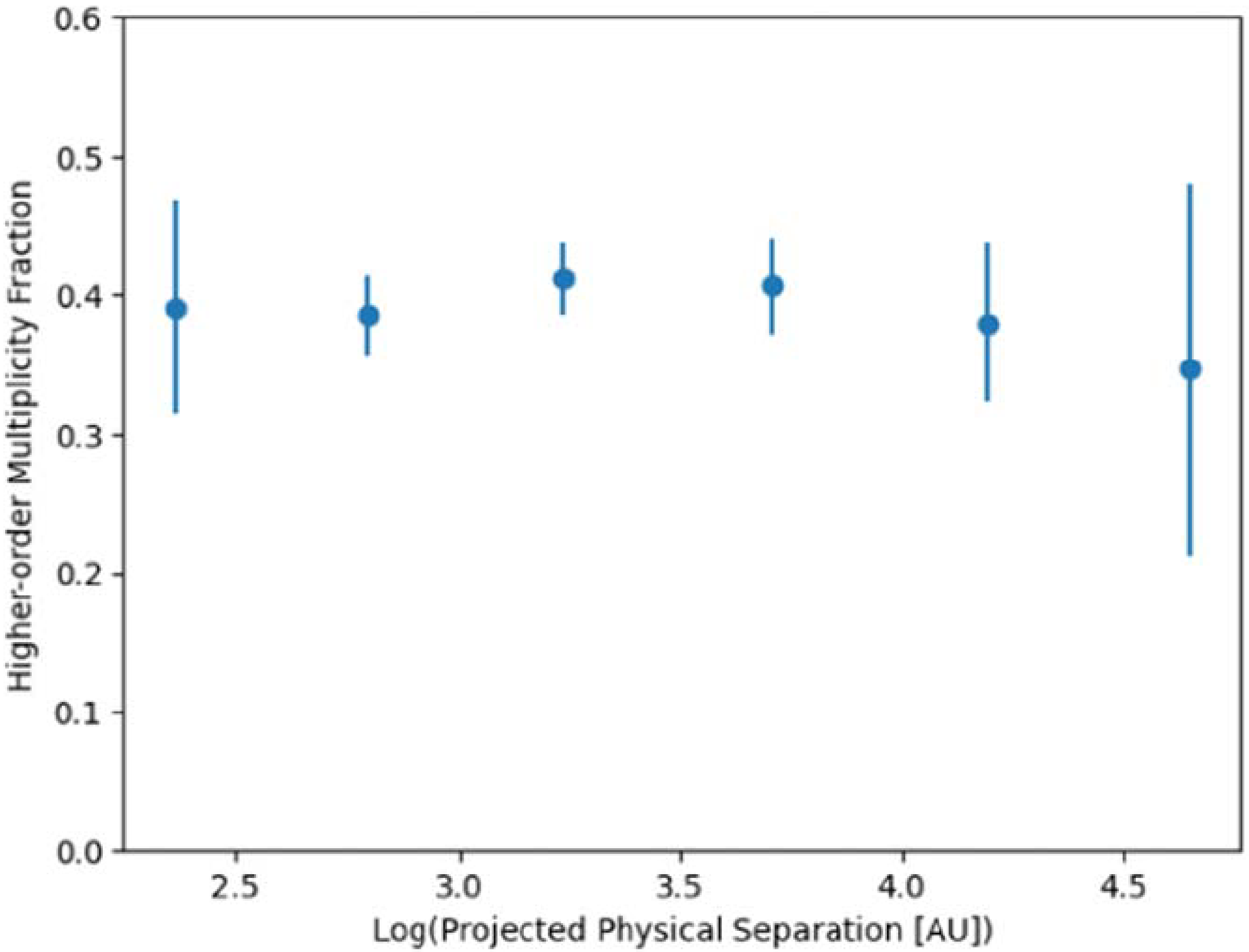
general statistics of multiplicity

Offner, Moe, Kratter, et al. (2022) arXiv:2203.10066 (ASP Conference Series, Vol. 534)

$$f_{\mathrm{multi}} = \frac{\mathrm{THF}}{\mathbf{MF}}$$



Observational constraint on the dependence of f_{multi} on separation (s) that are most relevant to the samples used for the recent gravity tests



most direct and relevant result available at the time of this talk

Hartman et al. (2022)
From 4947 K+K Gaia wide binaries

How to remove unresolved hierarchical systems to get a sample with $f_{\text{multi}} \rightarrow 0$

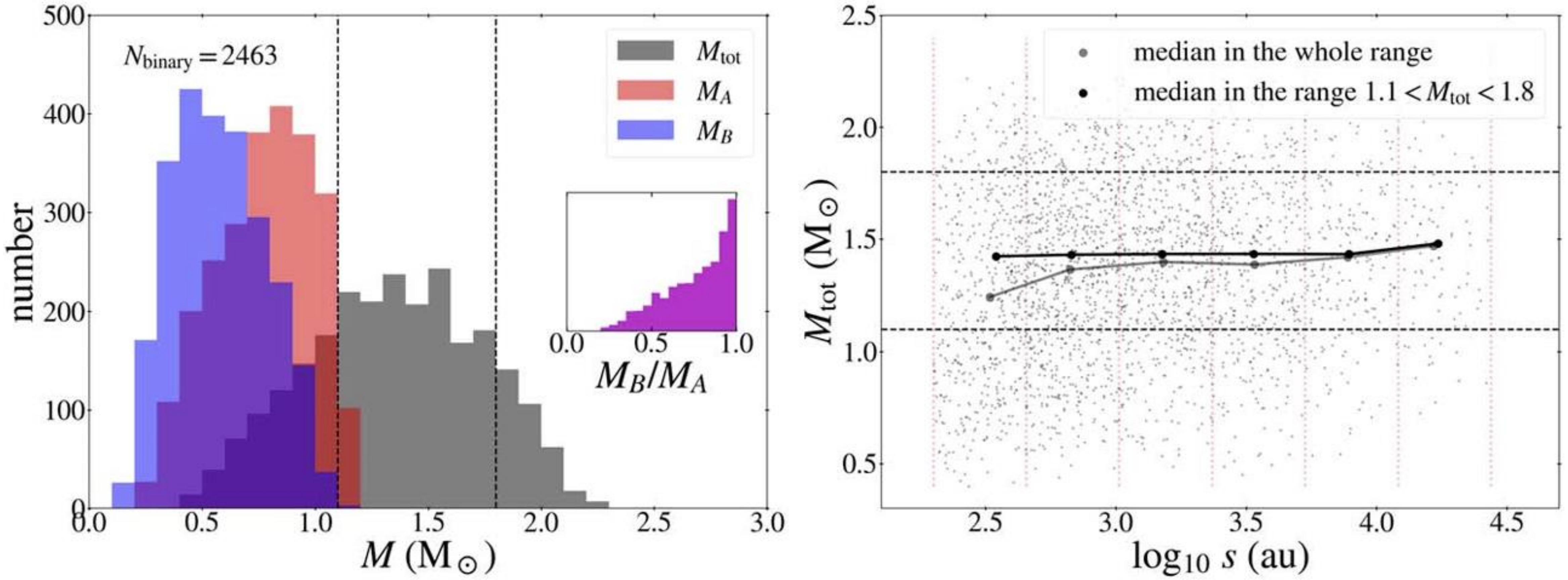
- Remove unresolved hierarchical systems using photometric, astrometric, and kinematic effects of the hidden components (as in the exoplanet detection): e.g. with the following stringent requirements (Chae 2024a)

- PM relative (fractional) errors < 0.005
- Distance relative errors < 0.005
- RV relative errors < 0.2
- Distance match: $|d_A - d_B| < \sqrt{4\left(\sigma_{d_A}^2 + \sigma_{d_B}^2\right) + (6s)^2}$
- RV match: $\left|v_{r,A} - v_{r,B}\right| < \sqrt{4\left(\sigma_{v_{r,A}}^2 + \sigma_{v_{r,B}}^2\right) + \left(\Delta v_{r,\text{orbit}}^{\max}\right)^2}$ with $\Delta v_{r,\text{orbit}}^{\max} = 0.9419 \text{ km s}^{-1} \sqrt{\frac{M_{\text{tot}}}{s}} \times 1.3 \times 1.2$
- N_{binary}: **up to $\sim$ 4000 within 200 pc.**

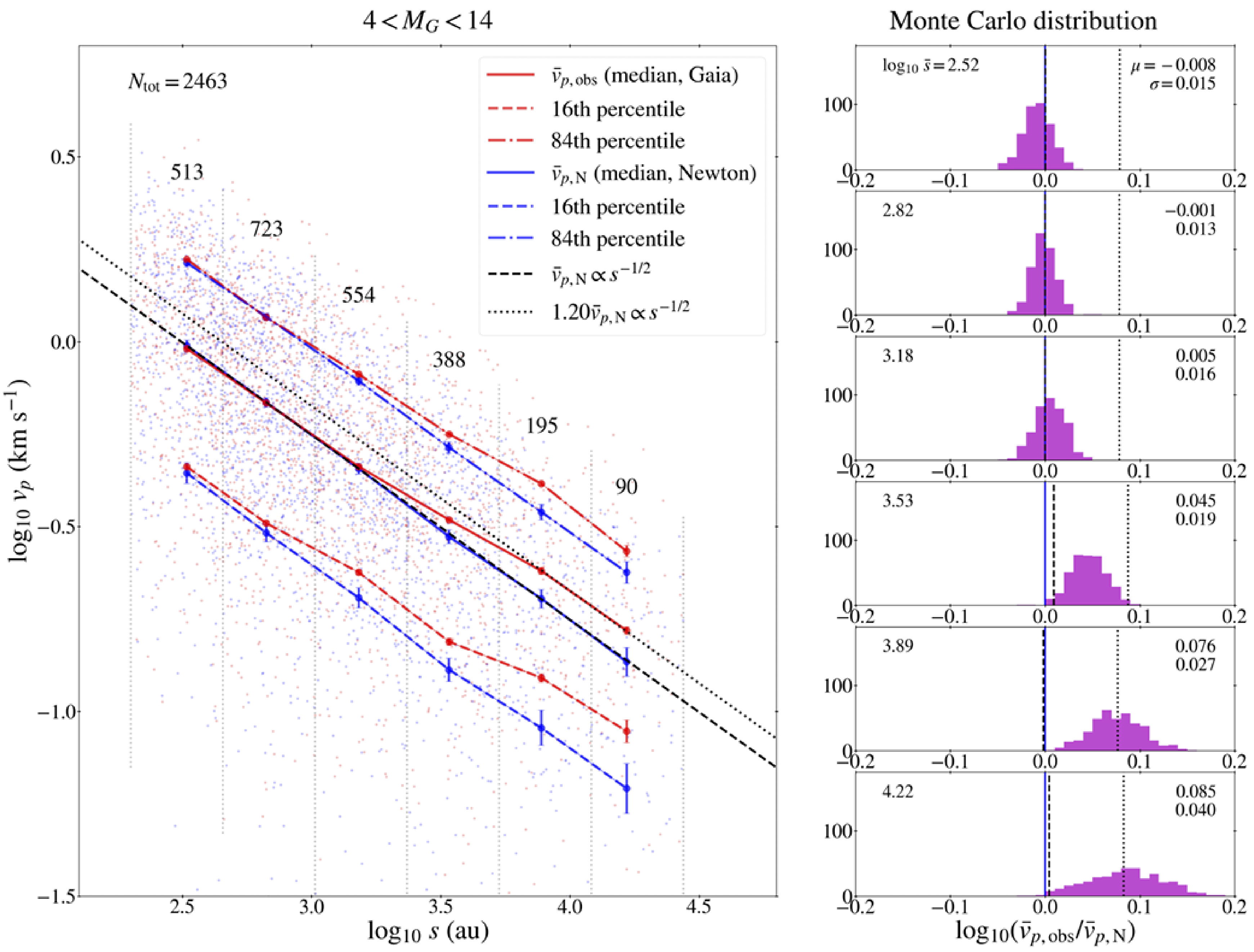
Test Result

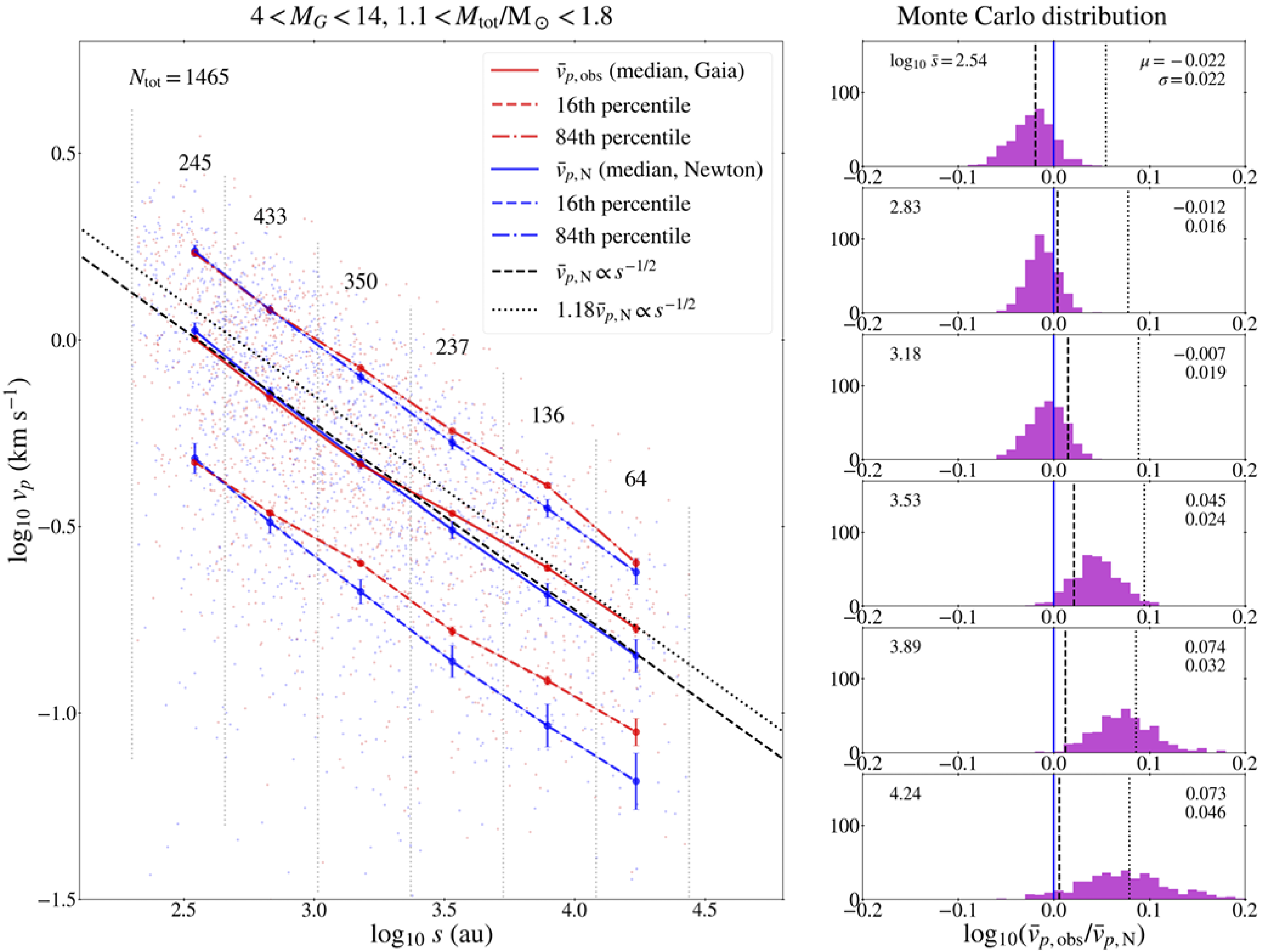
Stacked velocity profile test of pure binaries ($f_{\mathrm{multi}} \to 0$)

Chae (2024a)



5.0σ deviation from Newton in the three larger-s bins.





$4 < M_G < 14$, $1.1 < M_{tot}/M_\odot < 1.8$
Monte Carlo distribution
$N_{tot} = 1465$
245
433
350
237
136
64
$\bar{v}_{p,obs}$ (median, Gaia)
16th percentile
84th percentile
$\bar{v}_{p,N}$ (median, Newton)
16th percentile
84th percentile
$\bar{v}_{p,N} \propto s^{-1/2}$
$1.18\bar{v}_{p,N} \propto s^{-1/2}$
$\log_{10} v_p$ (km s^{-1})
$\log_{10} s$ (au)
$\log_{10} \bar{s} = 2.54$
$\mu = -0.022$
$\sigma = 0.022$
2.83
−0.012
0.016
3.18
−0.007
0.019
3.53
0.045
0.024
3.89
0.074
0.032
4.24
0.073
0.046
$\log_{10}(\bar{v}_{p,obs}/\bar{v}_{p,N})$

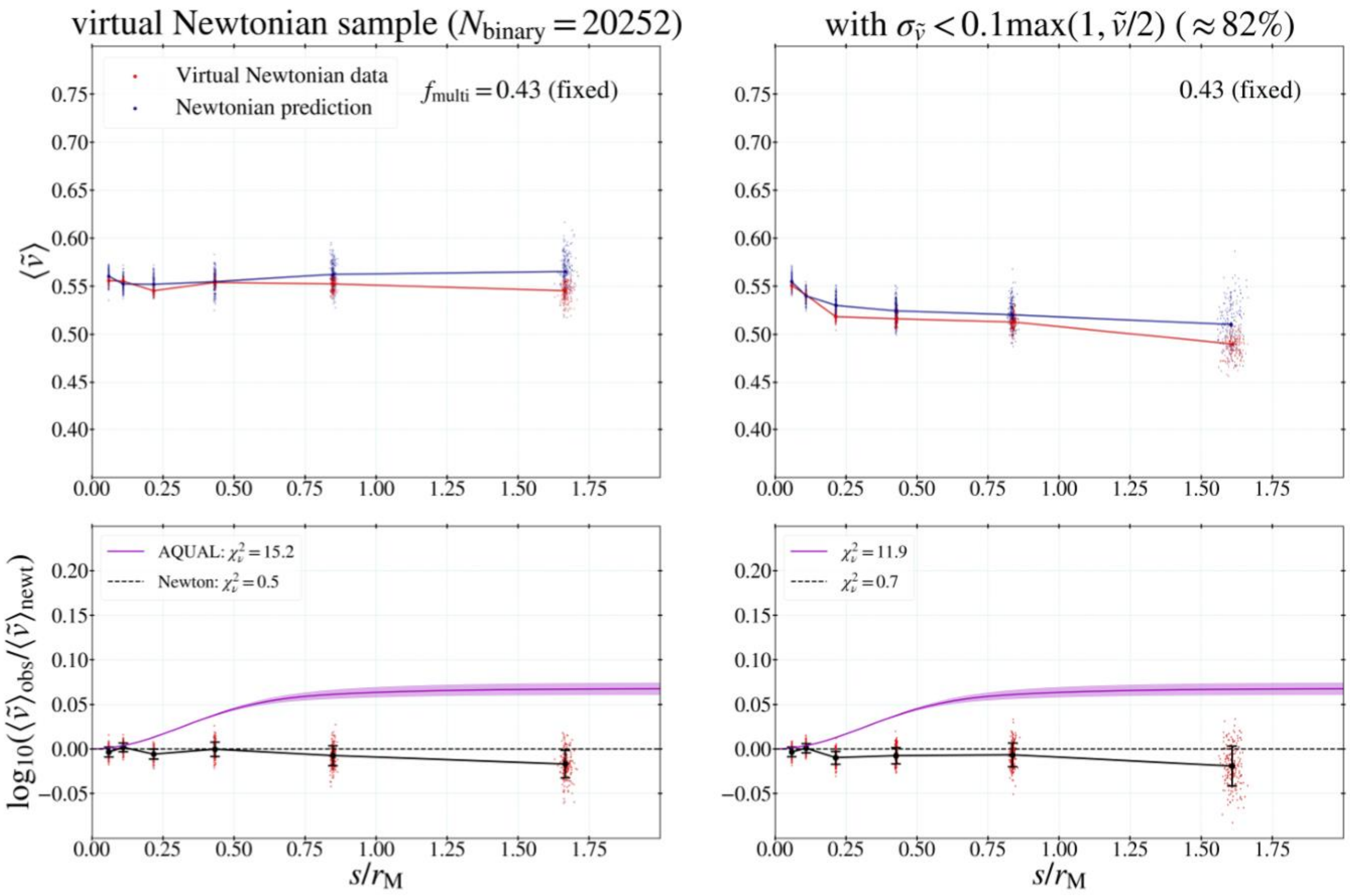
Other results for representative samples

Three (+ one) samples used in the most recent publication (Chae 2024b)

sample	$N_{\rm binary}$	key selection criteria	reference/comments
Chae (2023a)	19716	$\mathcal{R} < 0.01$, PM relative errors < 0.005	Chae (2023a)
new	6389	relative errors: PM < 0.005, dist < 0.01, RV < 0.5	$\mathcal{R}$ not used (this work)
pure binary	2463	$\mathcal{R} < 0.01$, relative errors: PM < 0.005, dist < 0.005, RV < 0.2	Chae (2024)
Chae (2023a) limited	5635	$\mathcal{R} < 0.01$, PM relative errors < 0.005, $2 < s < 30$ kau	limited range of s

Note. The Chae (2023a) limited sample is considered for the purpose of investigating/illustrating the effects of a limited dynamic range.

