# Ultralight cold dark matter

Jae-Weon Lee (Jungwon Univ.)

# Outline

1. Fuzzy DM and galaxies

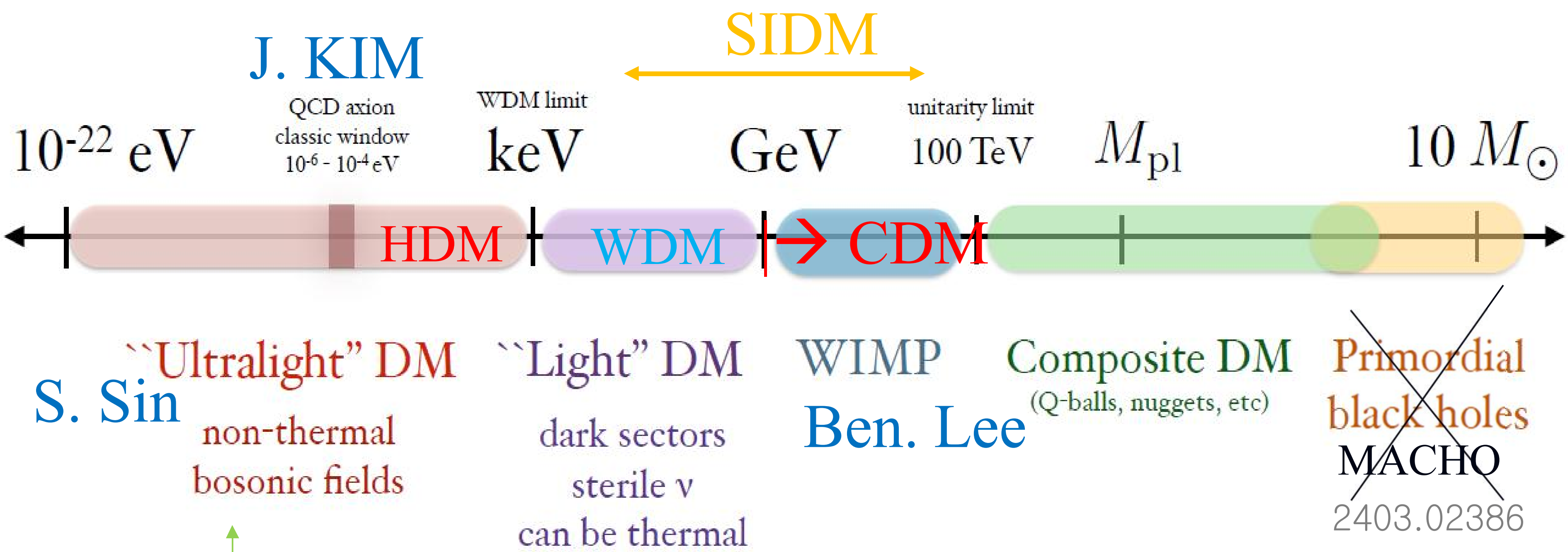
2. Self-interacting ULDM

3. ULDM and neutrinos

# Mass scale of dark matter

(not to scale)

TASI lectures by Lin arXiv:1904.07915

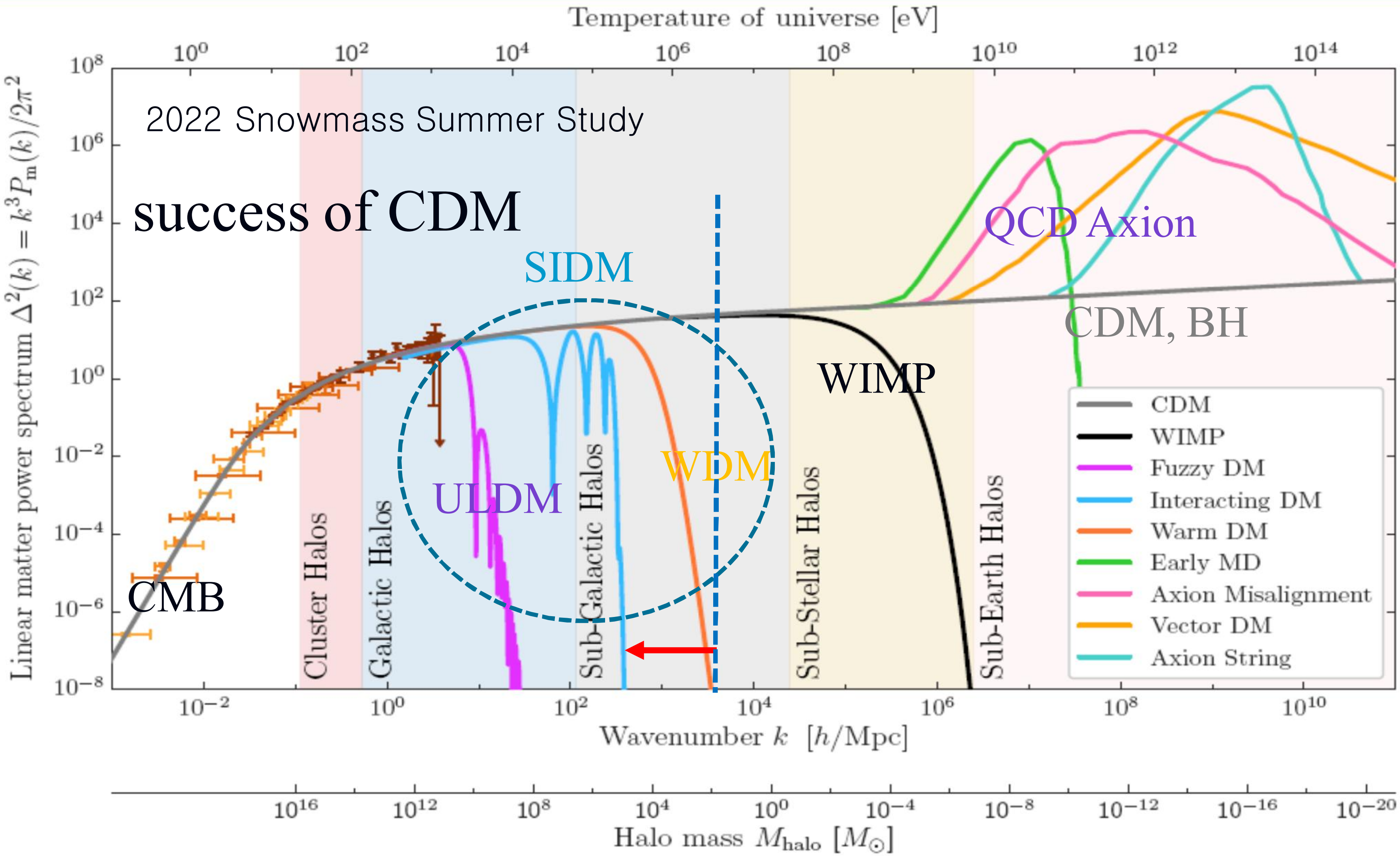



ULDM = Fuzzy, ULA, BEC, Wave, Scalar Field, ψ, Superfluid, quantum …

*Compact objects in the mass range from* $1.3 \times 10^{-5} M_{\odot}$ *to* $860\ M_{\odot}$ *cannot make up more than* 10% *of dark matter*. (2403.02386)

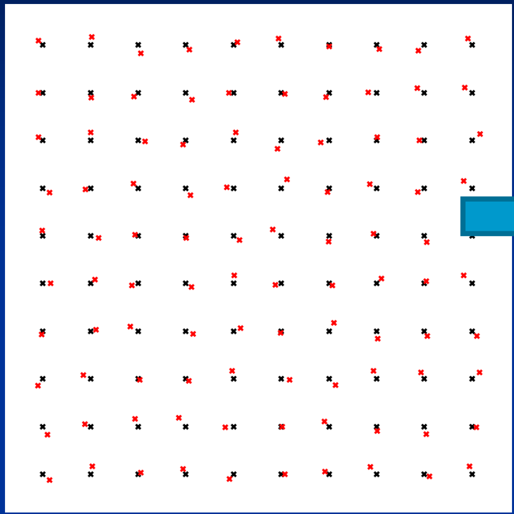
→ No DM star or planet observed in our galactic halo

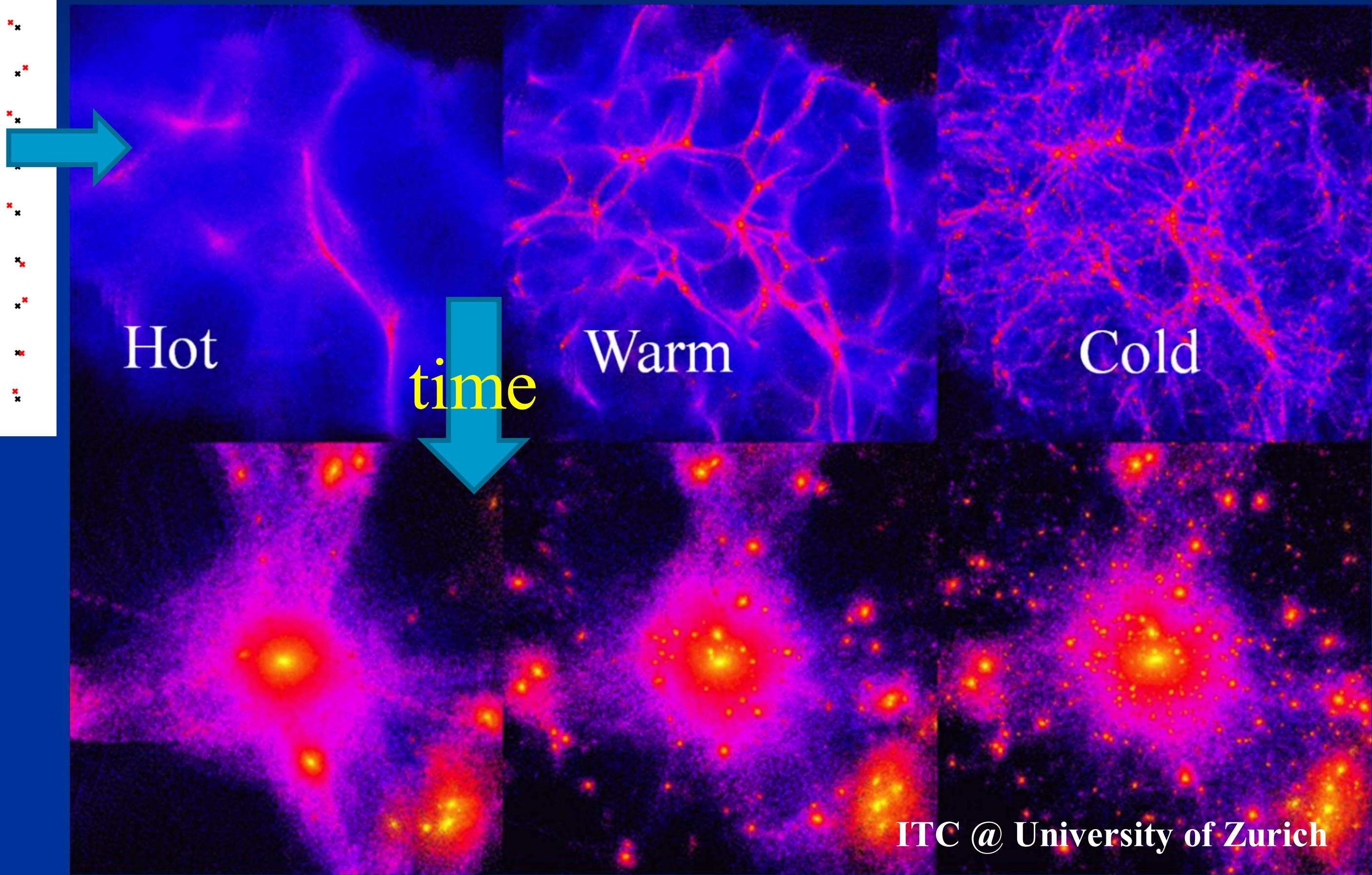
# Galaxies are DM dominated and seem to have ~ kpc size scale




No DM star or planet found so far → DM has kpc scale?

# Non-linear evolution (usually N-body)






N-body
ex) Gadget..

# Challenges for ΛCDM

2105.05208

•ΛCDM is very successful but encounters some tensions

1. Small scale crisis (at galaxy scale)
   predicts too many small structures not observed

2. Hubble parameter tension ~5$\sigma$:
   mismatch between Planck estimation and SN

3. $S_8$ tension ~2-3$\sigma$ : $S_8 \equiv \sigma_8\sqrt{\Omega_m/0.3}$
   mismatch between Planck estimation and WL & Cluster