What to expect for Newtonian gravity: From 200 MC results



$$\Gamma \equiv \log_{10} \gamma_{\tilde{v}} \equiv \log_{10} \left(\frac{\langle \tilde{v} \rangle_{\rm obs}}{\langle \tilde{v} \rangle_{\rm newt}} \right)$$

logarithmic velocity boost factor

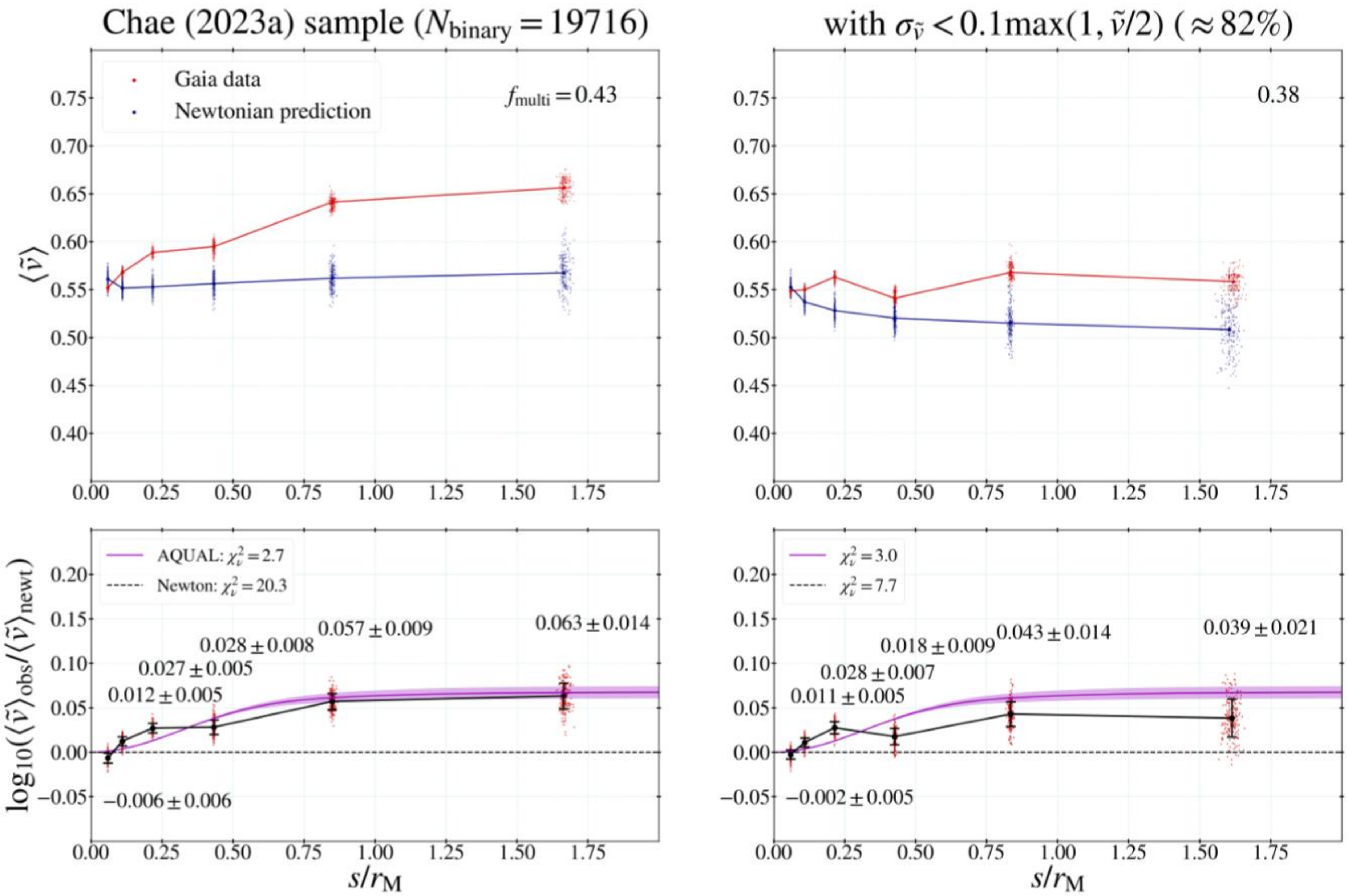
$$\gamma_g = 10^{2\Gamma}$$

gravity boost factor

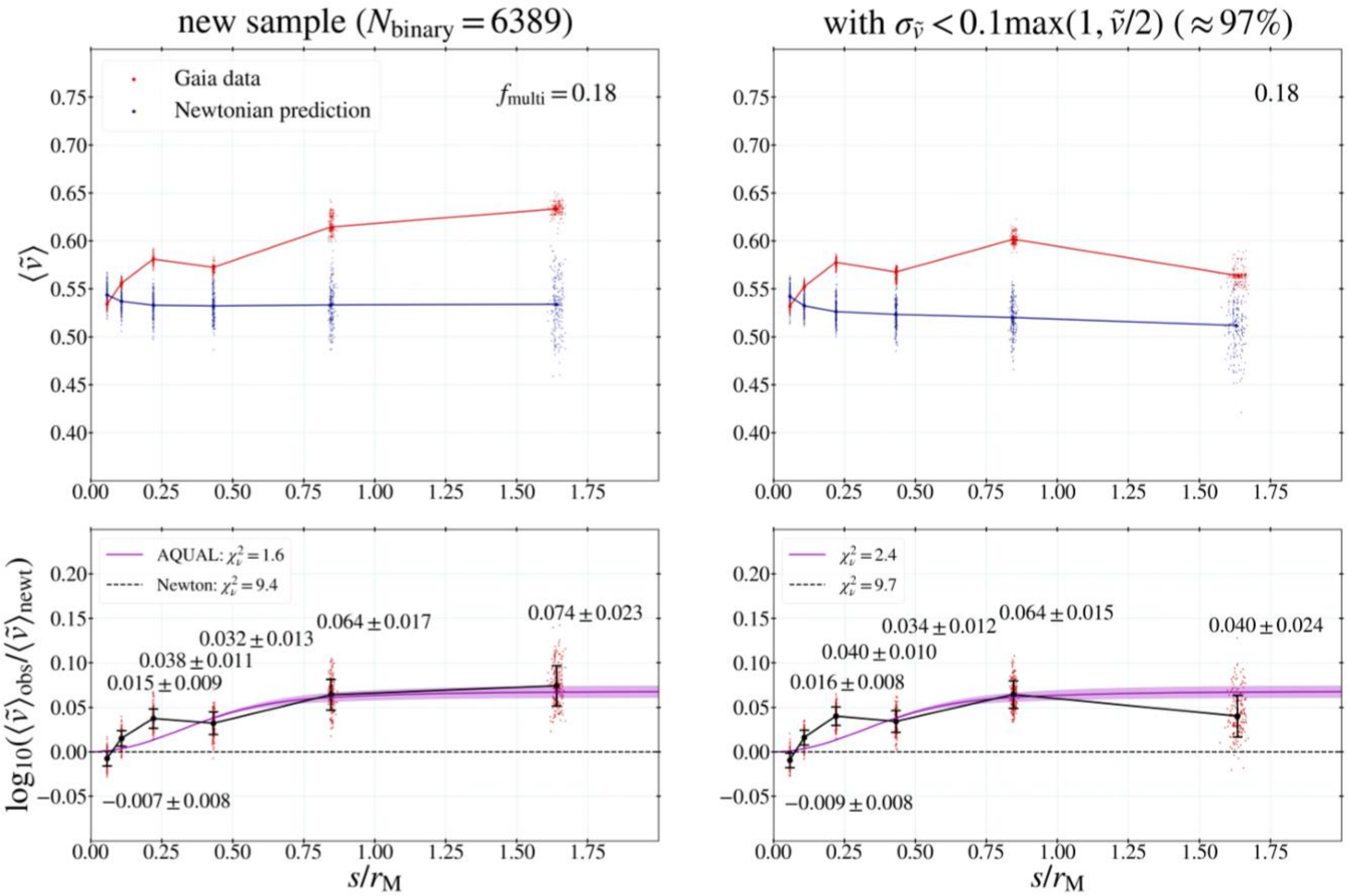
$$\chi^2_\nu \equiv \frac{1}{\nu} \sum_{i=1}^{N_{\rm bin}} \frac{\left(\mu_{\Gamma_i} - \log_{10} \gamma_{\tilde{v}_i}^{\rm model}\right)^2}{(\sigma_{\Gamma_i})^2 + (\sigma_i^{\rm model})^2}$$

reduced χ^2 statistic for the binned data of Γ

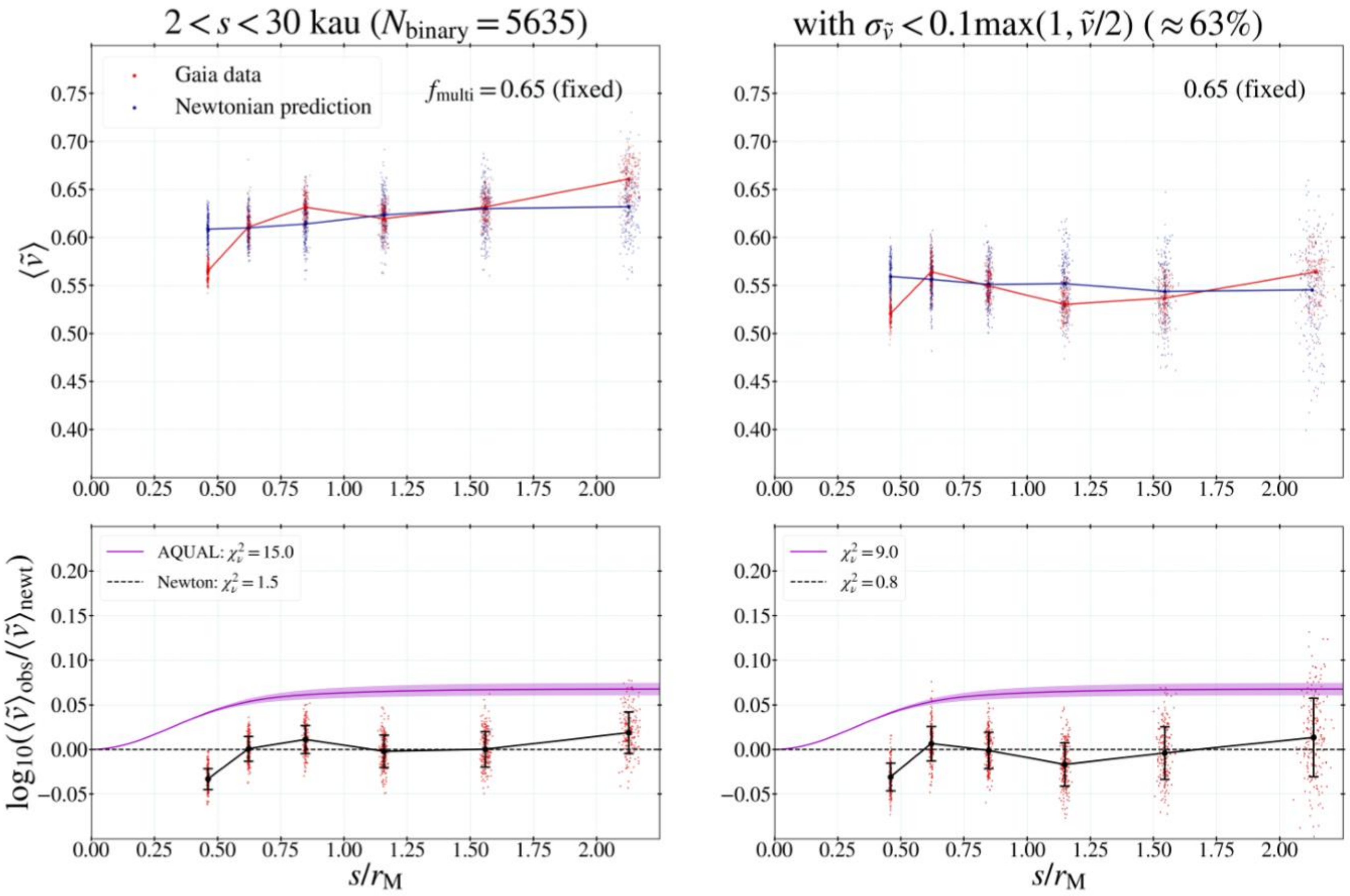
Chae (2023a) sample: almost exactly opposite to the expected result for standard gravity



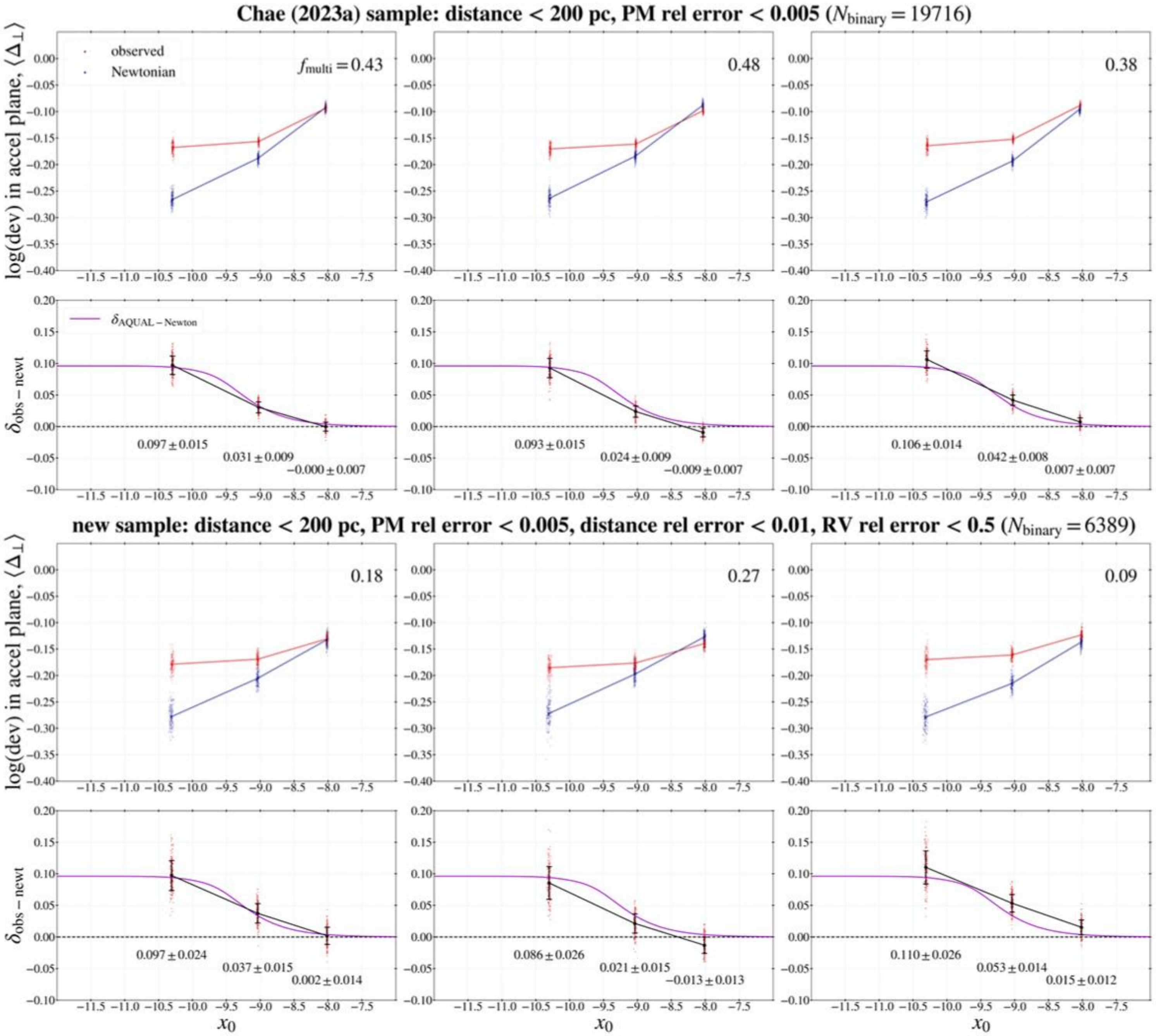
Similar result for the Chae (2024b) new sample



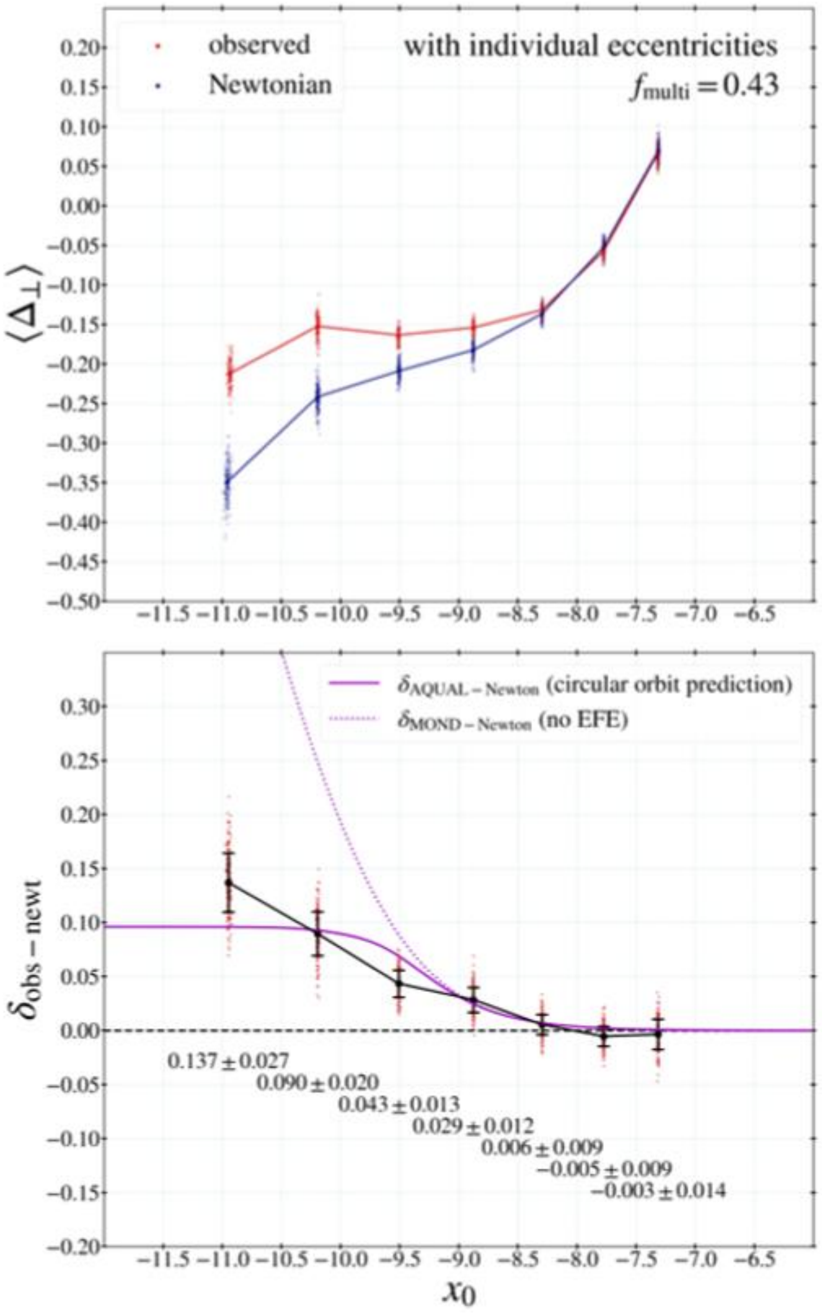
- ✓ For the limited dynamic range $2 < s < 30$ kau excluding the Newtonian regime, f_{multi} cannot be self-calibrated.
- ✓ With a high value of $f_{\mathrm{multi}} = 0.65$, one can make binaries appear agreeing with Newton! Actually, one can obtain whatever gravity they want by choosing a value of f_{multi}.