4. Early BH and galaxies (Webb)

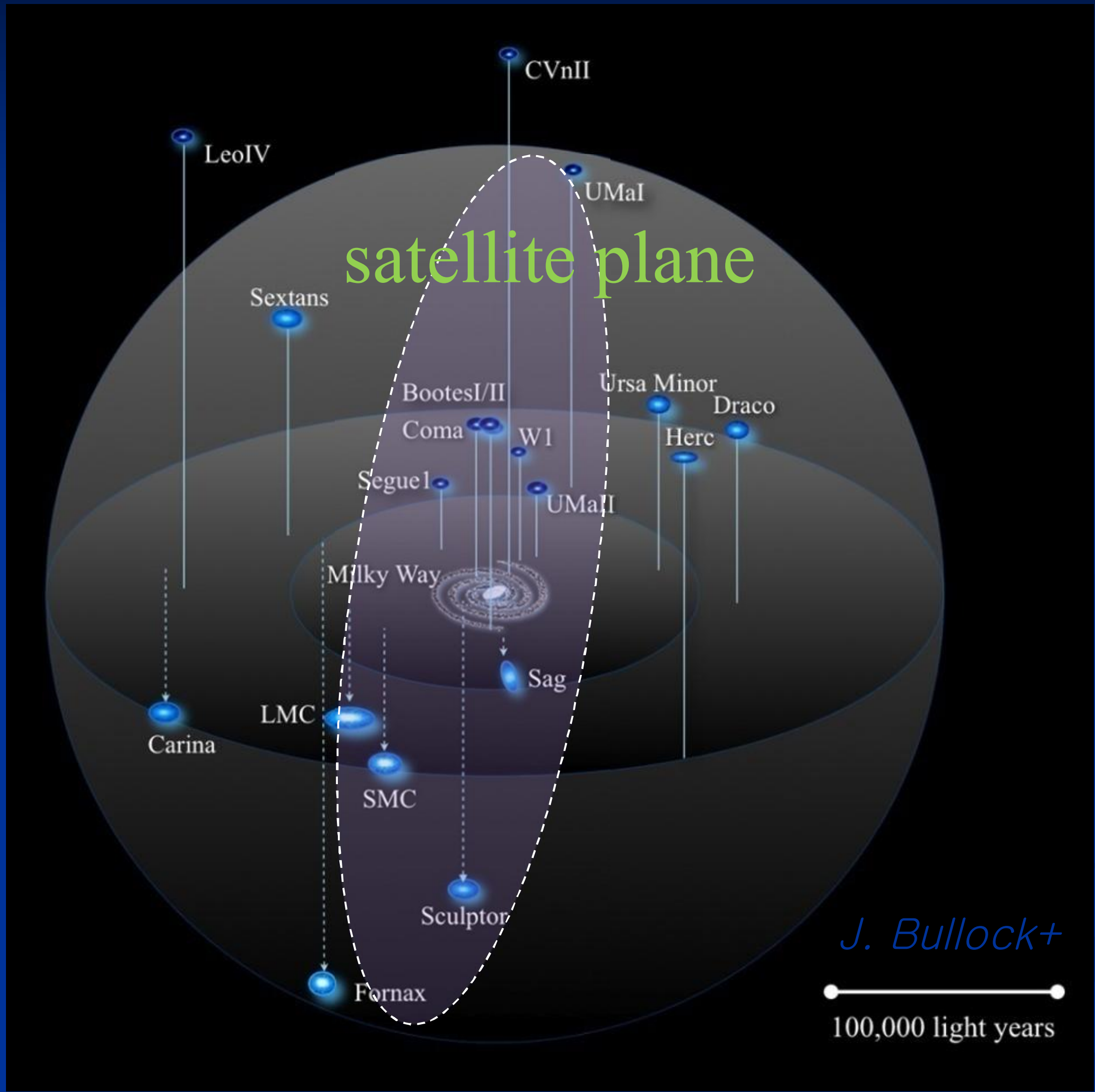

5. Cluster collision (collision speed & offset)

6. Li problem

…etc

→ Any good DM model should address these tensions

# Galaxies observed



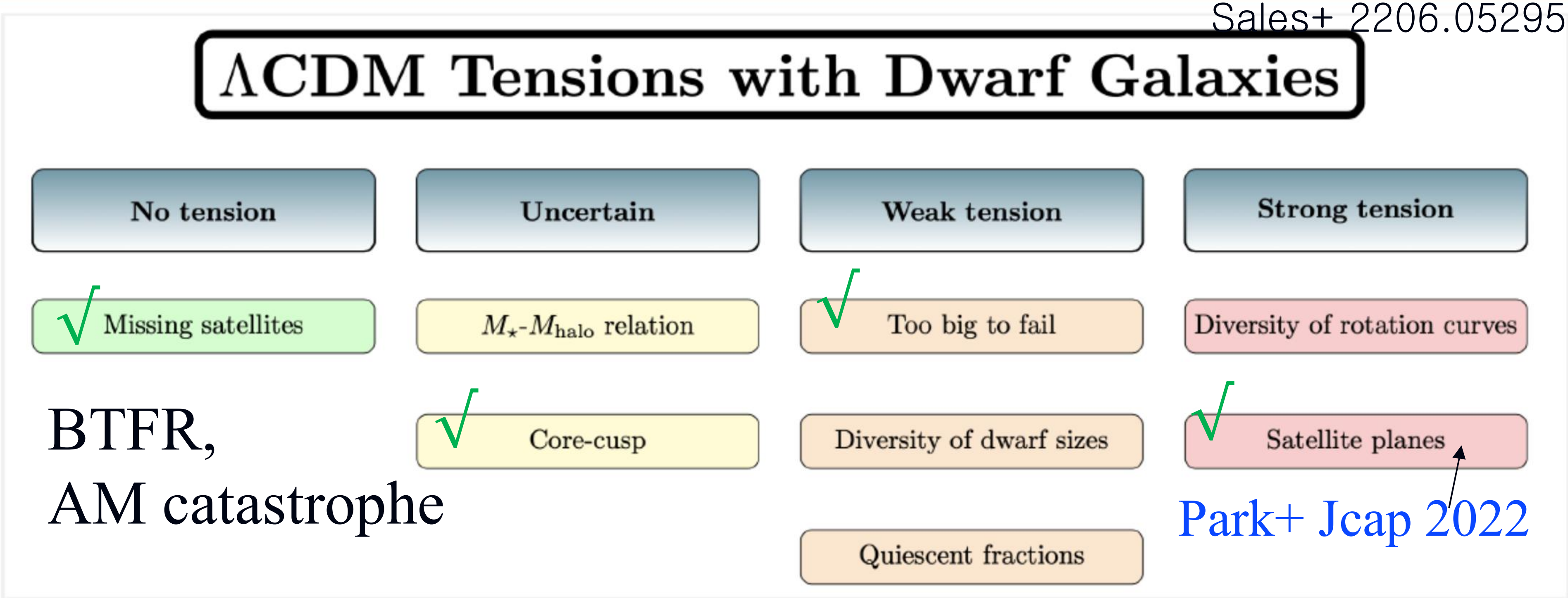

Minimum mass
~ $10^7$ Ms

Minimum size
~ kpc

Any good DM model should explain observed galaxies

# *Some* of small scale issues with CDM

Sales+ 2206.05295




- Key problem is how to suppress small scale structures < dwarf galaxies.
  → we need a new CDM → ULDM with m~ $10^{-22}$ eV can solve many of these
- Still unsolved problems seem to be related to Baryon-DM relation
- Can baryon physics + precise numerical simulation + more observations save CDM?

# Solutions to Small scale problems

*Key : How to generate the length scale ~ galaxy ~ kpc*
*→ How to give kinetic energy to DM*

- CDM: finer resolution, tidal effects, Baryon feedback: SN, BH jets & more observations…
- WDM: thermal kinetic energy from m ~ keV
- SIDM: $\sigma/m \sim 1\ cm^2/g$ (v dependent?)
- FDM: $m \sim 10^{-22}$eV, quantum pressure

[Q] Can these models solve all these problems with a *single set of parameters*?

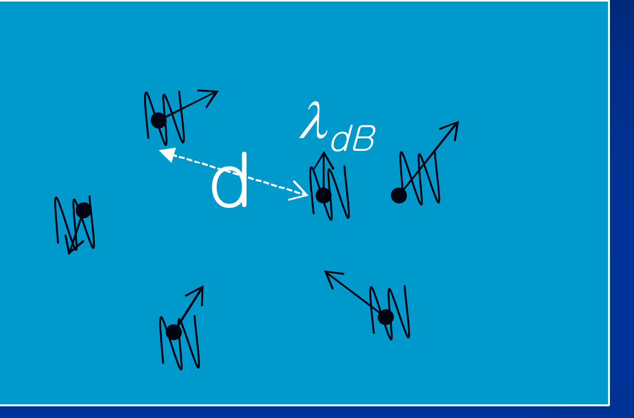
# ULDM: DM is in Bose-Einstein Condensate!