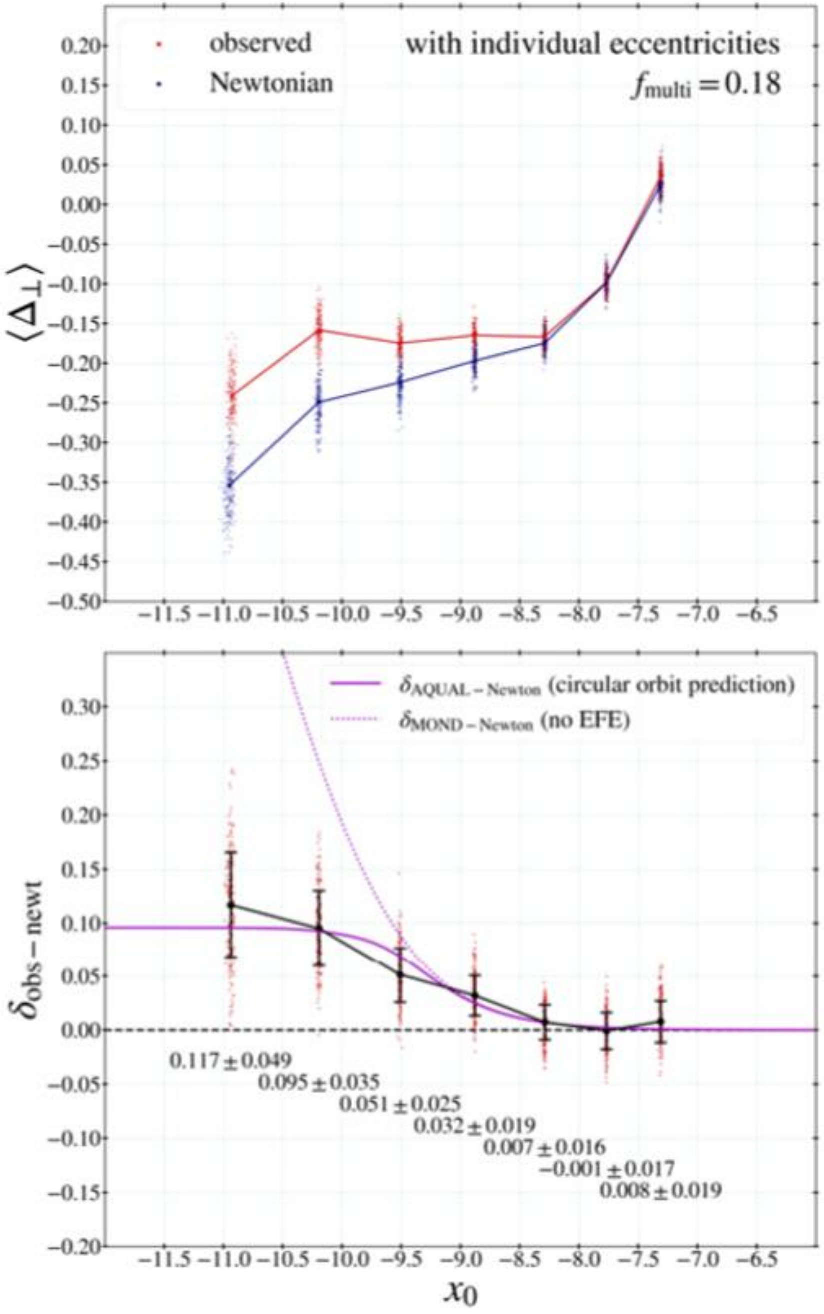
For general samples a constant (i.e. regardless of s) f_{multi} is assumed and fitted with the highest acceleration bin.



Chae (2023a) sample



Chae (2024b) new sample



VICE

‘Unbelievable’: Astronomer Claims ‘Direct Evidence’ of Gravity Breaking Down

A scientist has observed a “gravitational anomaly” in certain star systems that could potentially upend a fundamental assumption about the universe.

Aug 9, 2023

SciTechDaily

Conclusive Evidence for Modified Gravity: Collapse of Newton’s and Einstein’s Theories in Low Acceleration

A study on the orbital motions of wide binaries has uncovered evidence that standard gravity breaks down at low accelerations.

Aug 12, 2023

The Independent

Astronomer uncovers ‘direct evidence’ of gravity breaking down in the universe

A scientist claims to have discovered a “gravitational anomaly” that calls into question our fundamental understanding of the universe.

Aug 14, 2023

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AUGUST 8, 2023 | Editors' notes

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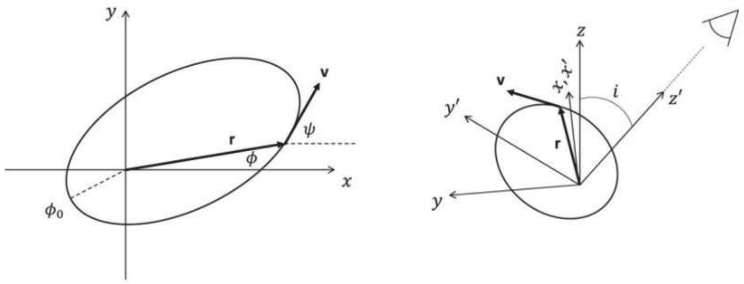
Smoking-gun evidence for modified gravity at low acceleration from Gaia observations of wide binary stars

by Sejong University



orbital plane (face-on view) | observer’s view (3D geometry)

The left panel shows an elliptical orbit in an orbital plane viewed face-on. Th...

Interpretations

- The measured gravitational anomaly shows that standard gravity breaks down at low acceleration.
- The gravitational anomaly is a pure measurement.
- The magnitude and trend of the anomaly are consistent with the generic prediction of MOND modified gravity theories with the external field effect (EFE) of the Milky Way.
- The gravitational anomaly is inconsistent with the algebraic MOND model without the EFE, and thus any modified gravity theory mimicking it (e.g. Moffat's MOG, Verlinde's emergent gravity?)

Algebraic MOND (Milgrom 1983)

- $\mu(g/a_0)g = g_N$ with $\begin{cases} \mu(x) \to 1 & \text{for } x \gg 1 \quad \text{(Newton regime)} \\ \mu(x) \to x & \text{for } x \ll 1 \quad \text{(MOND regime)} \end{cases}$

- $g = \nu(g_N/a_0)g_N$ with $\begin{cases} \nu(y) \to 1 & \text{for } y \gg 1 \quad \text{(Newton regime)} \\ \nu(y) \to 1/\sqrt{y} & \text{for } y \ll 1 \quad \text{(MOND regime)} \end{cases}$

$\mu(x)\nu(y) = 1$ (relation between interpolating functions)

$a_0 \approx 1.2 \times 10^{-10}\ \mathrm{m\,s^{-2}} = 0.12\ \mathrm{nm\,s^{-2}}$ (MOND acceleration constant)

A MOND model: AQUAL(Aquadratic Lagrangian) theory "nonrelativistic MOND-type gravity theory"

$$L = -\int d^3r \left\{ \rho\varphi + (8\pi G)^{-1} a_0^2 \mathscr{F}\left[\frac{(\nabla\varphi)^2}{a_0^2}\right]\right\}$$

(Lagrangian)

$$\nabla \cdot [\mu(|\nabla\varphi|/a_0)\nabla\varphi] = 4\pi G\rho$$

(modified Poisson equation)

$$\mu(x) = \mathcal{F}'(x^2)$$

THE ASTROPHYSICAL JOURNAL, **286**:7–14, 1984 November 1

DOES THE MISSING MASS PROBLEM SIGNAL THE BREAKDOWN OF NEWTONIAN GRAVITY?

JACOB BEKENSTEIN
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AND

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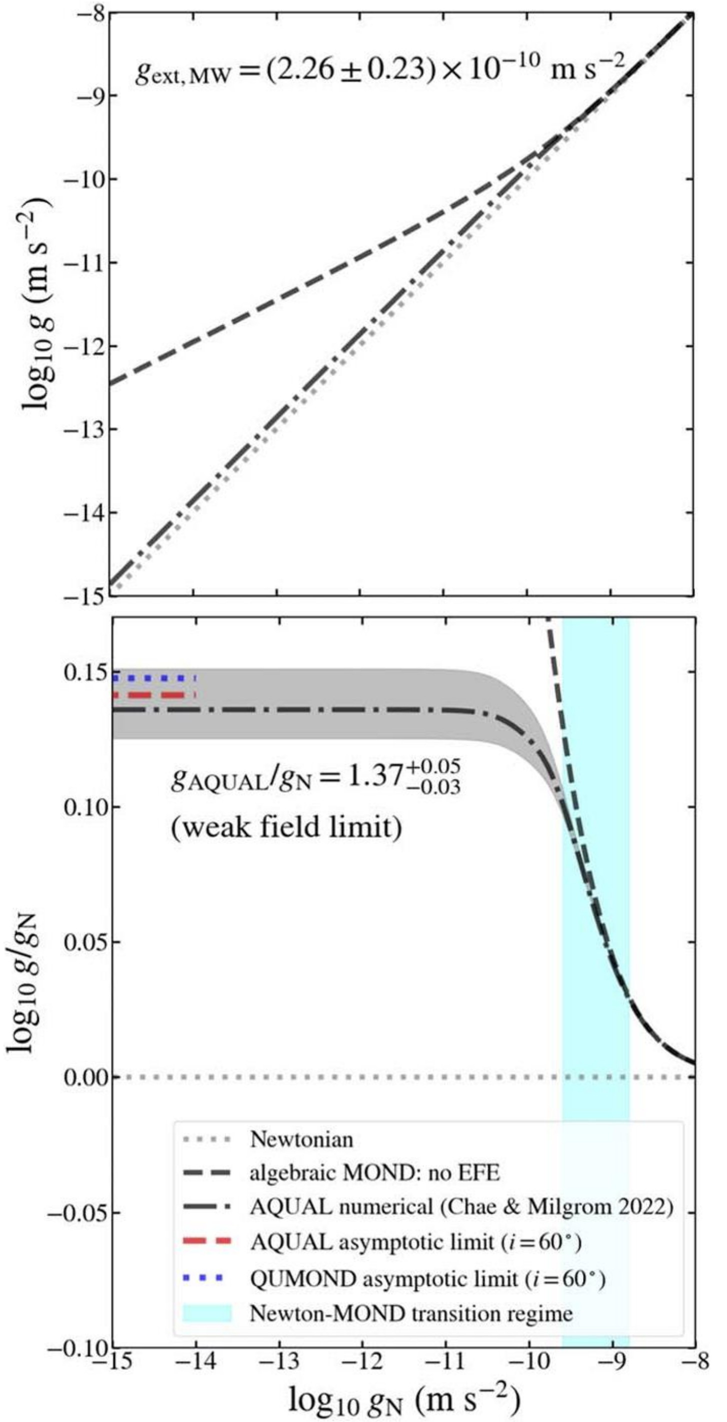
Department of Physics, Weizmann Institute of Science, Rehovot
Received 1984 March 28; accepted 1984 May 17

See also quasi-linear MOND (QUMOND) (Milgrom 2010, MNRAS).

Newton vs AQUAL for circular orbits

Based on Chae & Milgrom (2022) numerical solutions of the AQUAL modified Poisson equation



Bayesian 3D modeling: Towards an ultimate test with snapshot observations

- **Individual inferences** of gravity are derived.
- All **uncertainties** of the observational quantities (in particular, stellar masses) are **naturally reflected** in the inferences.
- Individual inferences for a similar gravity regime can be **consolidated** with a verifiable method.
- Can we know that the method will work? We can test the method with simulated data.
- What are the data requirements? We can use simulations to learn.
- Can even the current data give meaningful results?

The idea: use all available components of the relative displacement and the relative velocity

- Three observational constraints

$$\Delta x' = -3600 d_M \cos(0.5(\delta_A + \delta_B)\pi/180)\Delta\alpha \text{ au},$$
$$\Delta y' = 3600 d_M \Delta\delta \text{ au},$$

$$s = \sqrt{(\Delta x')^2 + (\Delta y')^2}.$$

$$v_{x'} = -4.7404 d_M (\mu^\star_{\alpha,B} - \mu^\star_{\alpha,A}) \text{ m s}^{-1},$$
$$v_{y'} = 4.7404 d_M (\mu_{\delta,B} - \mu_{\delta,A}) \text{ m s}^{-1},$$

$$v_p = \sqrt{v_{x'}^2 + v_{y'}^2}.$$

$$v_r = v_{z'} = -1000(\mathrm{RV}_B - \mathrm{RV}_A) \text{ m s}^{-1}.$$

$$v_{\mathrm{obs}} = \sqrt{v_p^2 + v_r^2}.$$ **(constraint 1)**

$$\beta_{p,\mathrm{obs}} \equiv \frac{v_{y'}}{v_{x'}},$$ **(constraint 2)**

$$\tau_{\mathrm{obs}} \equiv -\frac{v_r}{v_{y'}},$$ **(constraint 3)**

- Predictions of pseudo-Newtonian model ($G = \gamma_g\, G_{\mathrm{N}}$)

$$v_{\mathrm{mod}} = \sqrt{\frac{\gamma_g G_{\mathrm{N}} f_M M_{\mathrm{tot}}}{s/\sqrt{\cos^2\phi + \cos^2 i \sin^2\phi}} \left(2 - \frac{1-e^2}{1+e\cos(\phi-\phi_0)}\right)}.$$

$$\beta_{p,\mathrm{mod}} = -\cos i \frac{\cos\phi + e\cos\phi_0}{\sin\phi + e\sin\phi_0}.$$

$$\tau_{\mathrm{mod}} = \tan i$$

Bayesian inference

$$\ln p(\mathbf{\Theta}) = \ln \mathcal{L} + \sum_k \ln \Pr(\Theta_k),$$

$$\ln \mathcal{L} = -\frac{1}{2}\sum_j \left[\left(\frac{X_{j,\mathrm{obs}} - X_{j,\mathrm{mod}}(\mathbf{\Theta})}{\sigma_j}\right)^2 + \ln(2\pi\sigma_j^2)\right]$$

$$\mathbf{\Theta} = \{e, \phi_0, i, f_M, \Gamma\}$$

with $\Gamma \equiv \frac{1}{2}\log\gamma_g$,
f_M = mass parameter

Important priors

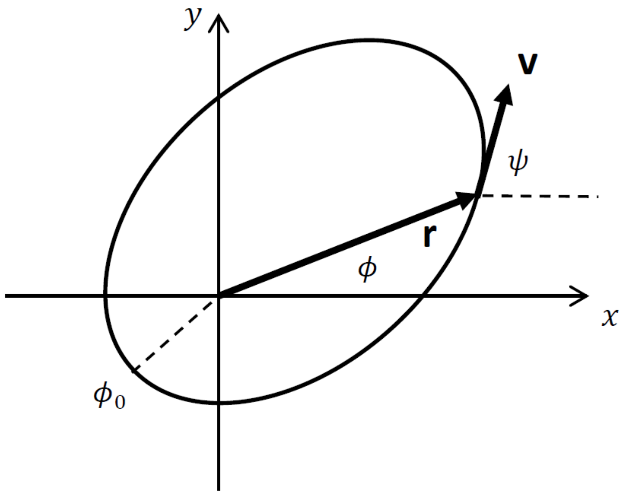
- Eccentricity: either flat or $\Pr(e) = (1+\alpha)e^{\alpha}$ (power-law)
  ($\alpha = 1$ is called "thermal")

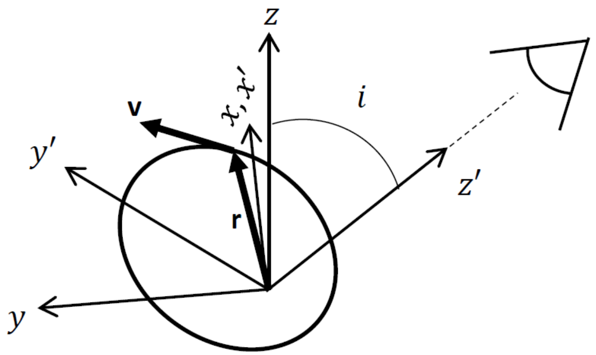
- ϕ_0: flat in time so

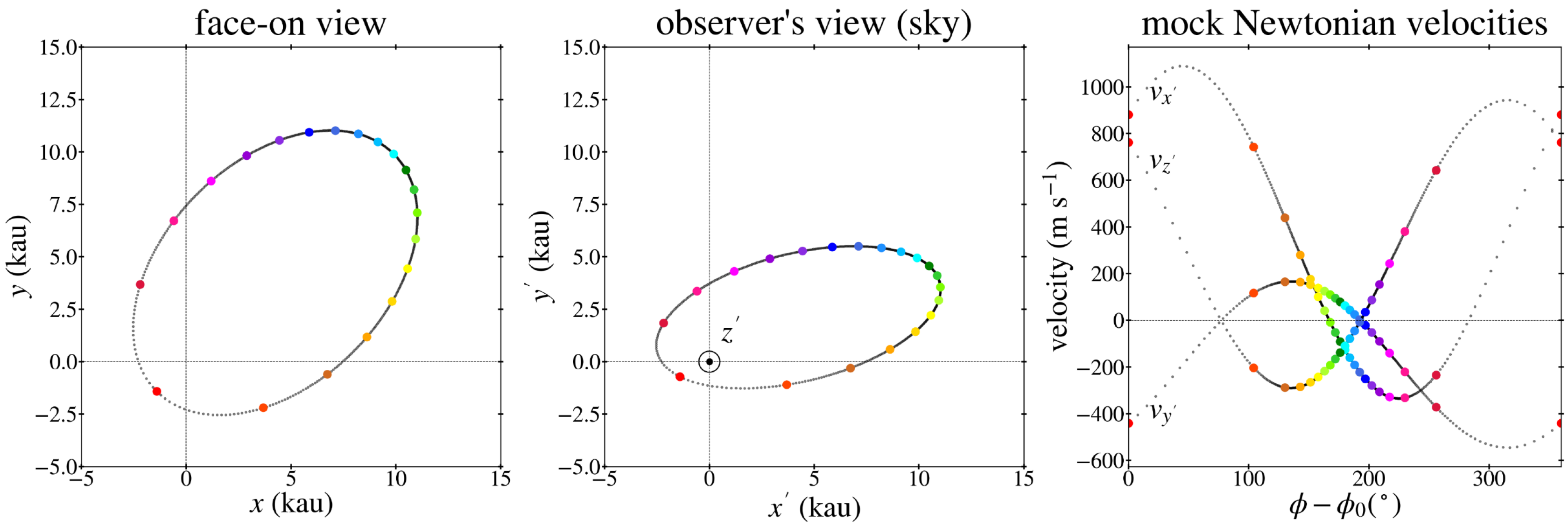
$$\Pr(\phi_0) \propto \frac{1}{[1 + e\cos(\phi - \phi_0)]^2}$$