A.K.A . Fuzzy, ULA , BEC, Wave, ψ, Fluid, Quantum, ELB,,...

m: DM particle mass

CDM (WIMP)

- *Heavy , m > GeV*
- *Particle-like*
- *d >> $\lambda_{dB}$ ~1/mv*
- *Newton's eq*
- *Random motion*
- *No scale practically*

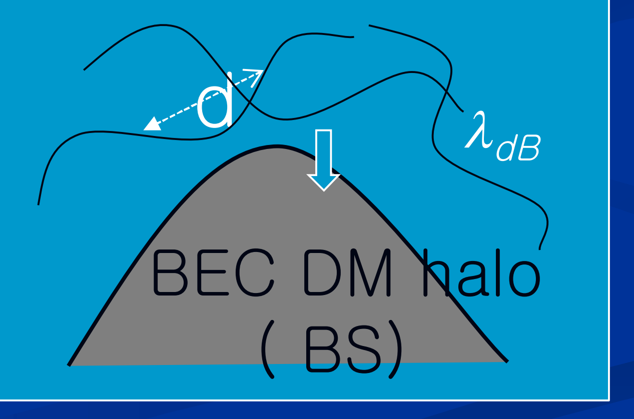



ULDM

- *Ultra-light, m ~$10^{-22}$ eV*
- *high # density n ~ $10^{25}$ /cm$^3$*

*→ (d << $\lambda_{dB}$) → wavfn. overlap*

- *wave-like*
- *SPE*
- *coherent motion*
- *Min. scale*




$$m \approx 10^{-22} eV$$

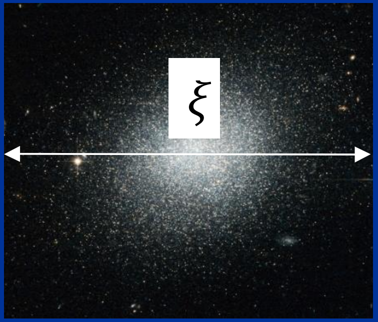
$$BEC \quad T_c = \left(\frac{3n}{m}\right)^{1/2} \sim 5\times10^{7}\, GeV,$$

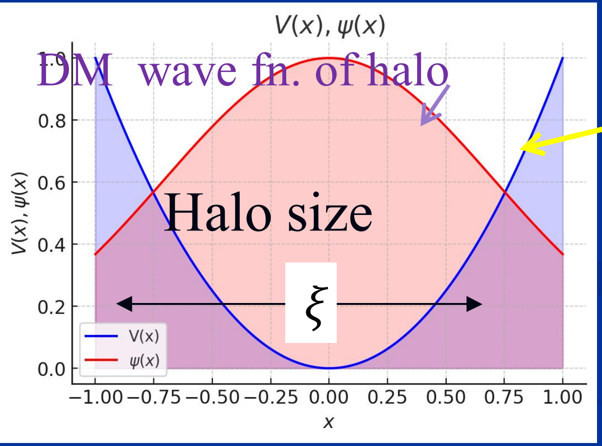
# ULDM

- Galactic DM halo is a BEC
- Quantum pressure (from uncertainty principle) prevents collapse
- Galaxy size ~ de Broglie wavelength of DM particles

→ $m \sim 10^{-22}$ eV

- Small m → high # density → overlap of wave fn. → classical wave




Self-gravitating potential well V

Schrodinger -Poisson (SP)

$$\begin{cases} i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi + int. \\ \nabla^2 V = 4\pi G(\rho_d + \rho_v) \end{cases}$$

visible, $\rho_d = m|\psi|^2$

SFDM $\phi(t,x) = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi(t,x) + e^{imt}\psi^*(t,x)\right]$

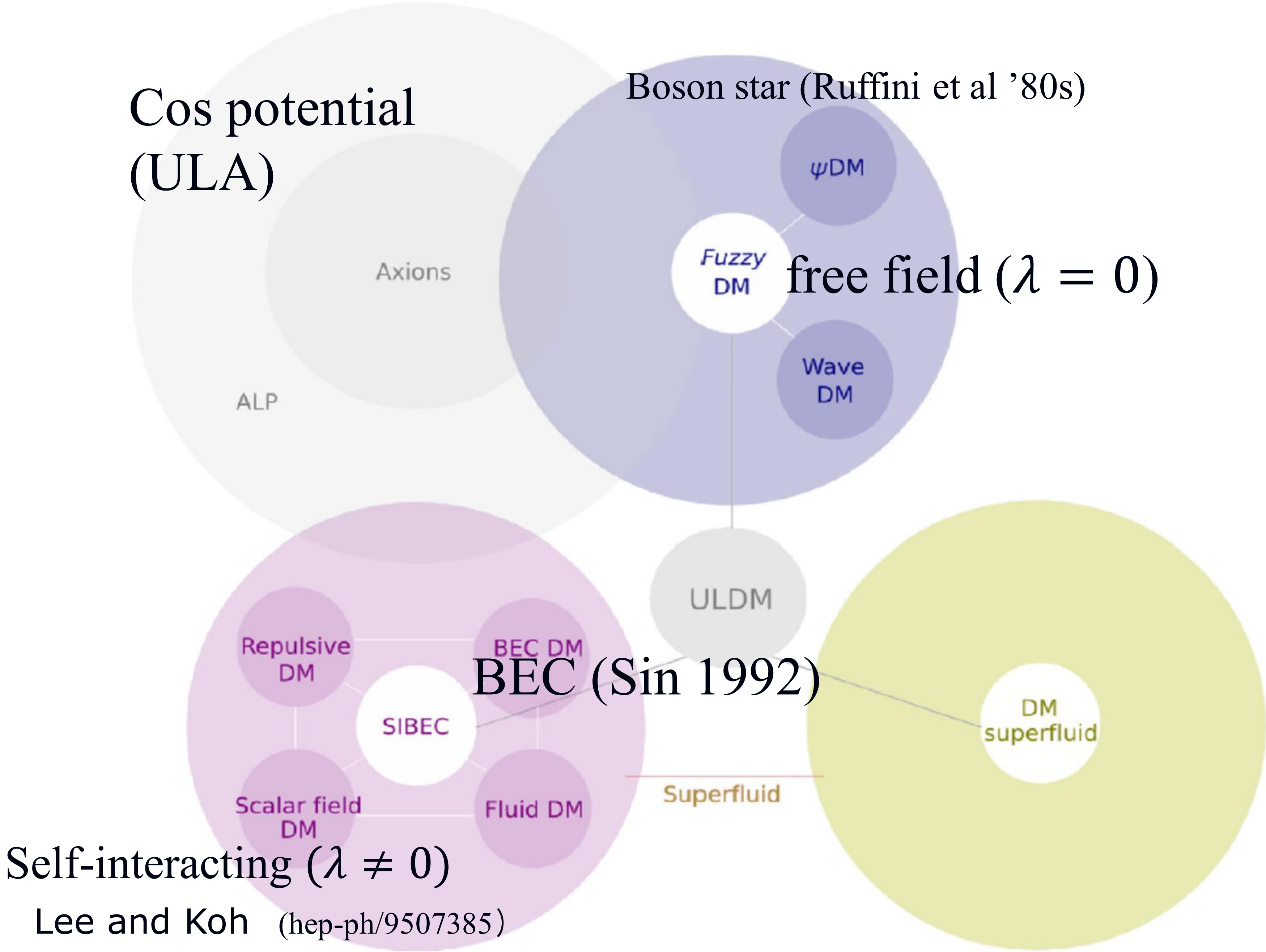



**Fig. 9** Map of the ULDM classes of models

Ferreira et al.

# Linear pert. Of ULDM

FDM has only 2 parameters   m and bg density $\rho_0$

(+ λ for $\phi^4$ self-interacting ULDM)

a=scale factor

Nonrelativistic →

$$i\hbar\left(\frac{\partial\psi}{\partial t}+\frac{3}{2}H\psi\right)=-\frac{\hbar^2}{2ma^2}\Delta\psi+mV\psi+\frac{\lambda|\psi|^2\psi}{2m^2}$$

Madelung representation

$$perturbation \quad \text{with } \psi=\sqrt{\rho}e^{iS}, \qquad v\equiv\frac{\hbar}{ma}\nabla S\Rightarrow$$

$$\begin{cases}\partial_t\rho+3H\rho+\frac{1}{a}\nabla\cdot(\rho v)=0\\ \partial_t v+\frac{1}{a}v\cdot\nabla v+Hv+\frac{1}{\rho a}\nabla p+\frac{1}{a}\nabla V+\frac{\hbar^2}{2m^2a^3}\nabla\left(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\right)=0\end{cases}$$

Quantum Pressure

Density contrast (k space)

$$\text{perturbation } \delta=\delta_k=\delta\rho/\rho_0$$

$$\Rightarrow\partial_t^2\delta+2H\partial_t\delta+\left(\left(\frac{\hbar^2k^2}{4m^2a^2}+c_s^2\right)\frac{k^2}{a^2}-4\pi G\rho_0\right)\delta=0$$

Hubble drag    gravity

Quantum Jeans length

$$\lambda_J=\frac{2\pi}{k_J}a=\pi^{3/4}\hbar^{1/2}(G\rho_0m^2)^{-1/4}\propto 1/\sqrt{mH}$$

- CDM-like on super-galactic scale (for a small $k < k_J$)
- Suppress sub-galactic structure (for a large $k > k_J$)

# Length scales of DM models

1) CDM, WIMP, WDM: $\partial_t^2\delta + 2H\partial_t\delta + \left(\left(\frac{\hbar^2 k^2}{4m^2 a^2} + c_s^2\right)\frac{k^2}{a^2} - 4\pi G\rho_0\right)\delta = 0$

→ use free streaming length instead

2) FDM: $\partial_t^2\delta + 2H\partial_t\delta + \left(\left(\frac{\hbar^2 k^2}{4m^2 a^2} + c_s^2\right)\frac{k^2}{a^2} - 4\pi G\rho_0\right)\delta = 0$

→ Q. Jeans length

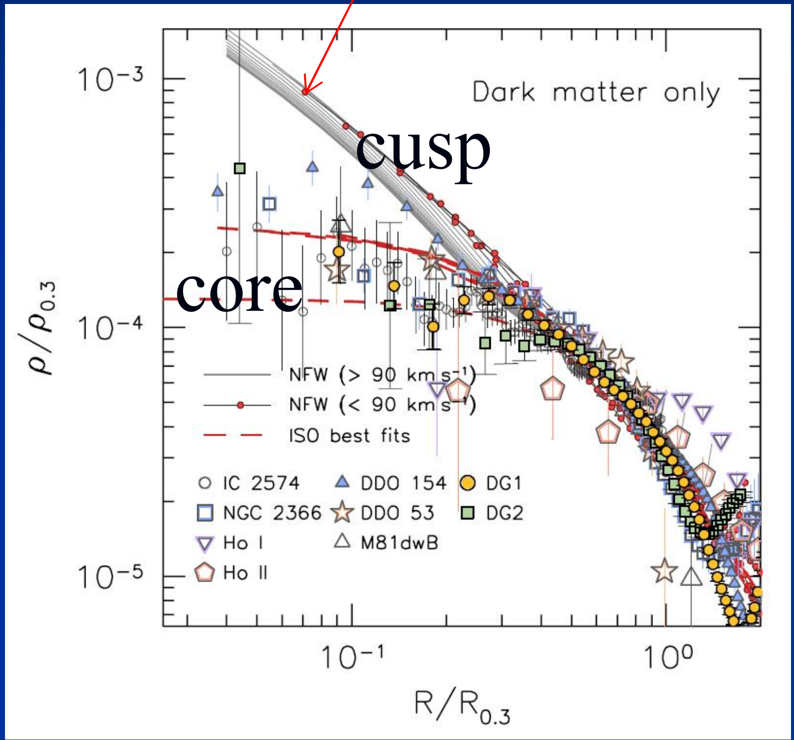
3) self-int ULDM: $\partial_t^2\delta + 2H\partial_t\delta + \left(\left(\frac{\hbar^2 k^2}{4m^2 a^2} + c_s^2\right)\frac{k^2}{a^2} - 4\pi G\rho_0\right)\delta = 0$

$p = \frac{2\pi a_s \hbar^2}{m^3}\rho^2, c_s^2 = p'(\rho) = \frac{4\pi a_s \hbar^2 \rho}{m^3}, \lambda_J = \sqrt{\frac{\pi\hbar^3\lambda}{2cGm^4}}, a_s = \frac{\lambda\hbar}{8\pi mc}$ *scattering length*

4) pt. SIDM: $\partial_t^2\delta + 2H\partial_t\delta + \left(\left(\frac{\hbar^2 k^2}{4m^2 a^2} + c_s^2\right)\frac{k^2}{a^2} - 4\pi G\rho_0\right)\delta = 0$

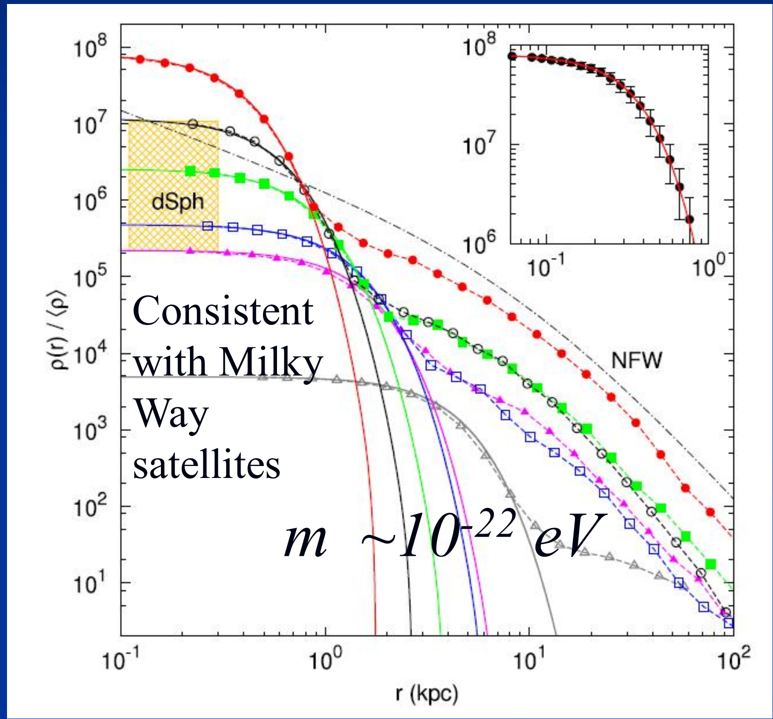
# Core/Cusp problem of CDM

CDM (NFW) → Cusp




ULDM→

Core ~ de Broigle wave len.




Schive etal , Nature physics 2014

Density profile of small galaxies disfavors CDM

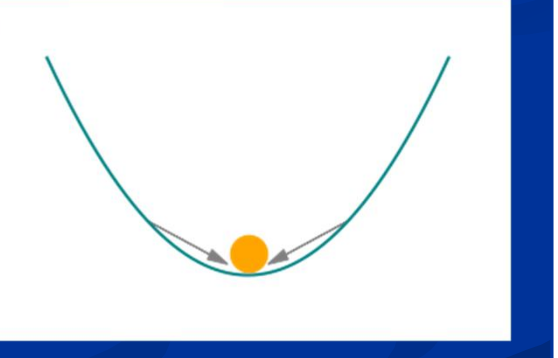
→ ULDM well explains the core profile

# Features of ULDM

$$\phi(t,x) = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi(t,x) + e^{imt}\psi^*(t,x)\right]$$

fast (bg), slow (galaxy)

- Typical galaxy size ~ $\lambda_{dB}$~ kpc
- wave nature → gravitational cooling
- small dynamical friction
- bg oscillation with m ~ nHz
- explain DM density
  → GUT scale field value



→ explain many mysteries of galaxies

# Typical scales of FDM

JLee 2310.01442

time dependent and functions of $\frac{\hbar}{m}$

1) time

$t_c \simeq (G\bar{\rho})^{-1/2}$ : Hubble time

2) length (q. Jeans length) → explain size evolution (JLee PLB 2016)

$$x_c = \lambda_{dB} = \left(\frac{\hbar}{m}\right)^2 \frac{1}{GM} = 854.8\, pc \left(\frac{10^{-22} eV}{m}\right)^2 \frac{10^8 M_\odot}{M} = \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{-1/4}$$