Testing the method with mock Newtonian data

$$M_{\rm tot} = 2M_\odot$$
$$a = 8\text{kau}$$
$$e = 0.75$$
$$i = 60^\circ$$

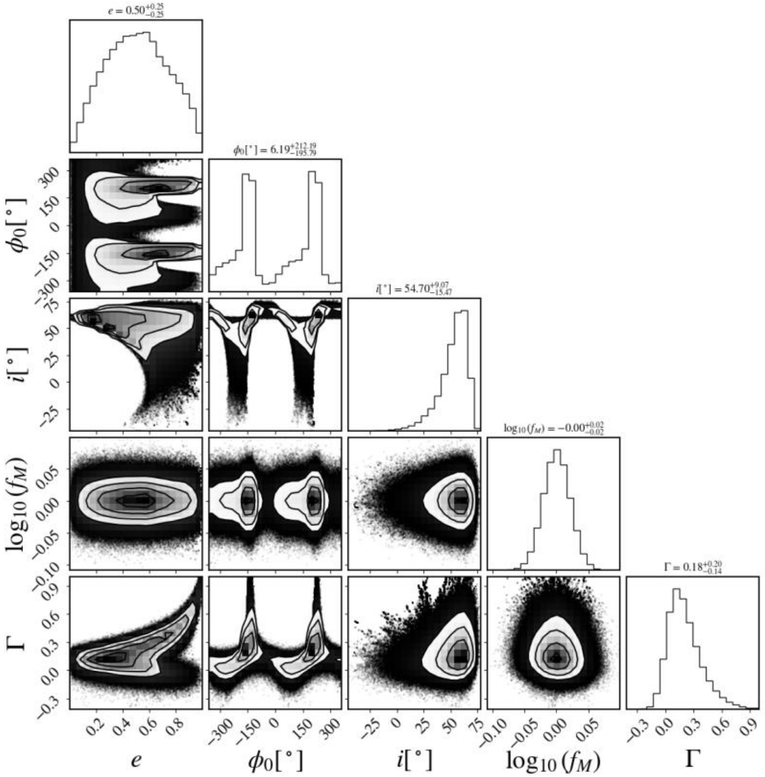




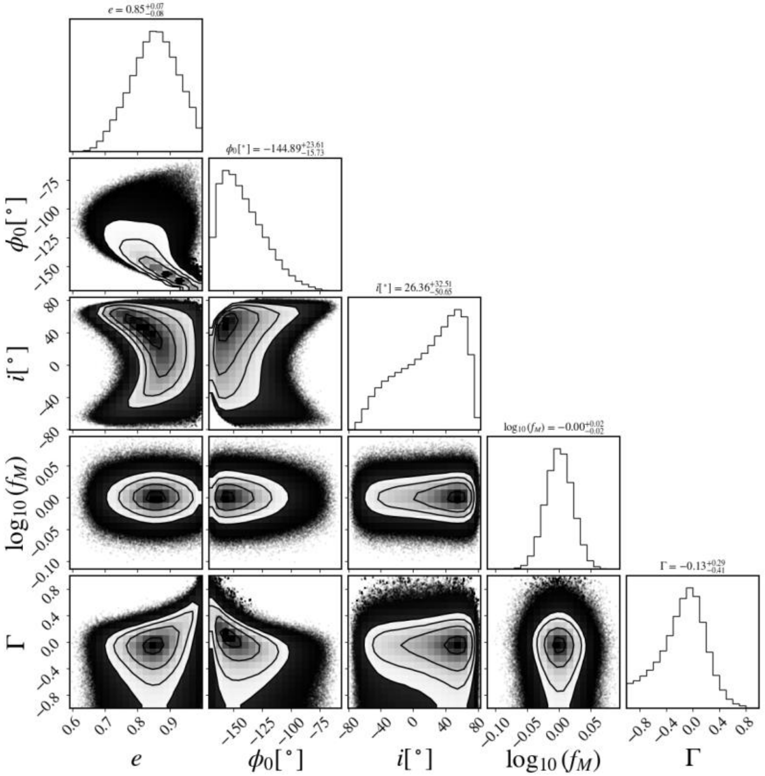


Individual Bayesian inferences with RV uncertainties of 200 m/s (but assuming the values are accurate)

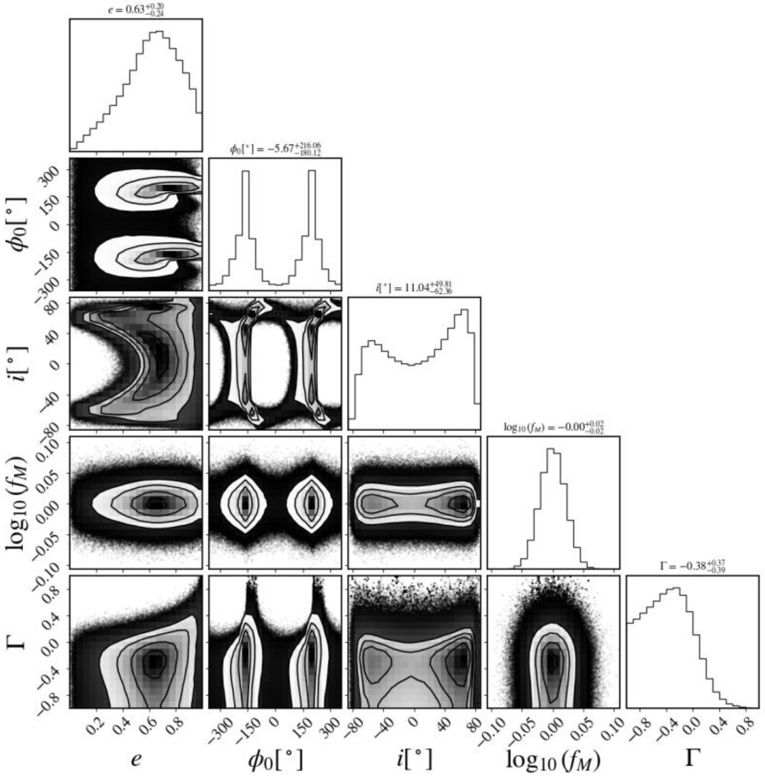
$$\frac{t}{P} = 0$$



$$\frac{t}{P} = 0.2$$



$$\frac{t}{P} = 0.5$$



How to consolidate individual inferences?

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An Interdisciplinary Journal of Nonlinear Science

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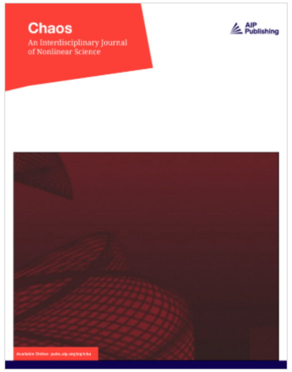
Volume 21, Issue 3
September 2011



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RESEARCH ARTICLE | JULY 20 2011

How to combine independent data sets for the same quantity

Theodore P. Hill; Jack Miller

Check for updates

Chaos 21, 033102 (2011)
https://doi.org/10.1063/1.3593373   Article history

Share ⌄   Tools ⌄

This paper describes a new mathematical method called conflation for consolidating data from independent experiments that measure the same physical quantity. Conflation is easy to calculate and visualize and minimizes the maximum loss in Shannon information in consolidating several independent distributions into a single distribution. A formal mathematical treatment of conflation has recently been published. For the benefit of experimenters wishing to use this technique, in this paper we derive the principal basic properties of conflation in the special case of normally distributed (Gaussian) data. Examples of applications to measurements of the fundamental physical constants and in high energy physics are presented, and the conflation operation is generalized to weighted conflation for cases in which the underlying experiments are not uniformly reliable.

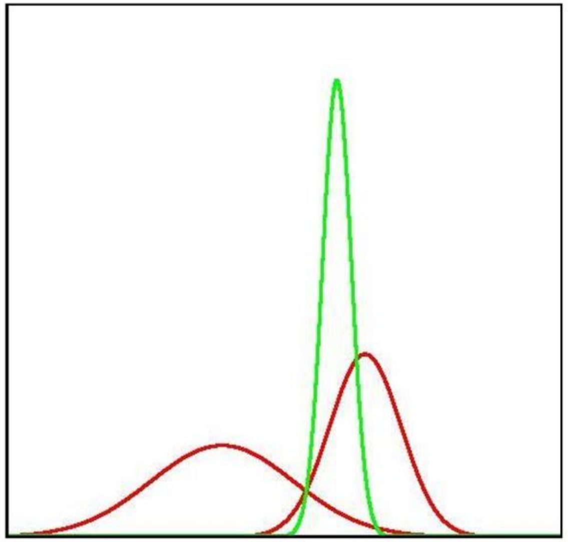


Figure 3. Conflating Distributions

This is the correct method!

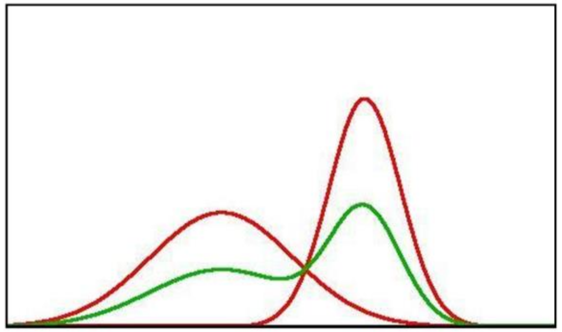
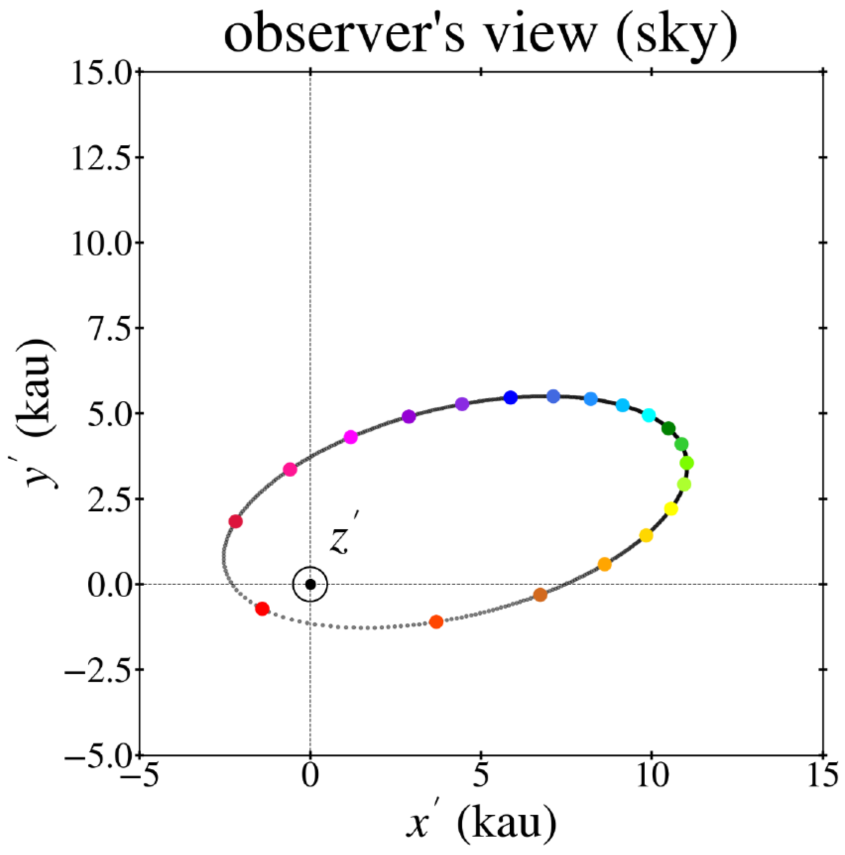
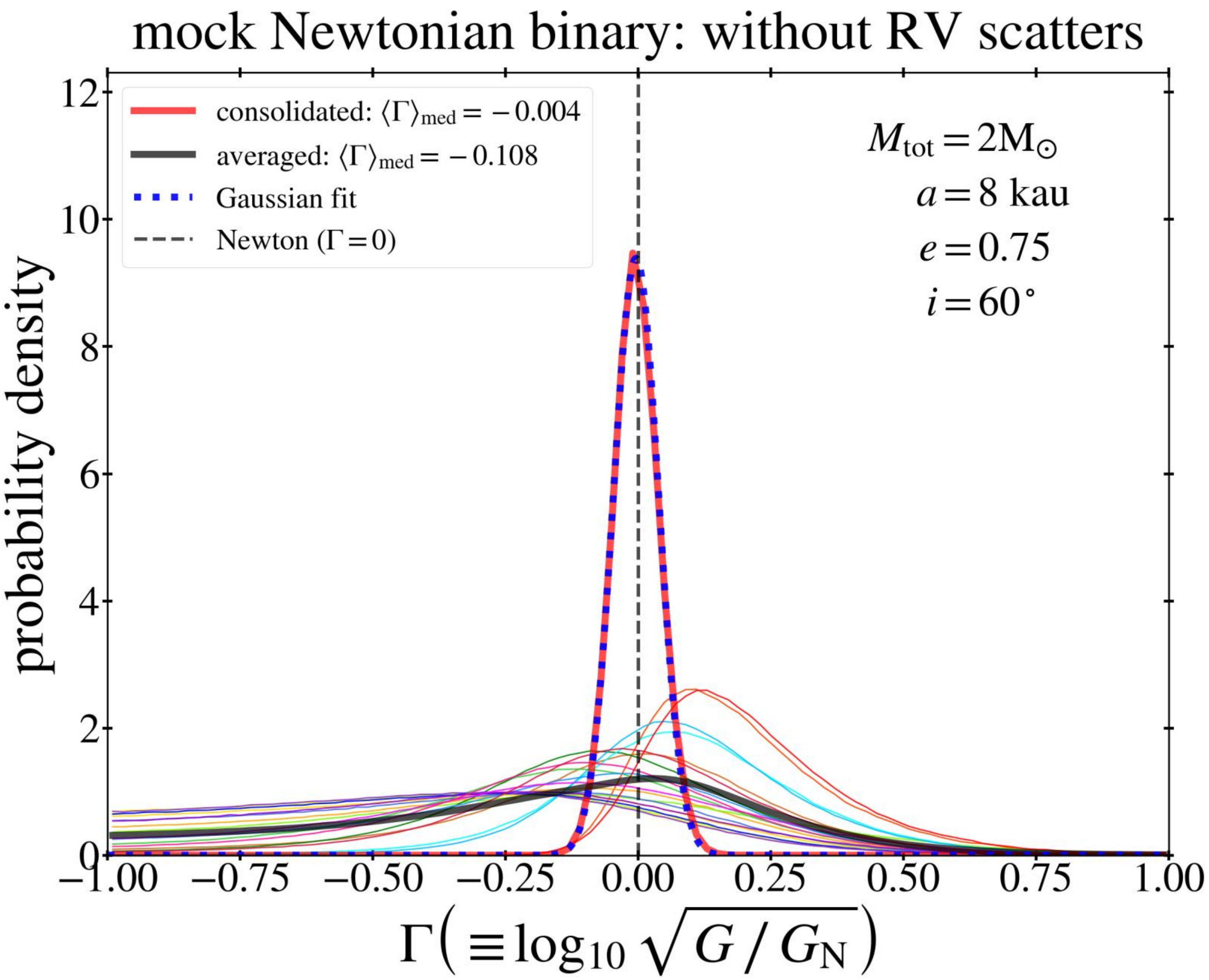


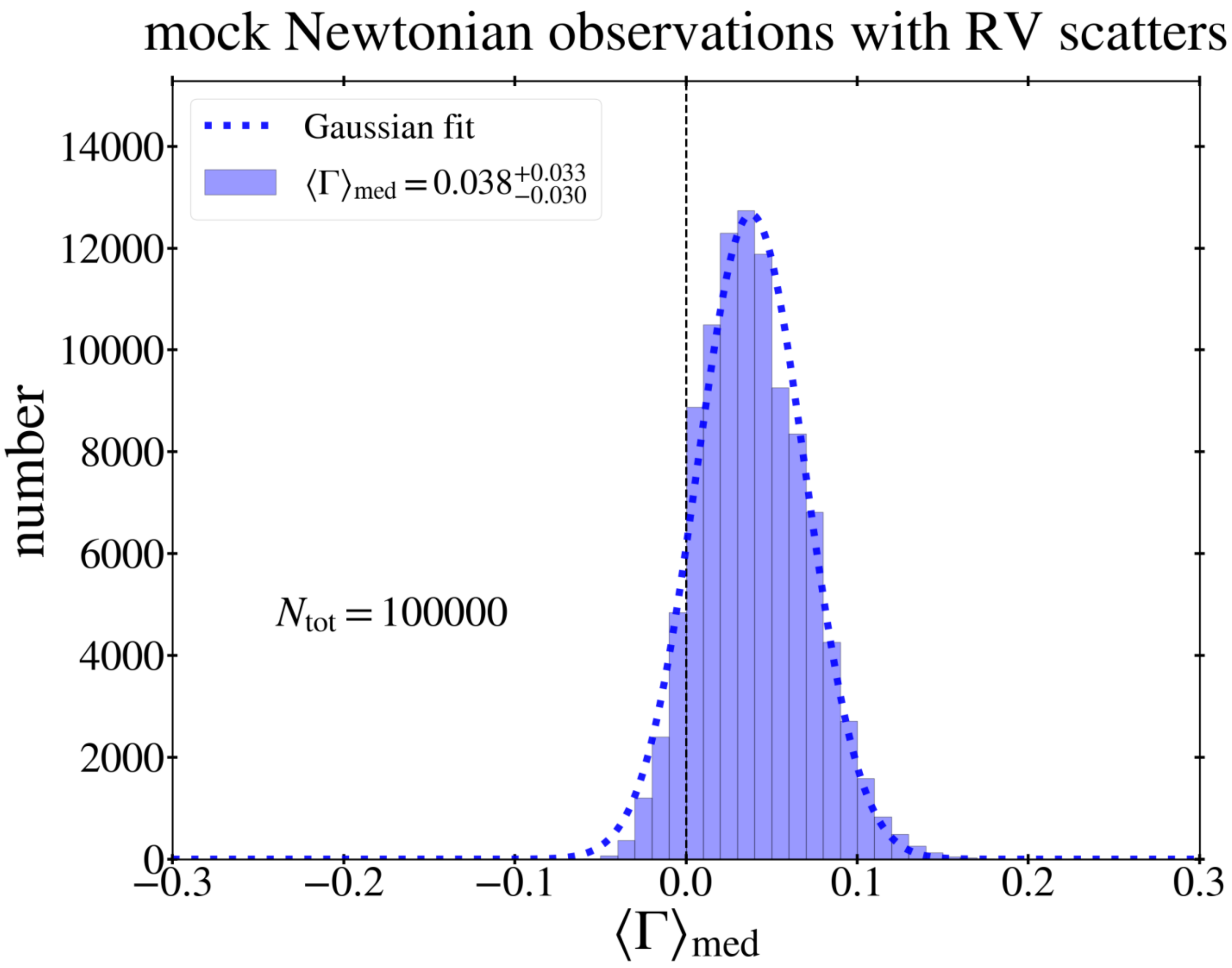
Figure 1. Averaging the Probabilities.


observer's view (sky)
15.0
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−5
0
5
10
15
y′ (kau)
x′ (kau)
z′


mock Newtonian binary: without RV scatters
consolidated: ⟨Γ⟩med = −0.004
averaged: ⟨Γ⟩med = −0.108
Gaussian fit
Newton (Γ = 0)
Mtot = 2M⊙
a = 8 kau
e = 0.75
i = 60°
probability density
0
2
4
6
8
10
12
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
Γ(≡ log10 √(G/GN))

Estimating the uncertainty of the consolidated value

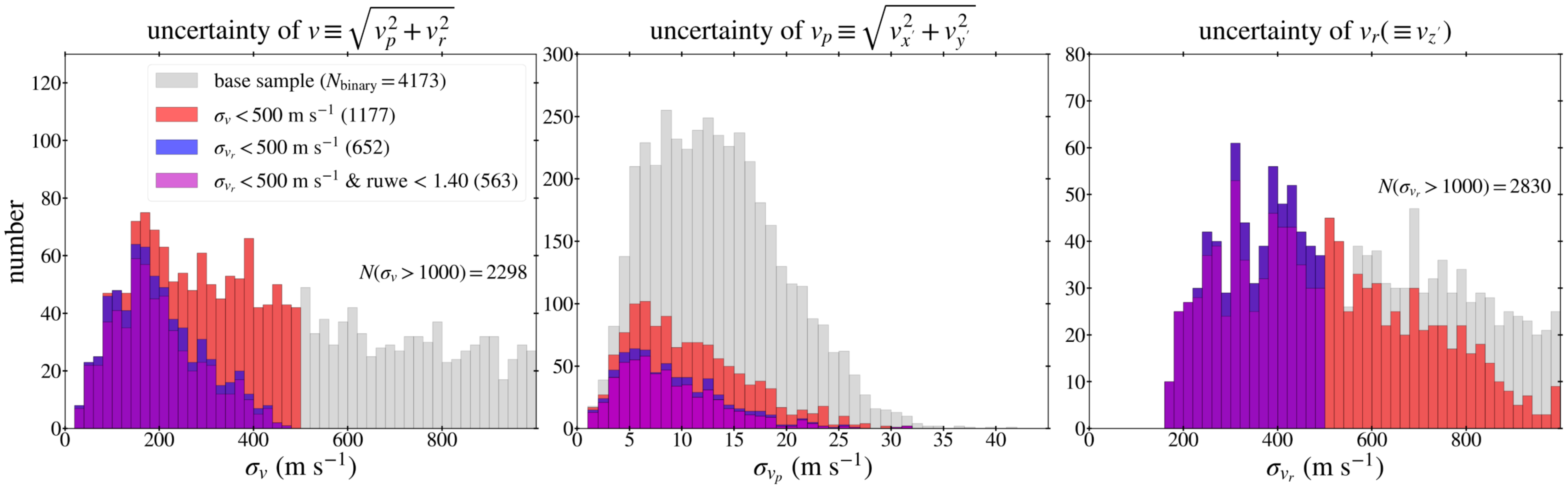
- Newton predicted values of radial velocities are scattered with the assumed uncertainty of 200 m/s.
- The consolidated value is biased: $\langle\Gamma\rangle_{\rm med} = 0.038^{+0.033}_{-0.030}$.
- The expected bias depends on the properties of the data.