~ Gravitational Bohr radius ~ de Broglie wavelength

3) velocity

$$v_c \equiv x_c/t_c = GM\, m/\hbar = 22.4\, km/s \left(\frac{M}{10^8 M_\odot}\right)\left(\frac{m}{10^{-22} eV}\right) \simeq \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{1/4},$$

4) mass

$$M_Q = \frac{4}{3}\left(\frac{\lambda_Q}{2}\right)^3 \bar{\rho} = \frac{4}{3}\pi^{\frac{13}{4}} \left(\frac{\hbar}{G^{\frac{1}{2}} m}\right)^{\frac{3}{2}} \bar{\rho}(z)^{\frac{1}{4}} = 1.54 \times 10^8 M_\odot \left(\frac{m}{10^{-22} eV}\right)^{-3/2} \left(\frac{\bar{\rho}}{10^{-7} M_\odot/pc^3}\right)^{1/4}$$

also explain max. mass of galaxies

5) Angular momentum

$$L_c = M x_c v_c = \hbar \frac{M}{m} = N\hbar,$$ (L eigenstates?)

$$= 1.1 \times 10^{96} \hbar \left(\frac{M}{10^8 M_\odot}\right)\left(\frac{10^{-22} eV}{m}\right) \simeq \frac{\left(\frac{\hbar}{m}\right)^{5/2} \bar{\rho}^{1/4}}{G^{3/4}}$$

6) acceleration → MOND (LKL, PLB 2019)

$$a_c = x_c/t_c^2 = G^3 m^4 M^3/\hbar^4$$

$$= 1.9 \times 10^{-11} meter/s^2 \left(\frac{m}{10^{-22} eV}\right)^4 \left(\frac{M}{10^8 M_\odot}\right)^3 \simeq \sqrt{\frac{\hbar}{m}} (G\bar{\rho})^{3/4}$$

cf) MOND scale $a_0 = 1.2 \times 10^{-10} meter/s^2$

7) potential $V_c$ =1 gives Max. Galaxy mass M = $10^{12} M_\odot$

$$V_c = \frac{m^2}{\hbar^2}(4\pi G M)^2 = 8.8 \times 10^{-7} c^2 \sim \left(\frac{m}{10^{-22} eV}\right)^2 \left(\frac{M}{10^8 M_\odot}\right)^2$$ Nonrelativistic

# ULA miracle

$$I = \int d^4x\sqrt{g}\left[\frac{1}{2}F^2 g^{\mu\nu}\partial_\mu a\partial_\nu a - \mu^4(1-\cos a)\right]$$

$$m = \frac{\mu^2}{F}$$

$$\ddot{a} + 3H\dot{a} + m^2 \sin a = 0$$

oscillation starts at $\mathrm{H}\sim\frac{T_{osc}^2}{M_P} = m$

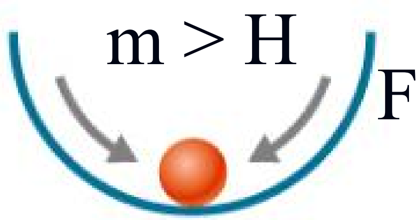
MDE starts at $T_1 \sim 1eV \rightarrow \frac{\mu^4(DM)}{T_{osc}^4(rad)} \rightarrow \frac{\mu^4 T_{osc}}{T_{osc}^4 T_1} \sim 1$

$$F = \frac{\mu^2}{m} \sim \frac{M_P^{3/4} T_1^{1/2}}{m^{1/4}} \sim 10^{17} GeV$$

$$\Omega_a \sim 0.1\left(\frac{F}{10^{17}GeV}\right)^2\left(\frac{m}{10^{-22}eV}\right)^{1/2}$$

ULA miracle?

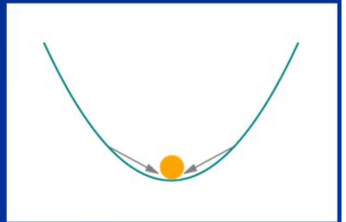
Hui et al 2017



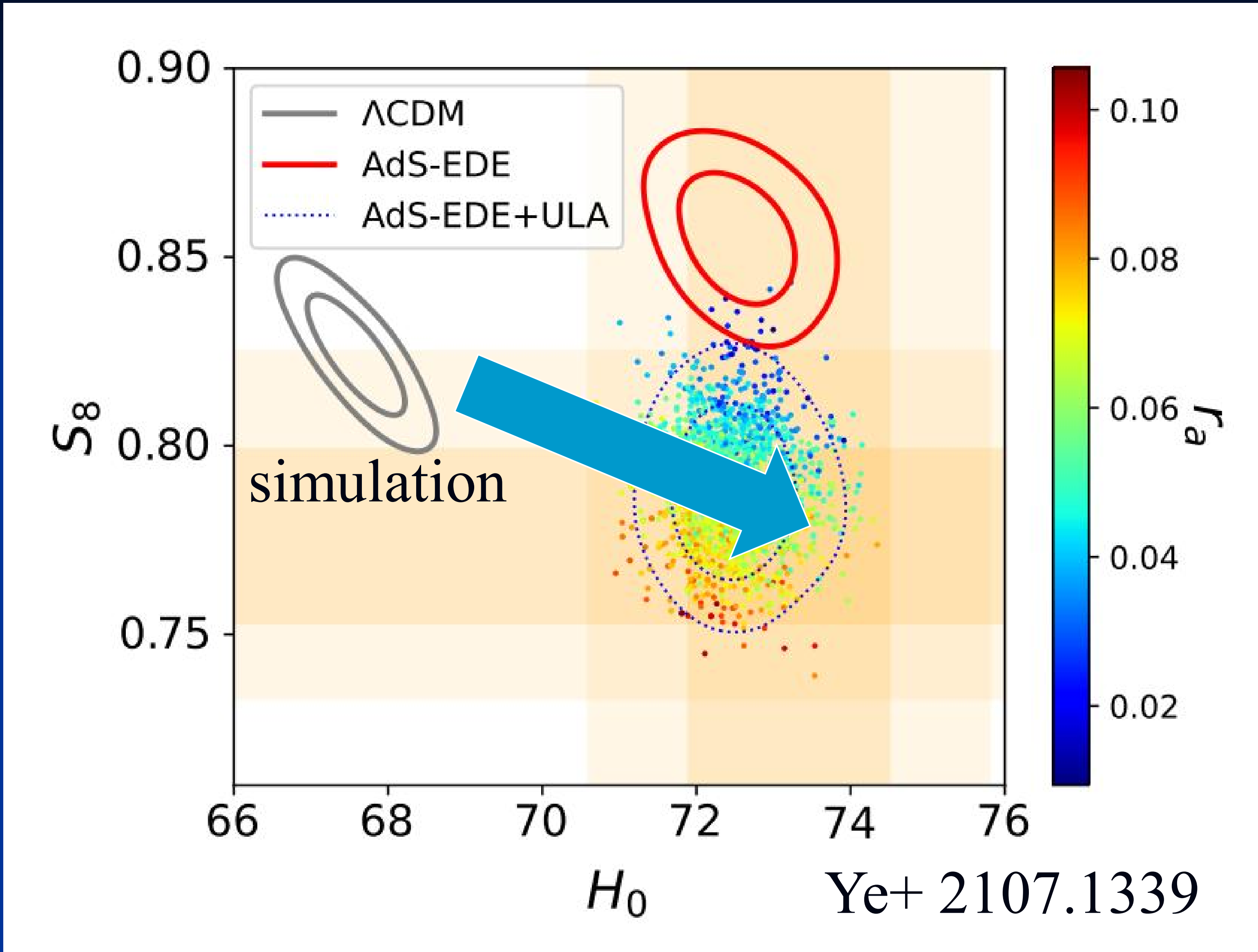

ULDM naturally explains DM density with GUT scale.
This holds for generic ULDM with a quadratic pot.

# Thermal history

| | |
|---|---|
| GUT | $T_c \simeq F \sim \frac{M_{\mathrm{P}}^{3/4}\, T_1^{1/2}}{m^{1/4}} \sim 10^{17}$ GeV |
| Oscillation starts | $T_{osc} \sim (M_P m)^{1/2} \sim 10^3$eV |
| mde | $T_1 \sim 1$ eV |
| Now | $T_0 \sim 10^{-4}$ eV |

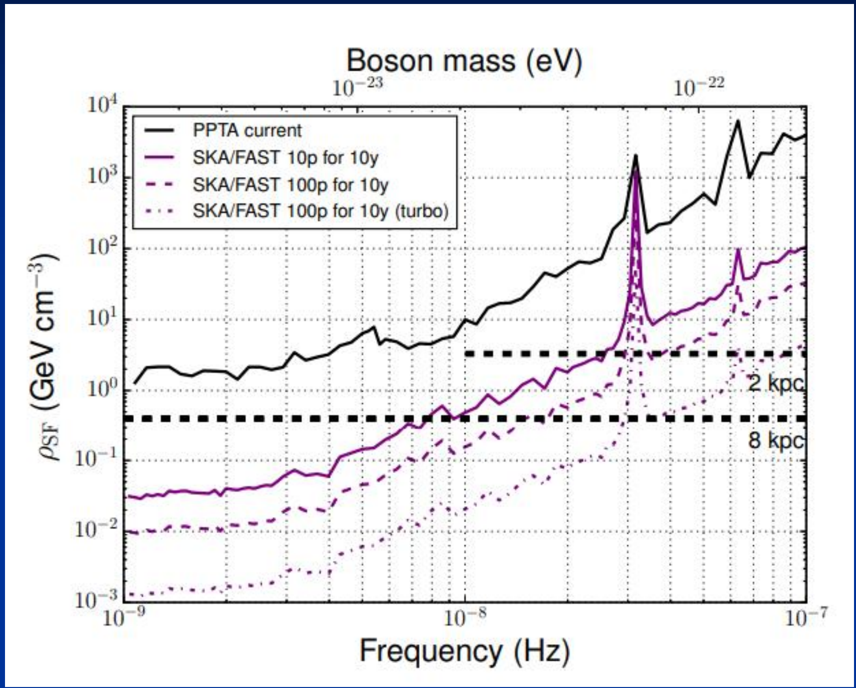


$$\Omega_\phi \sim 0.1\left(\frac{F}{10^{17}GeV}\right)^2\left(\frac{m}{10^{-22}\,eV}\right)^{1/2}$$



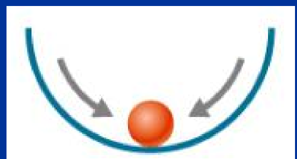

Can ULDM solve both $H_0$ and $S_8$ ?

# GW background detected by pulsar timing array




Porayko+ 1810.03227


ULDM has

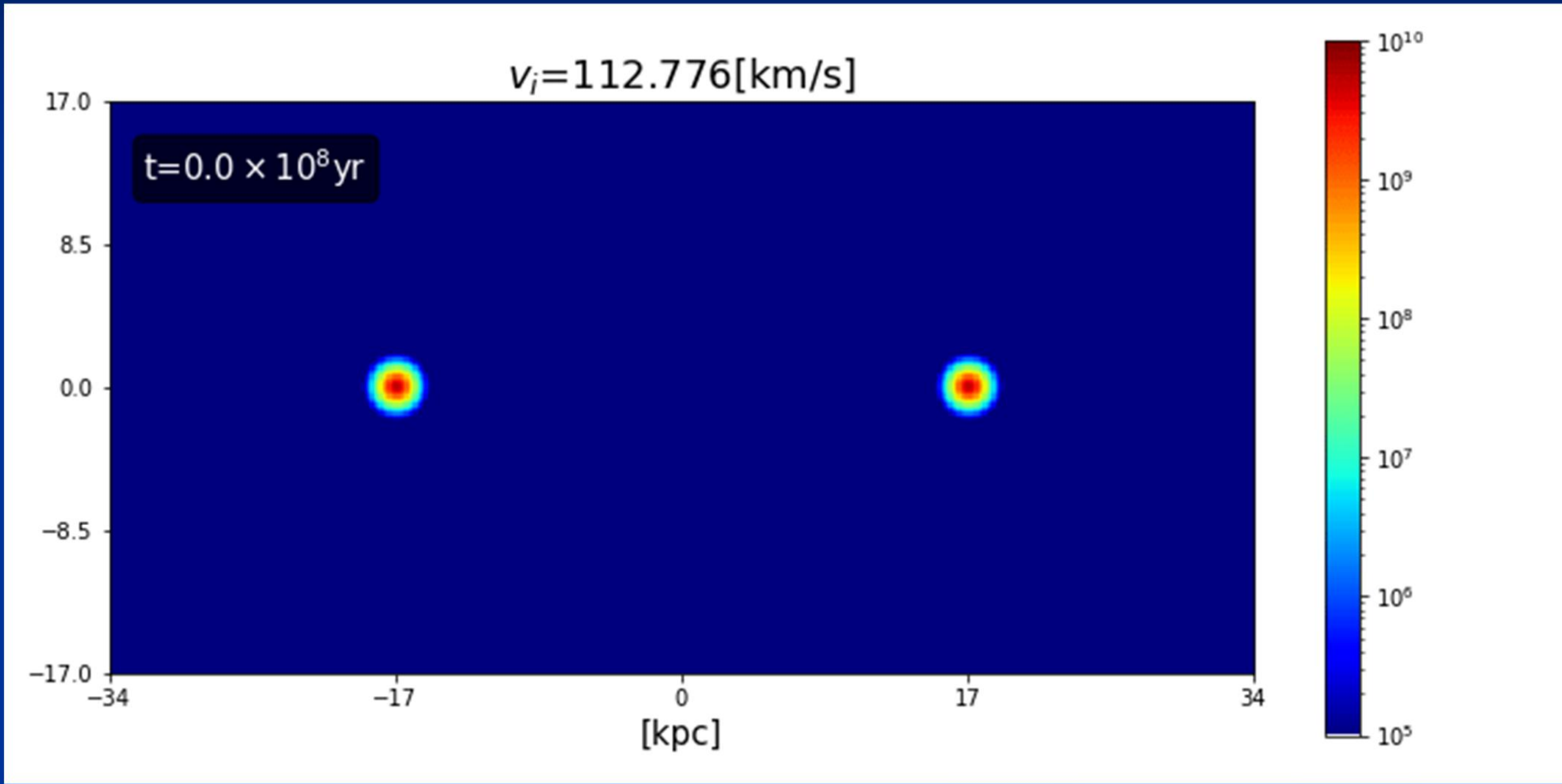


intrinsic oscillation time scale
1/m ~ yrs [f ~ nHz]