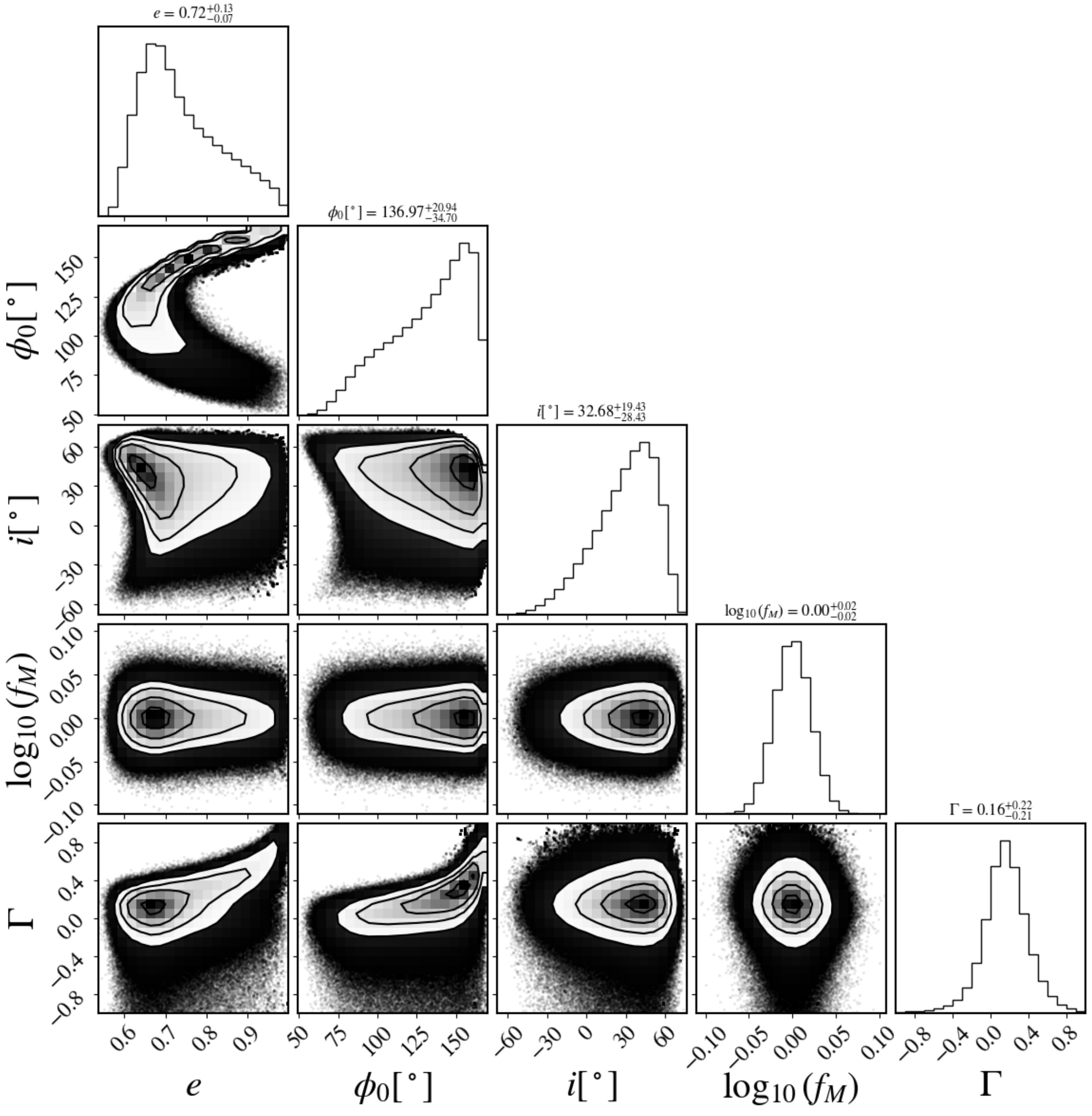
Selecting binaries from the Gaia database for Bayesian modeling

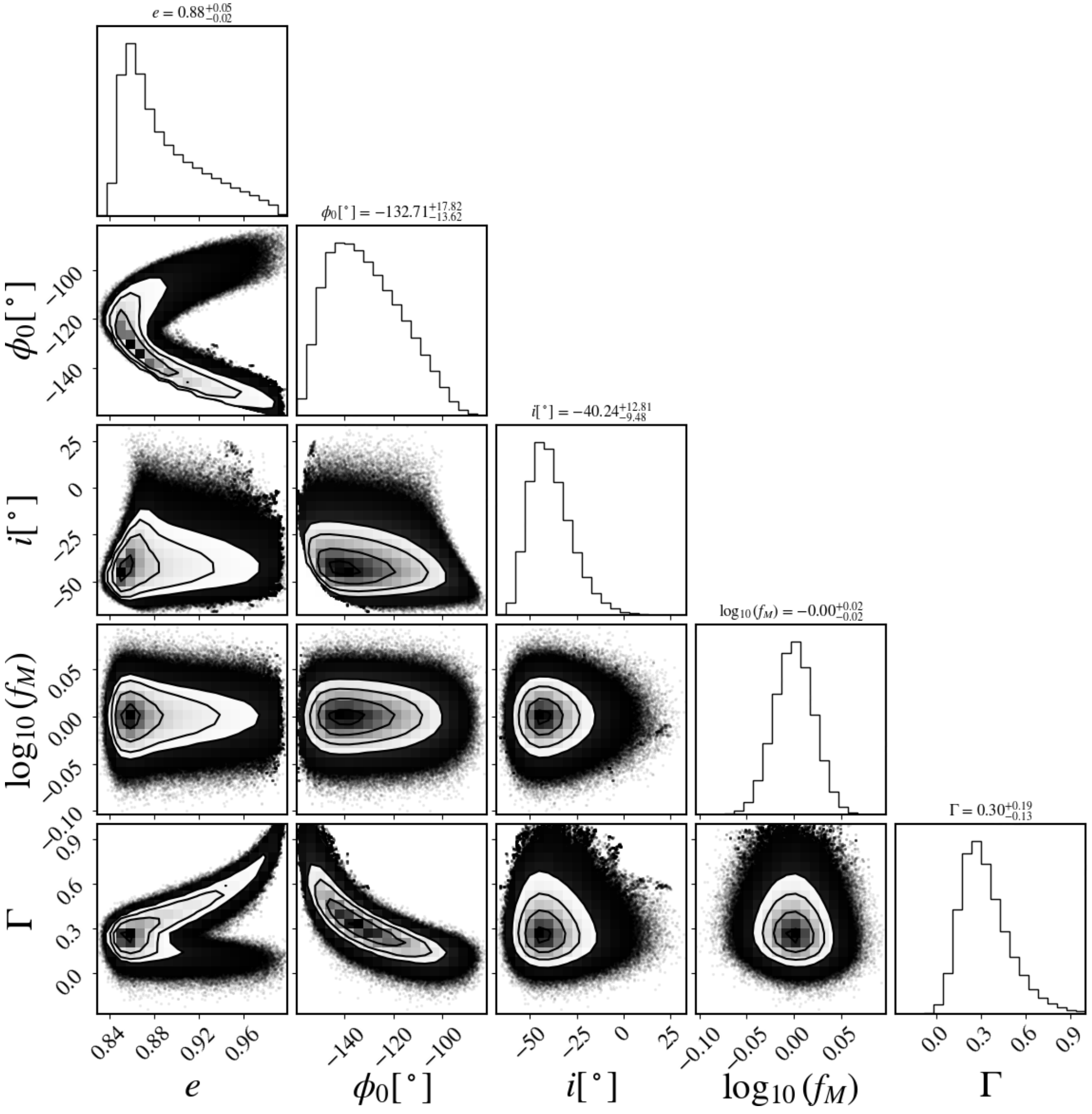
- Initial selection from the El-Badry et al. (2021) catalogue following the strategy of Chae (2024a). But, the following changes are made:
- $3.8 < M_G < 13.4$
- $d < 200$ pc and Decl. $> -28°$ with dust extinction information
- $d < 100$ pc or $|b| > 60°$ without dust extinction information
- 4276 binaries are selected.
- 35 cases are removed as resolved multiples and 68 as chance alignments based on a stricter criterion than El-Badry et al. leaving 4173 binaries (i.e. 2.4% are excluded).
- 1177 from them have $\sigma_v < 500$ m/s.
- 652 from them have $\sigma_{v_r} < 500$ m/s.
- 563 from them have ruwe < 1.4.

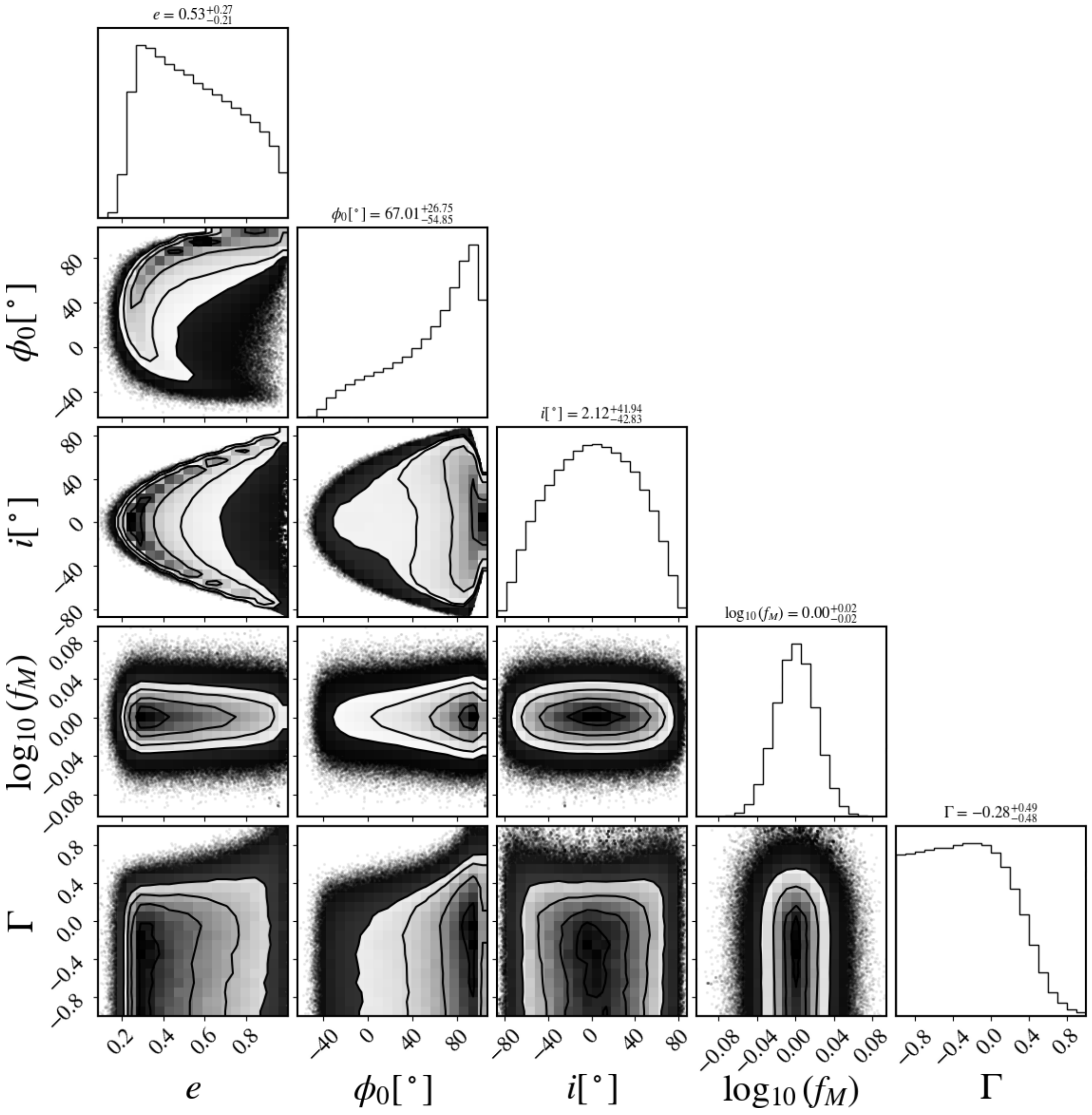
Distribution of velocity uncertainties in the selected sample

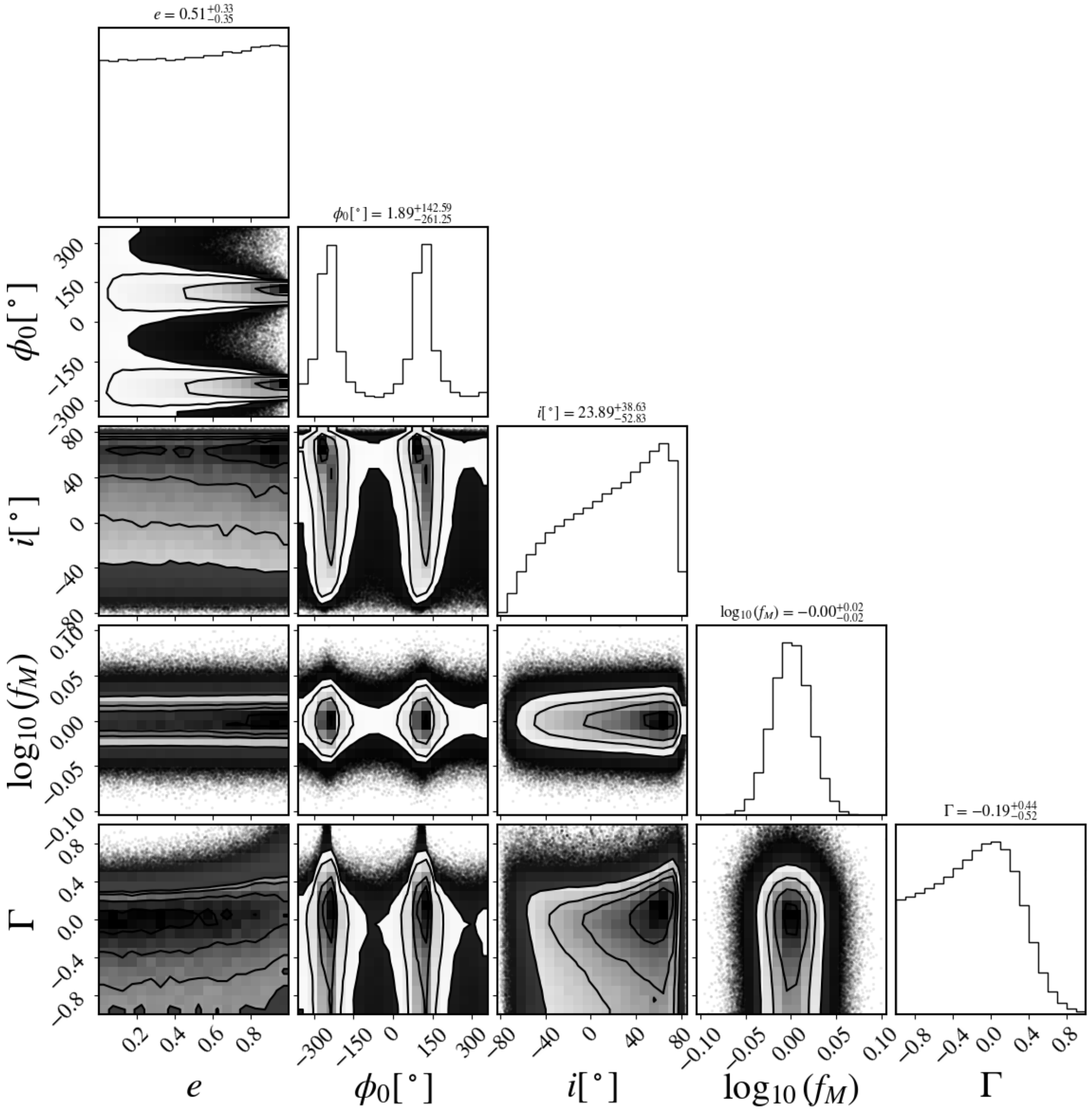


Results for the selected Gaia binaries: examples





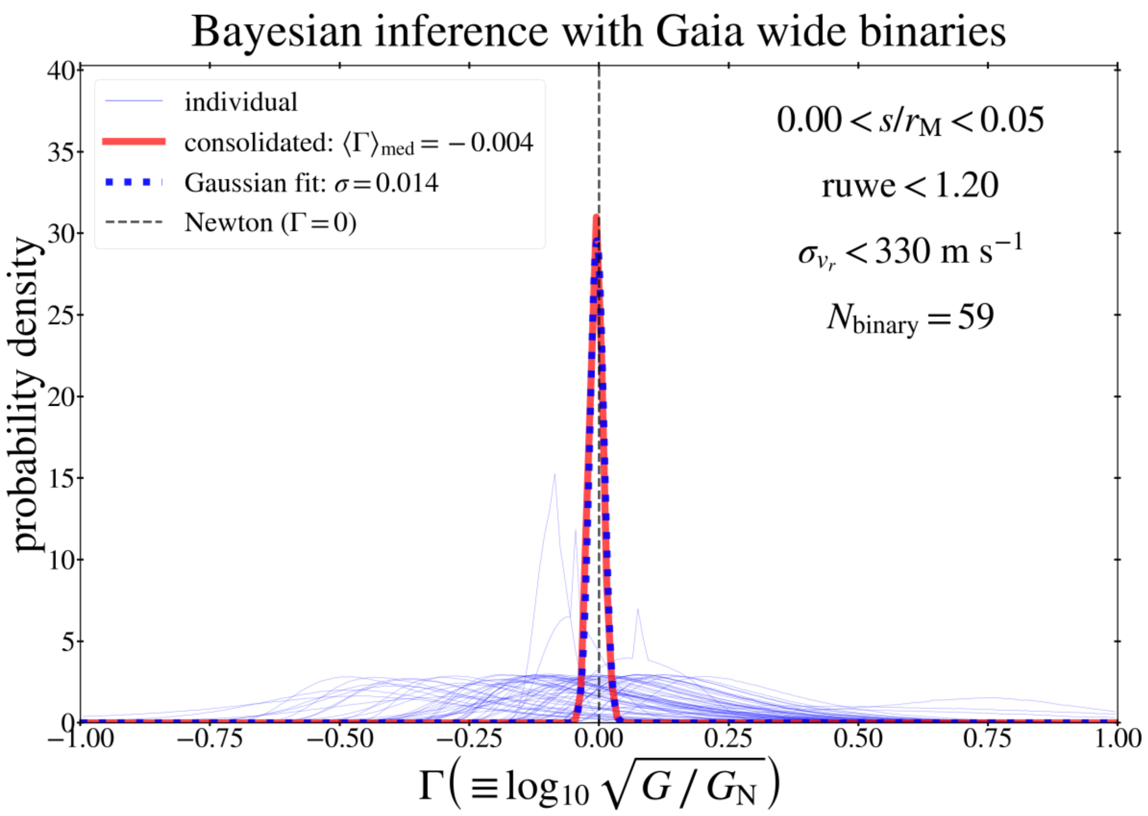

$e = 0.53^{+0.27}_{-0.21}$
$\phi_0[^\circ] = 67.01^{+26.75}_{-54.85}$
$i[^\circ] = 2.12^{+41.94}_{-42.83}$
$\log_{10}(f_M) = 0.00^{+0.02}_{-0.02}$
$\Gamma = -0.28^{+0.49}_{-0.48}$
e
$\phi_0[^\circ]$
$i[^\circ]$
$\log_{10}(f_M)$
Γ

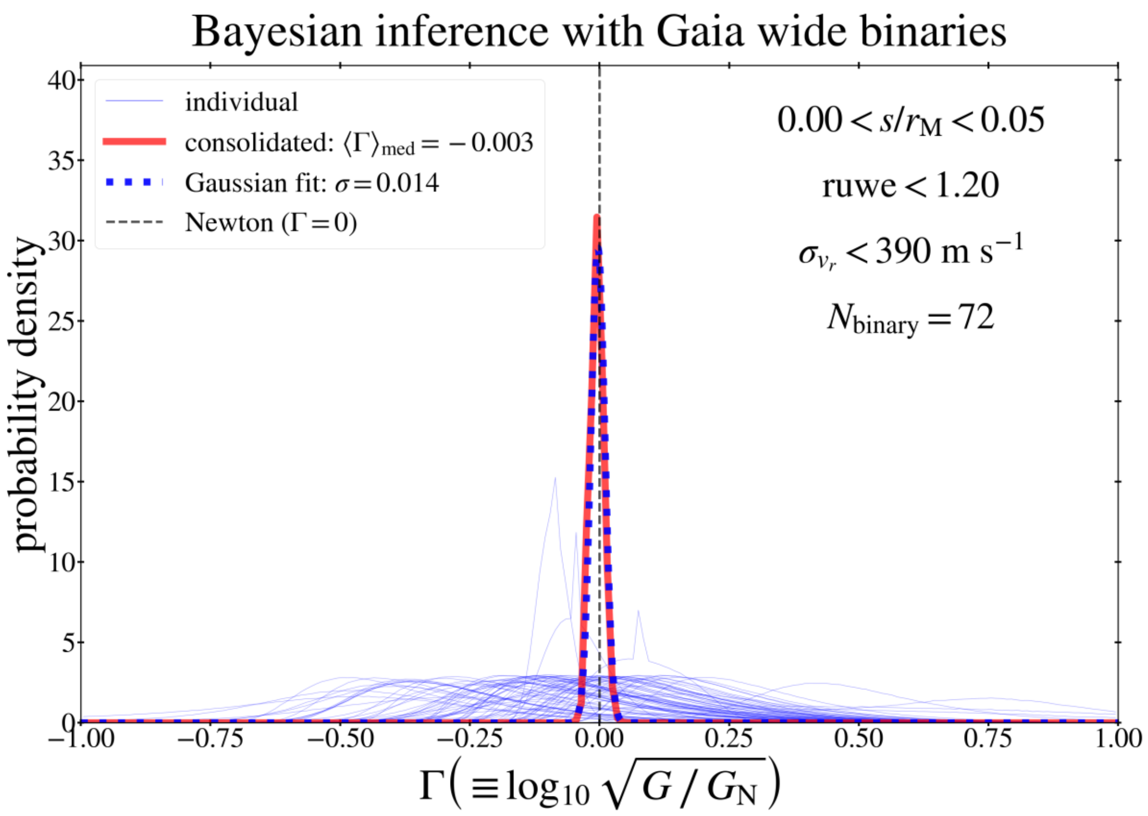

$e = 0.51^{+0.33}_{-0.35}$
$\phi_0[^\circ] = 1.89^{+142.59}_{-261.25}$
$i[^\circ] = 23.89^{+38.63}_{-52.83}$
$\log_{10}(f_M) = -0.00^{+0.02}_{-0.02}$
$\Gamma = -0.19^{+0.44}_{-0.52}$
e
$\phi_0[^\circ]$
$i[^\circ]$
$\log_{10}(f_M)$
Γ

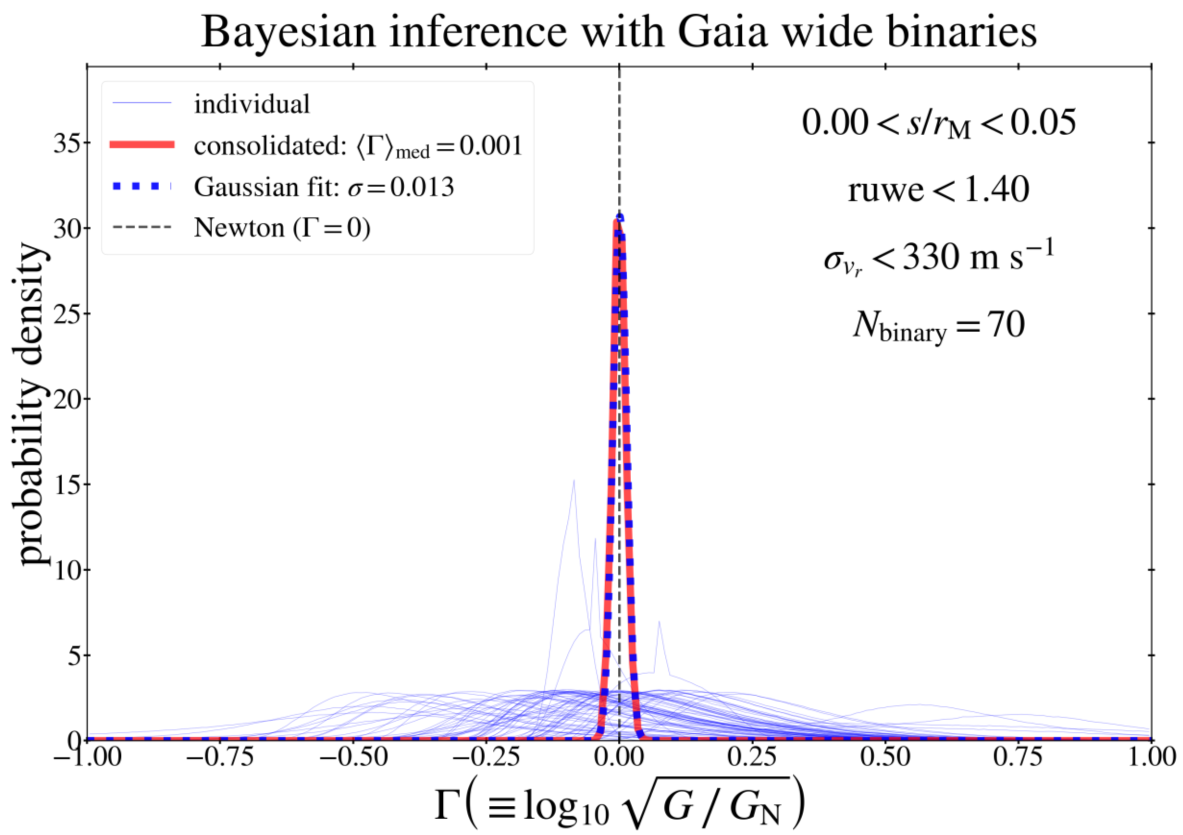
How to remove potentially biased results

- Uses Gaia's ruwe parameter to remove potentially problematic astronomical solutions.
- Requires that individual PDF includes the currently likely gravity range within 3σ. I.e., if a PDF is too off from the Newtonian value of $\Gamma = 0$, we suspect that the system may be kinematically contaminated, e.g. due to hidden close faint stars or Jovian planets. This ensures that the consolidated PDF is not dominated by a few exceptions.

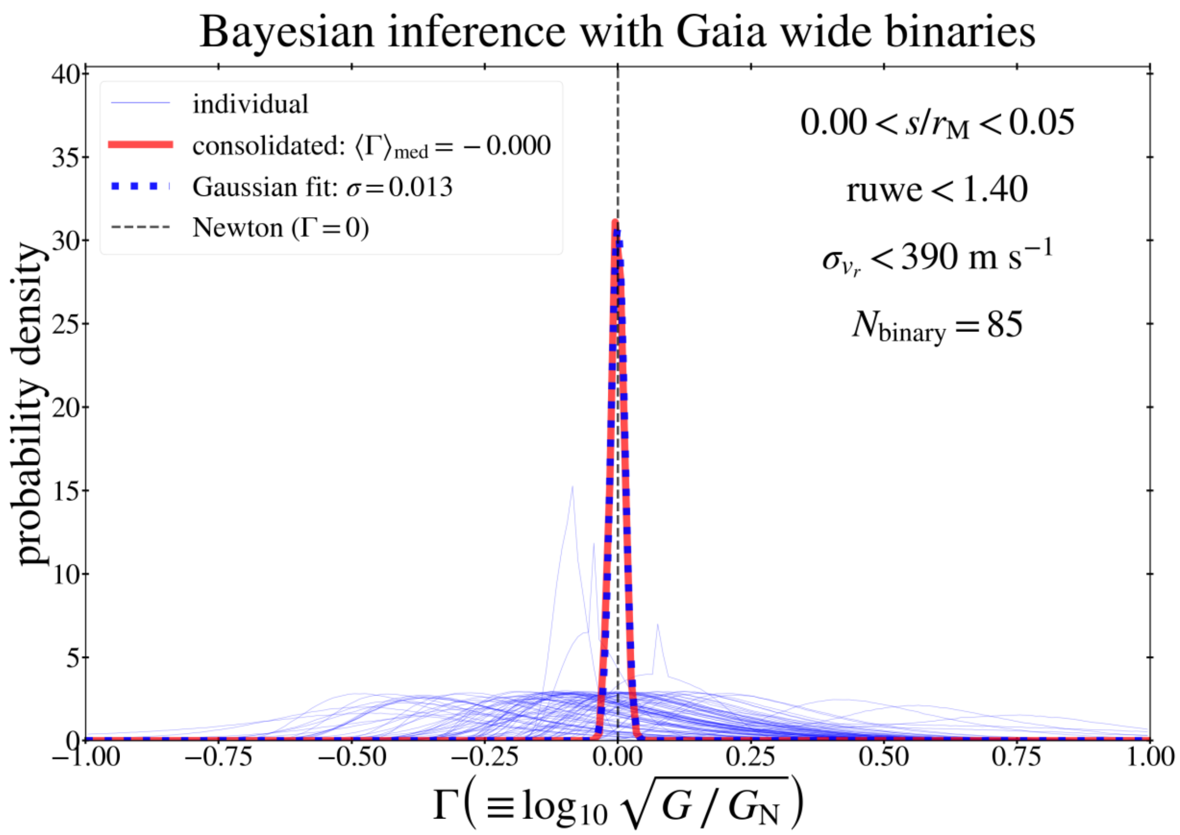
Results for the Newtonian regime





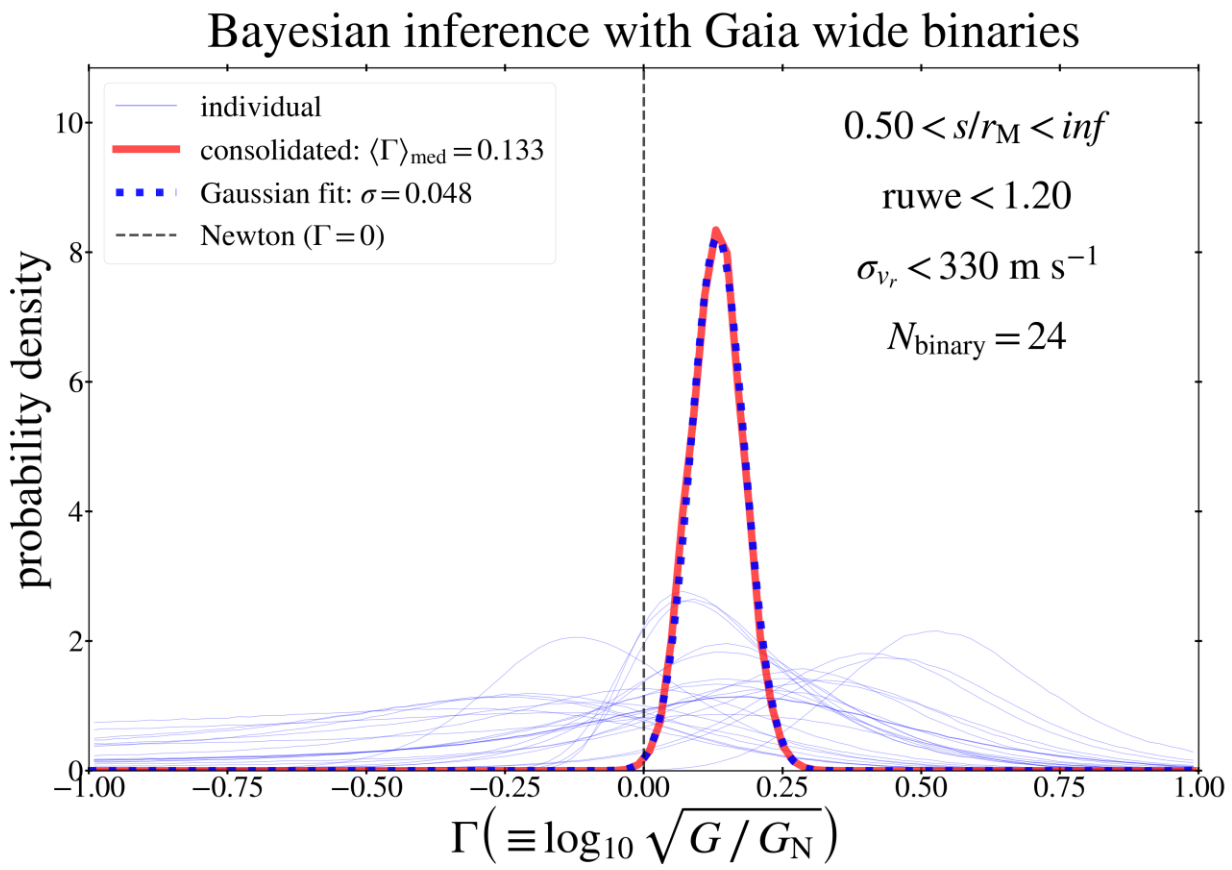


Bayesian inference with Gaia wide binaries
individual
consolidated: $\langle\Gamma\rangle_{\rm med}=0.001$
Gaussian fit: $\sigma=0.013$
Newton ($\Gamma=0$)
$0.00<s/r_{\rm M}<0.05$
ruwe <1.40
$\sigma_{v_r}<330$ m s^{-1}
$N_{\rm binary}=70$
probability density
$\Gamma\left(\equiv\log_{10}\sqrt{G/G_{\rm N}}\right)$

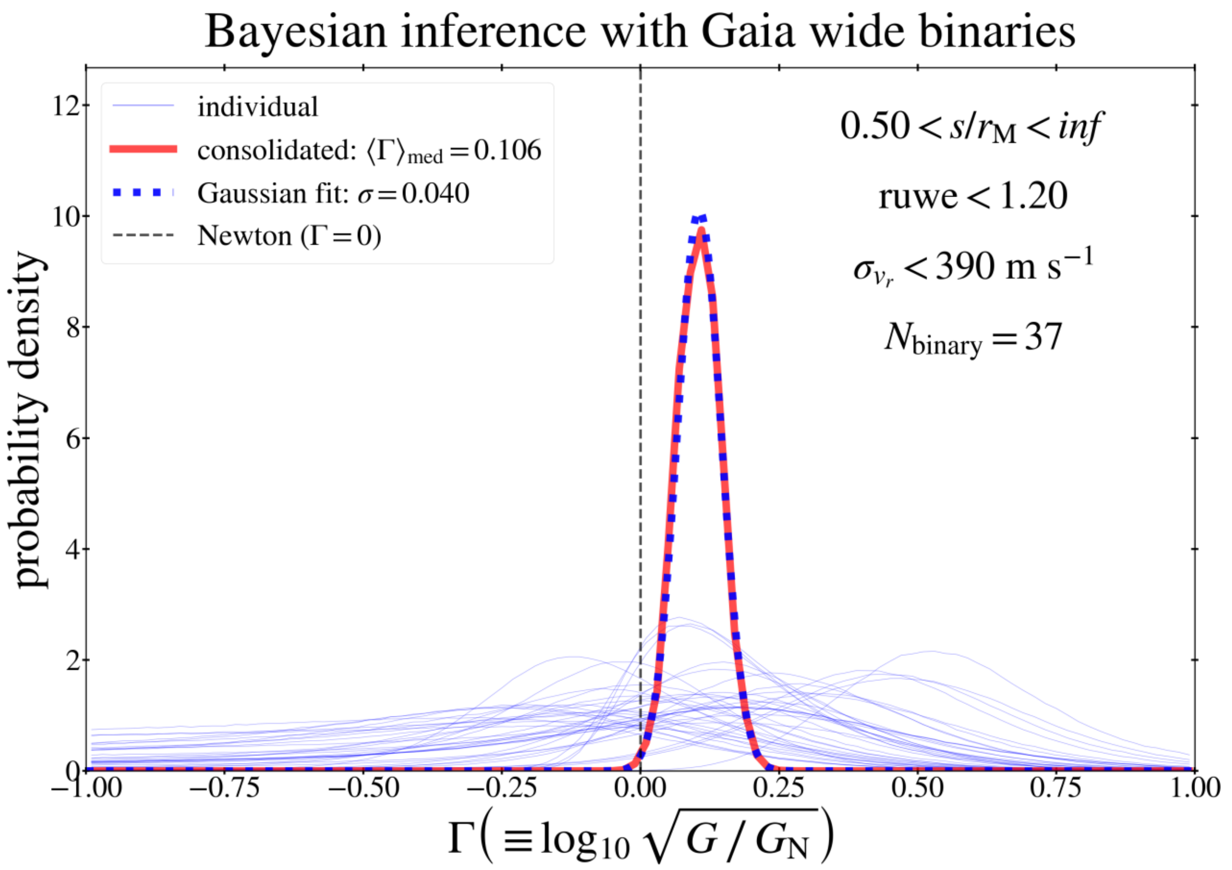


Bayesian inference with Gaia wide binaries
individual
consolidated: $\langle\Gamma\rangle_{\rm med}=-0.000$
Gaussian fit: $\sigma=0.013$
Newton ($\Gamma=0$)
$0.00<s/r_{\rm M}<0.05$
ruwe <1.40
$\sigma_{v_r}<390$ m s^{-1}
$N_{\rm binary}=85$
probability density
$\Gamma\left(\equiv\log_{10}\sqrt{G/G_{\rm N}}\right)$

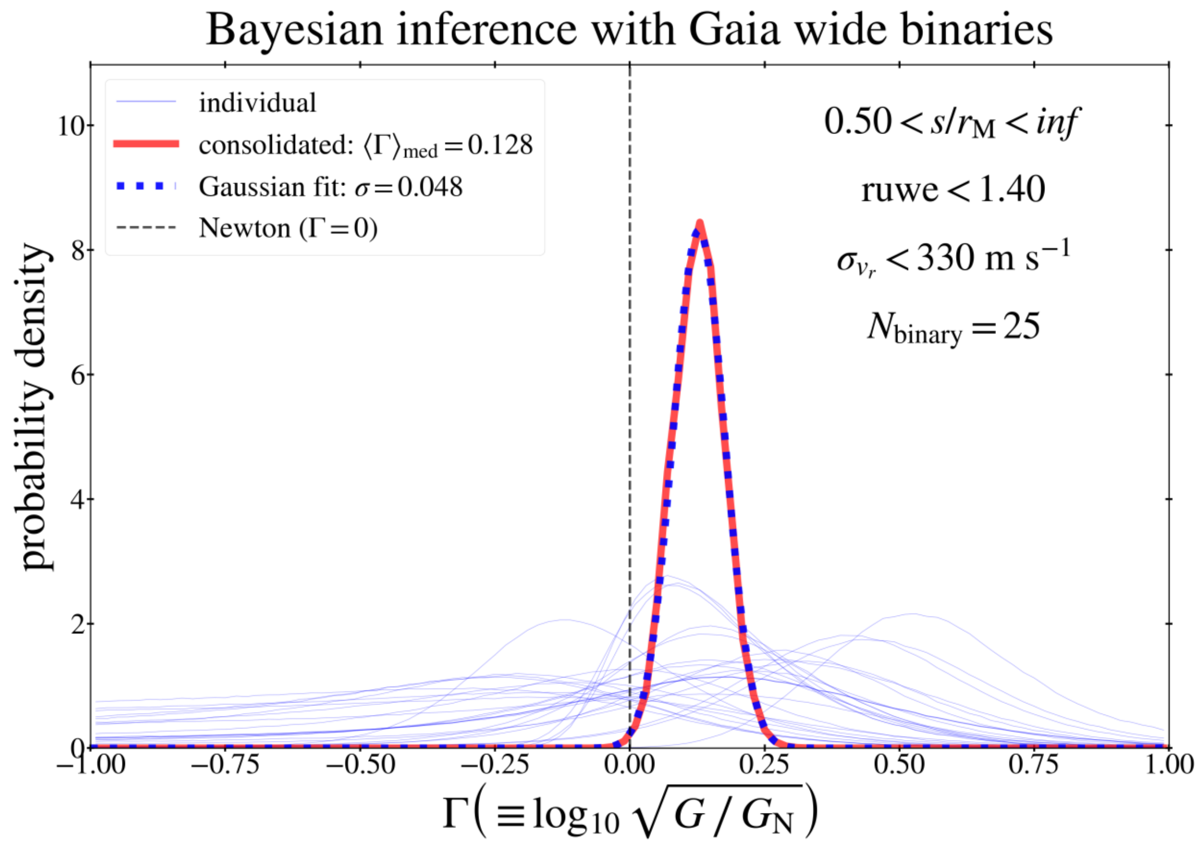
MOND regime

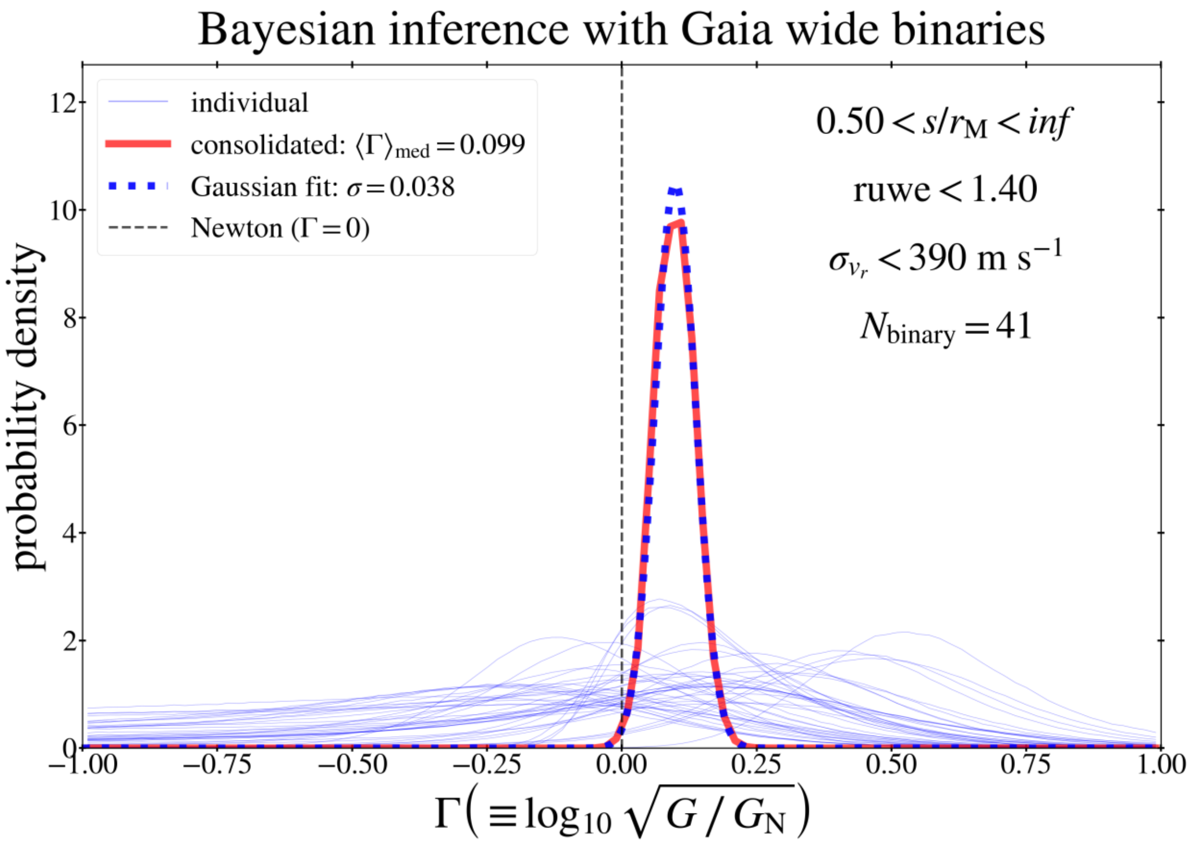


Bayesian inference with Gaia wide binaries
individual
consolidated: $\langle\Gamma\rangle_{med} = 0.133$
Gaussian fit: $\sigma = 0.048$
Newton ($\Gamma = 0$)
$0.50 < s/r_M < inf$
ruwe < 1.20
$\sigma_{v_r} < 330$ m s^{-1}
$N_{binary} = 24$
probability density
$\Gamma\left(\equiv \log_{10}\sqrt{G/G_N}\right)$

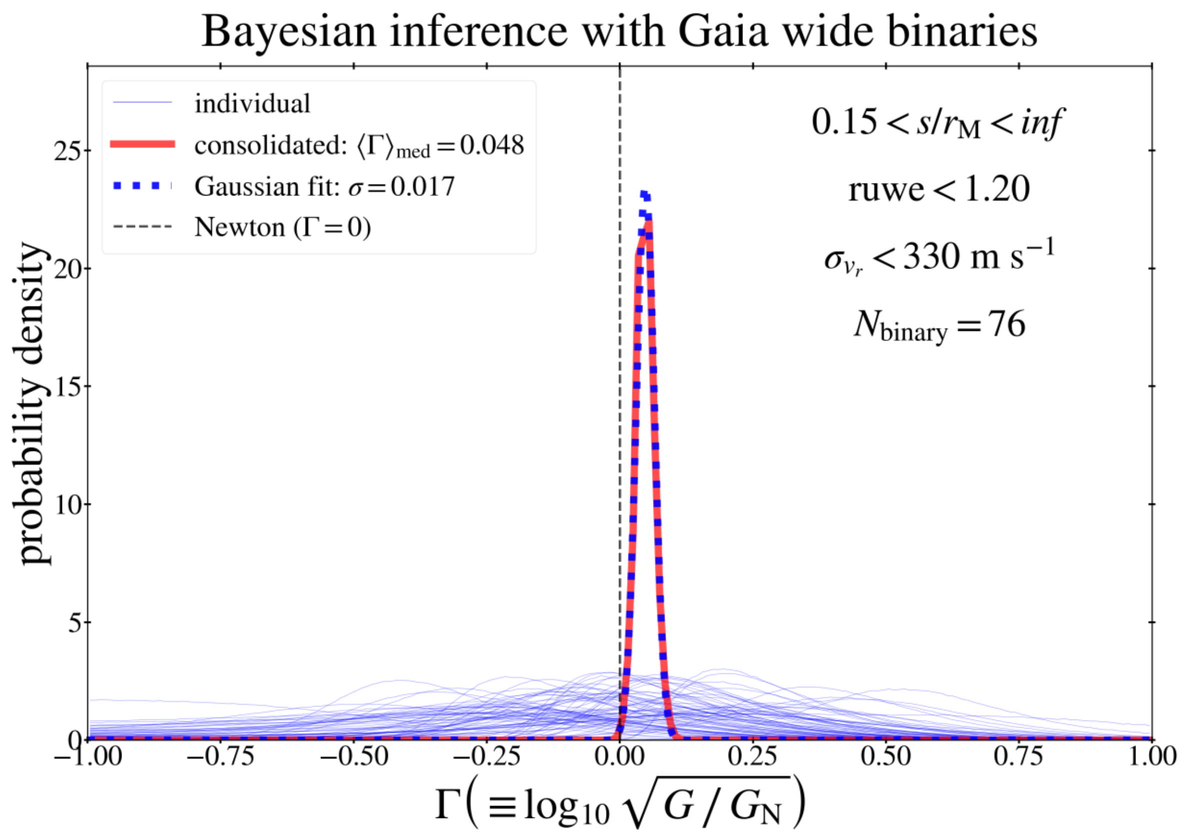


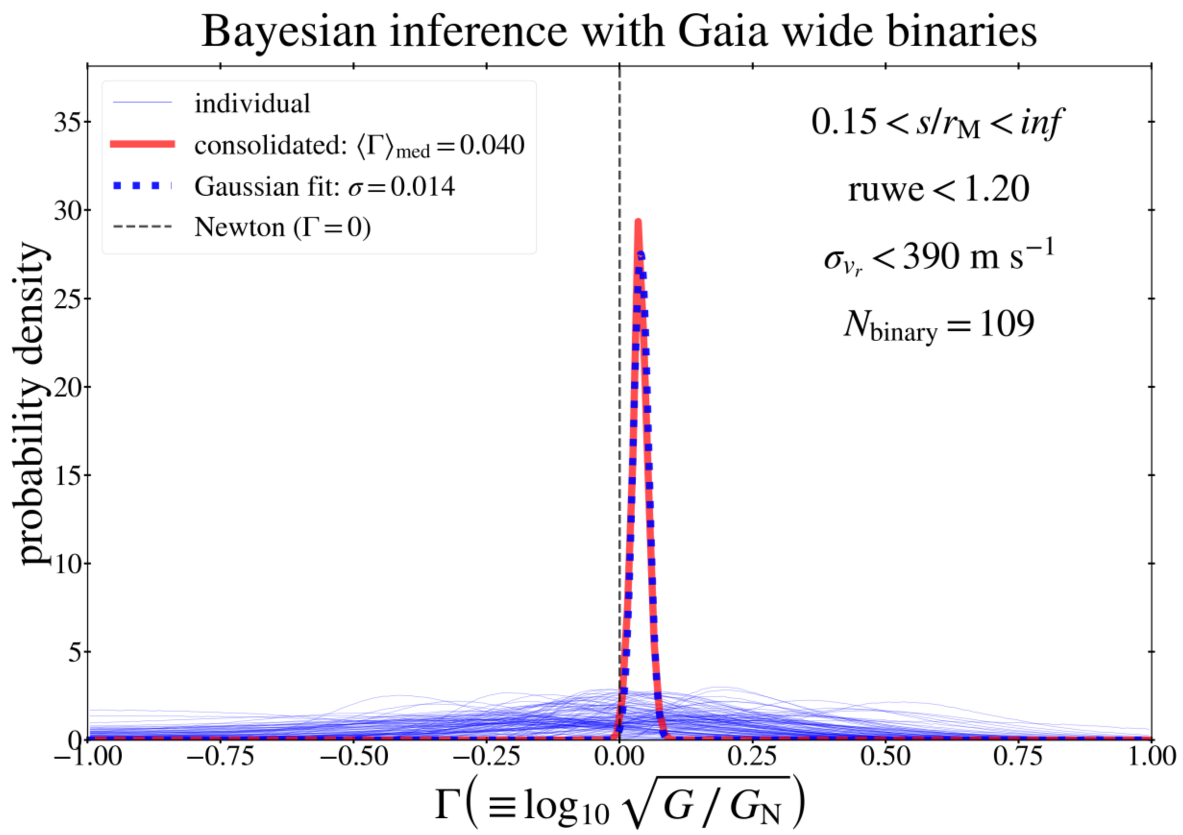
Bayesian inference with Gaia wide binaries
individual
consolidated: $\langle\Gamma\rangle_{med} = 0.106$
Gaussian fit: $\sigma = 0.040$
Newton ($\Gamma = 0$)
$0.50 < s/r_M < inf$
ruwe < 1.20
$\sigma_{v_r} < 390$ m s^{-1}
$N_{binary} = 37$
probability density
$\Gamma\left(\equiv \log_{10}\sqrt{G/G_N}\right)$

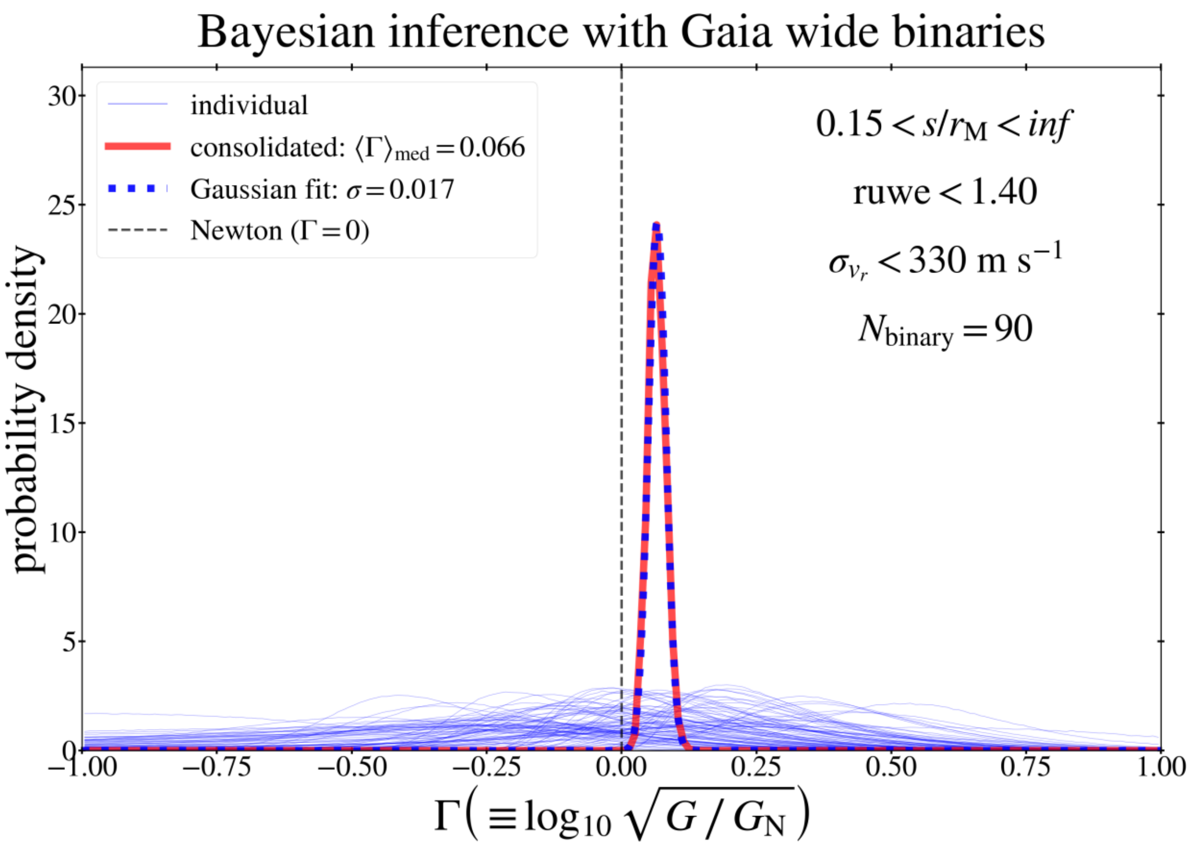

Bayesian inference with Gaia wide binaries
individual
consolidated: $\langle\Gamma\rangle_{med} = 0.128$
Gaussian fit: $\sigma = 0.048$
Newton ($\Gamma = 0$)
$0.50 < s/r_M < inf$
ruwe < 1.40
$\sigma_{v_r} < 330$ m s^{-1}
$N_{binary} = 25$
probability density
$\Gamma\left(\equiv \log_{10}\sqrt{G/G_N}\right)$

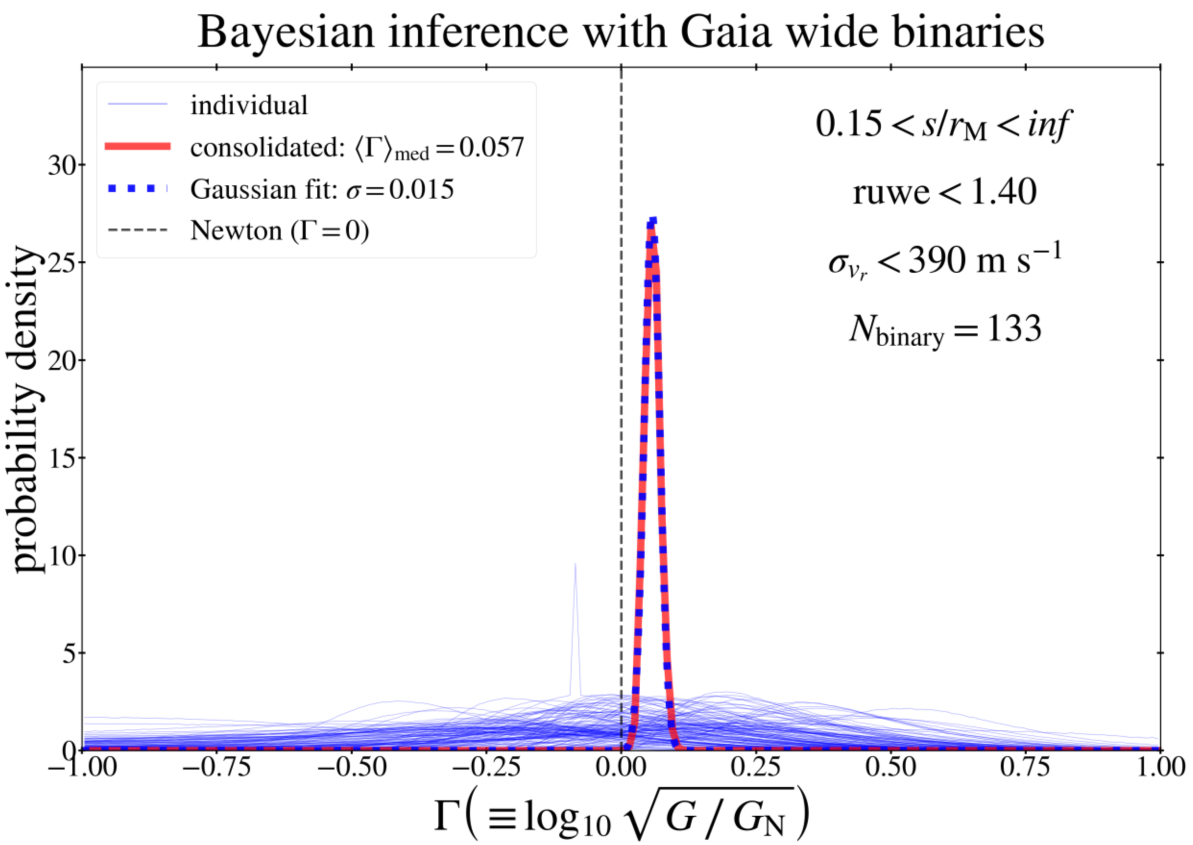

Bayesian inference with Gaia wide binaries
individual
consolidated: $\langle\Gamma\rangle_{med} = 0.099$
Gaussian fit: $\sigma = 0.038$
Newton ($\Gamma = 0$)
$0.50 < s/r_M < inf$
ruwe < 1.40
$\sigma_{v_r} < 390$ m s^{-1}
$N_{binary} = 41$
probability density
$\Gamma\left(\equiv \log_{10}\sqrt{G/G_N}\right)$

transition + MOND regime

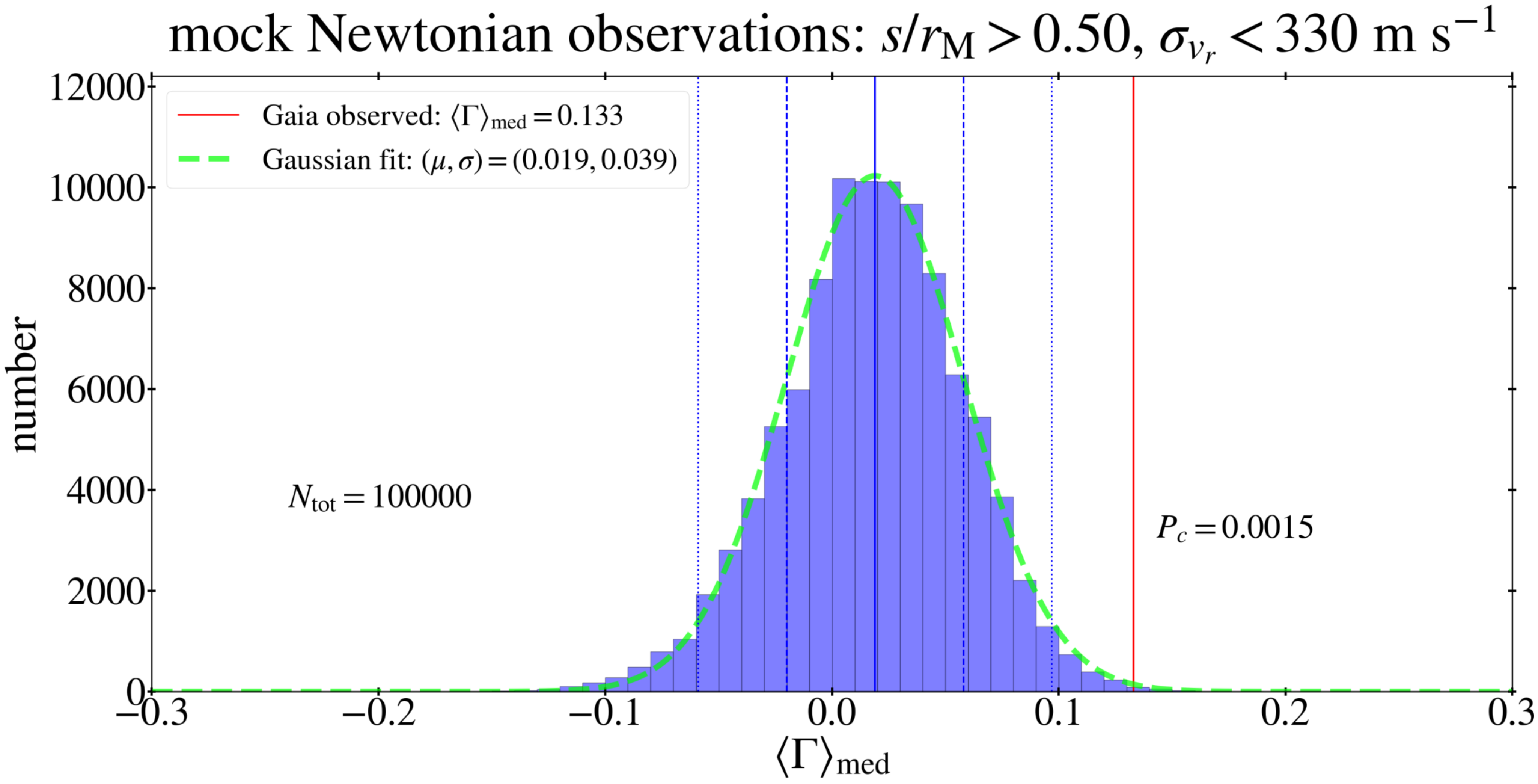


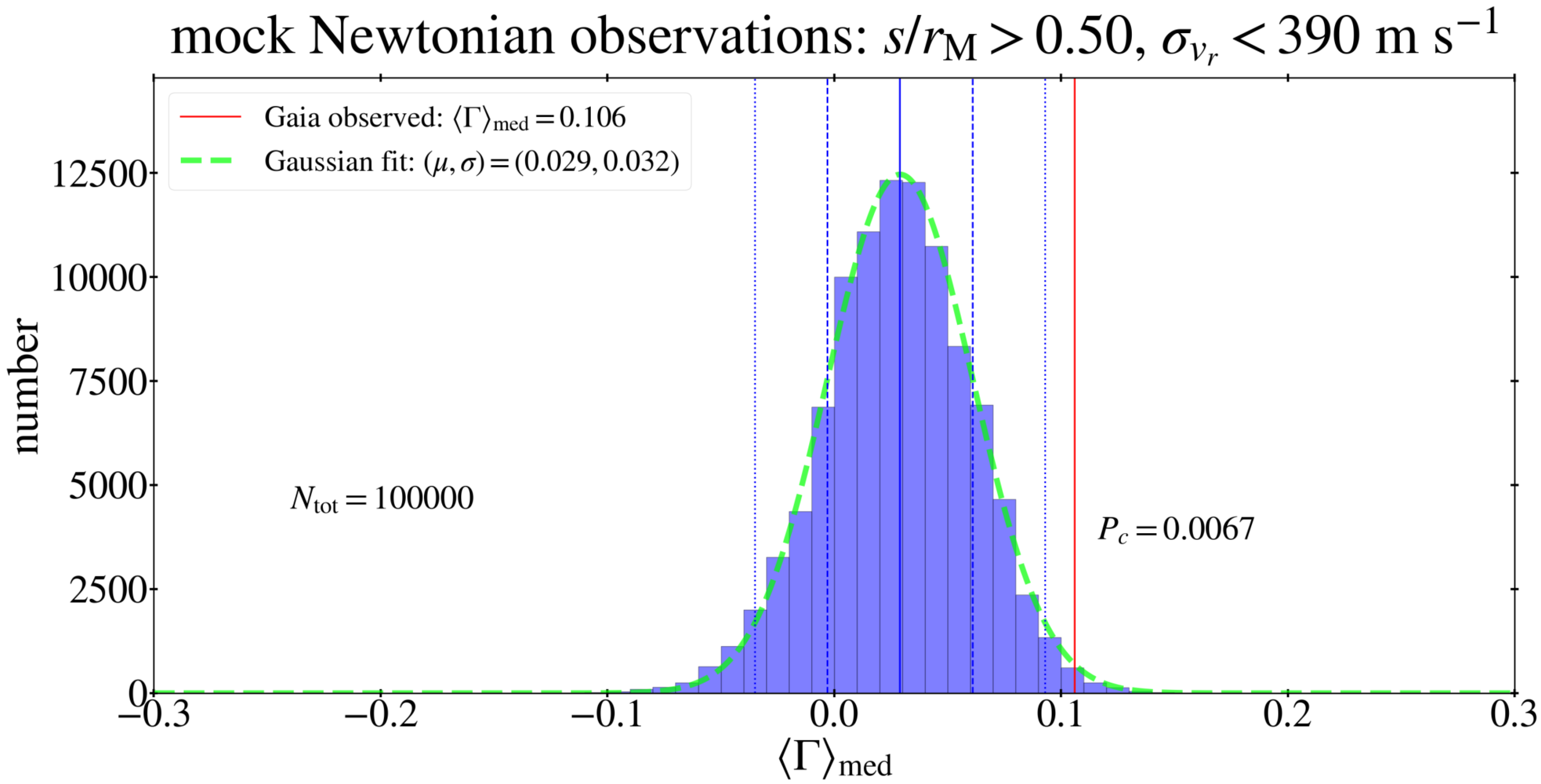


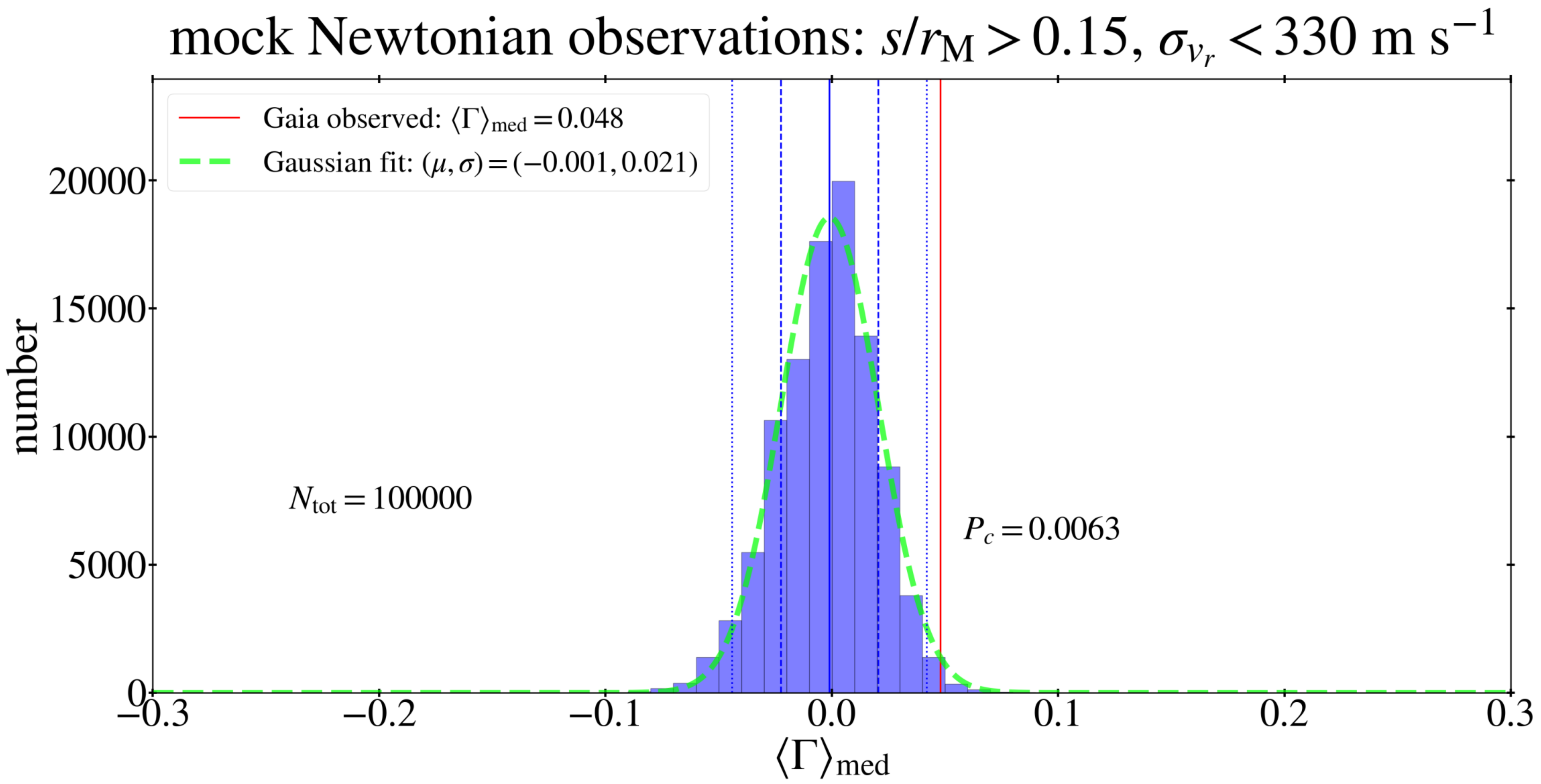

Bayesian inference with Gaia wide binaries
individual
consolidated: $\langle\Gamma\rangle_{\rm med} = 0.066$
Gaussian fit: $\sigma = 0.017$
Newton ($\Gamma = 0$)
$0.15 < s/r_{\rm M} < inf$
ruwe < 1.40
$\sigma_{v_r} < 330$ m s^{-1}
$N_{\rm binary} = 90$
probability density
$\Gamma\left(\equiv \log_{10}\sqrt{G/G_{\rm N}}\right)$

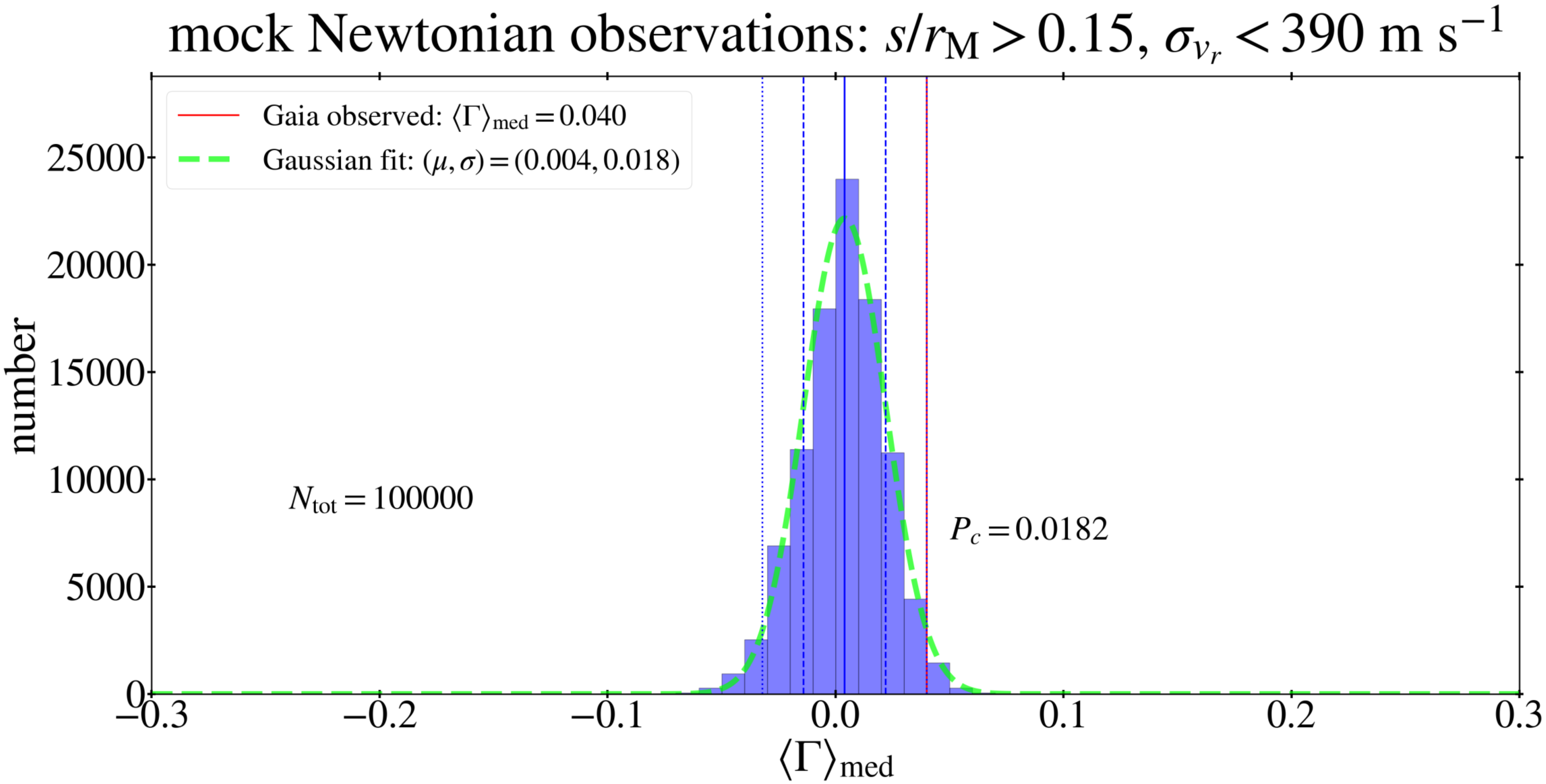

Bayesian inference with Gaia wide binaries
individual
consolidated: $\langle\Gamma\rangle_{\rm med} = 0.057$
Gaussian fit: $\sigma = 0.015$
Newton ($\Gamma = 0$)
$0.15 < s/r_{\rm M} < inf$
ruwe < 1.40
$\sigma_{v_r} < 390$ m s^{-1}
$N_{\rm binary} = 133$
probability density
$\Gamma\left(\equiv \log_{10}\sqrt{G/G_{\rm N}}\right)$

Estimating the bias due to the RV uncertainties




mock Newtonian observations: $s/r_M > 0.50$, $\sigma_{v_r} < 390$ m s^{-1}
Gaia observed: $\langle\Gamma\rangle_{med} = 0.106$
Gaussian fit: $(\mu, \sigma) = (0.029, 0.032)$
$N_{tot} = 100000$
$P_c = 0.0067$
number
12500
10000
7500
5000
2500
0
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
$\langle\Gamma\rangle_{med}$


mock Newtonian observations: $s/r_M > 0.15$, $\sigma_{v_r} < 330$ m s^{-1}
Gaia observed: $\langle\Gamma\rangle_{med} = 0.048$
Gaussian fit: $(\mu, \sigma) = (-0.001, 0.021)$
number
$N_{tot} = 100000$
$P_c = 0.0063$
$\langle\Gamma\rangle_{med}$
0
5000
10000
15000
20000
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3


mock Newtonian observations: $s/r_M > 0.15$, $\sigma_{v_r} < 390$ m s^{-1}
Gaia observed: $\langle\Gamma\rangle_{med} = 0.040$
Gaussian fit: $(\mu, \sigma) = (0.004, 0.018)$
$N_{tot} = 100000$
$P_c = 0.0182$
number
$\langle\Gamma\rangle_{med}$
25000
20000
15000
10000
5000
0
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3

main references (2023 – 2024 work)

- Chae (2023a): “**Breakdown of the Newton–Einstein Standard Gravity at Low Acceleration in Internal Dynamics of Wide Binary Stars**” (**ApJ**, 952, 128) *(>65k downloads, the #1 most read article in ApJ for three months*)
- Chae (2023b): “**Python scripts to test gravity with the dynamics of wide binary stars**” (**Zenodo** v5 as of Mar 2024, continually updated/improved) *(>2k downloads)*
- Chae (2024a): “**Robust Evidence for the Breakdown of Standard Gravity at Low Acceleration from Statistically Pure Binaries Free of Hidden Companions**” (**ApJ**, 960, 114) *(>8.7k downloads, the #1 most read article in ApJ for three weeks*)
- Chae (2024b): “**Measurements of the Low-Acceleration Gravitational Anomaly from the Normalized Velocity Profile of Gaia Wide Binary Stars and Statistical Testing of Newtonian and Milgromian Theories**” (**ApJ**, 2024c, 172, 186)
- Chae+ in preparation: Bayes 3D modeling results
- Hernandez, Chae, & Aguayo-Ortiz (2024b): “**A critical review of recent Gaia wide binary gravity tests**” (**MNRAS**, 2024, 533, 729)
- Hernandez (2023), Hernandez et al. (2024a): independent results agreeing with Chae (2023-2024) results.

Conclusions & Prospects

- There appears an immovable gravitational anomaly when all factors are properly taken into account through various methods based on various samples of different f_{multi}.
- The currently estimated property of the gravitational anomaly naturally agrees with the generic prediction of MOND-type modified gravity.
- Accurate and precise radial velocities to be observed in the coming years can make the current evidence a true scientific fact.
- Theoretical developments need to be based on correct experimental/observational evidence and correct use/interpretation of it.