$$g_{ij} = -1 - 2\,\Psi(t)\delta_{ij}$$

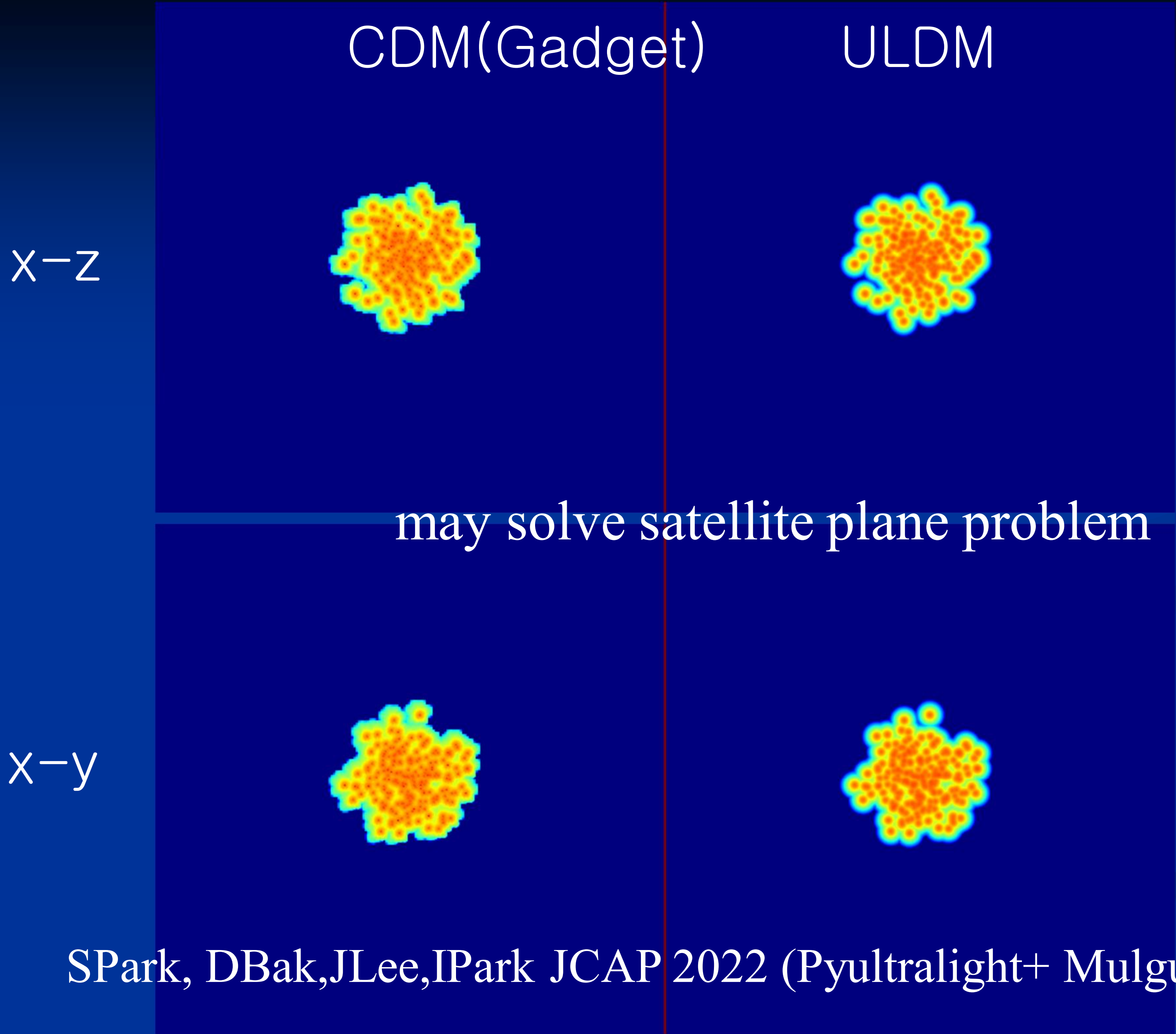
time-dependent metric
→ frequency shift δν
→ timing residuals

# Gravitational cooling




Seoul city univ.

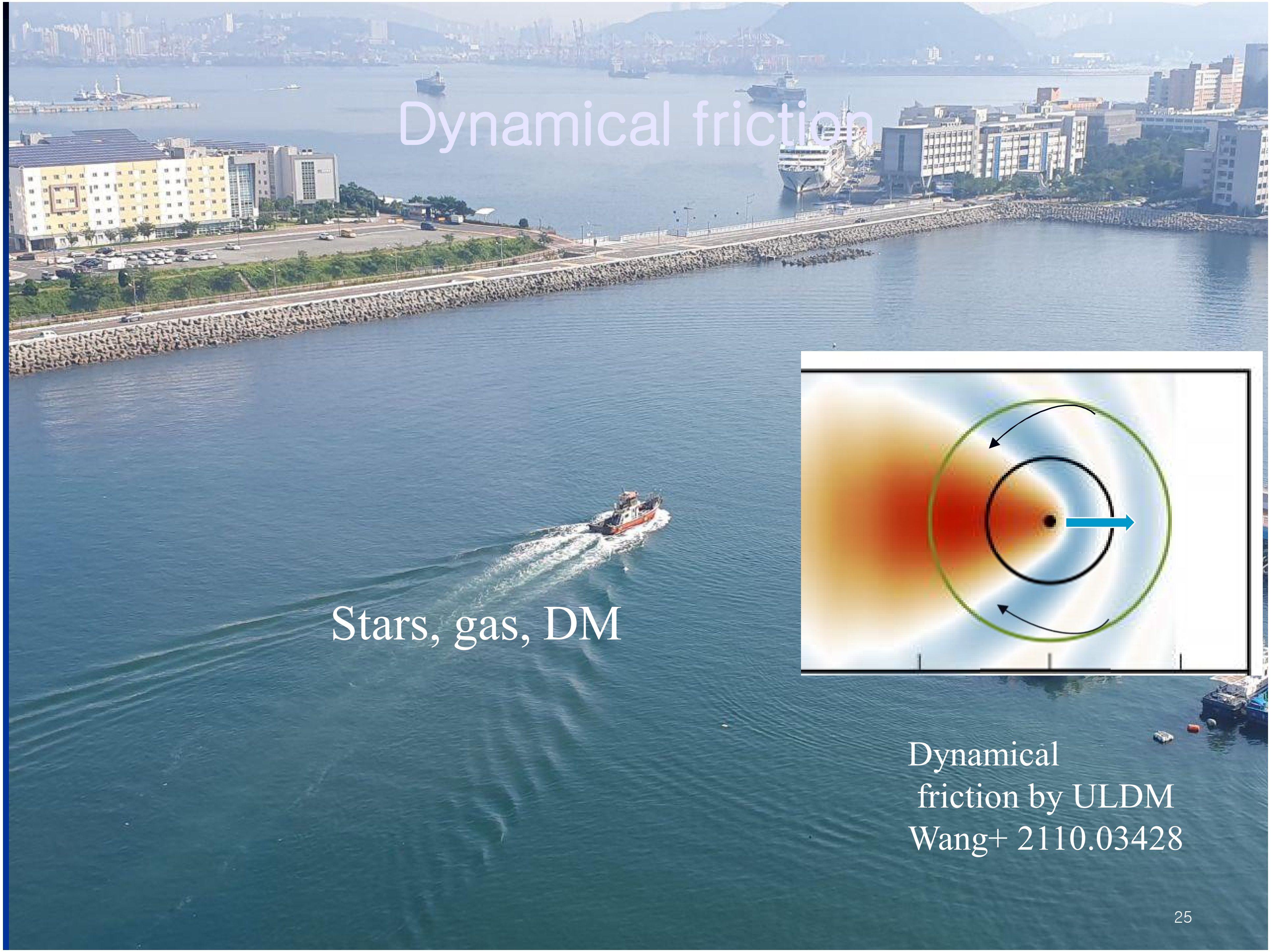
GC gives effective friction even for FDM




may solve satellite plane problem

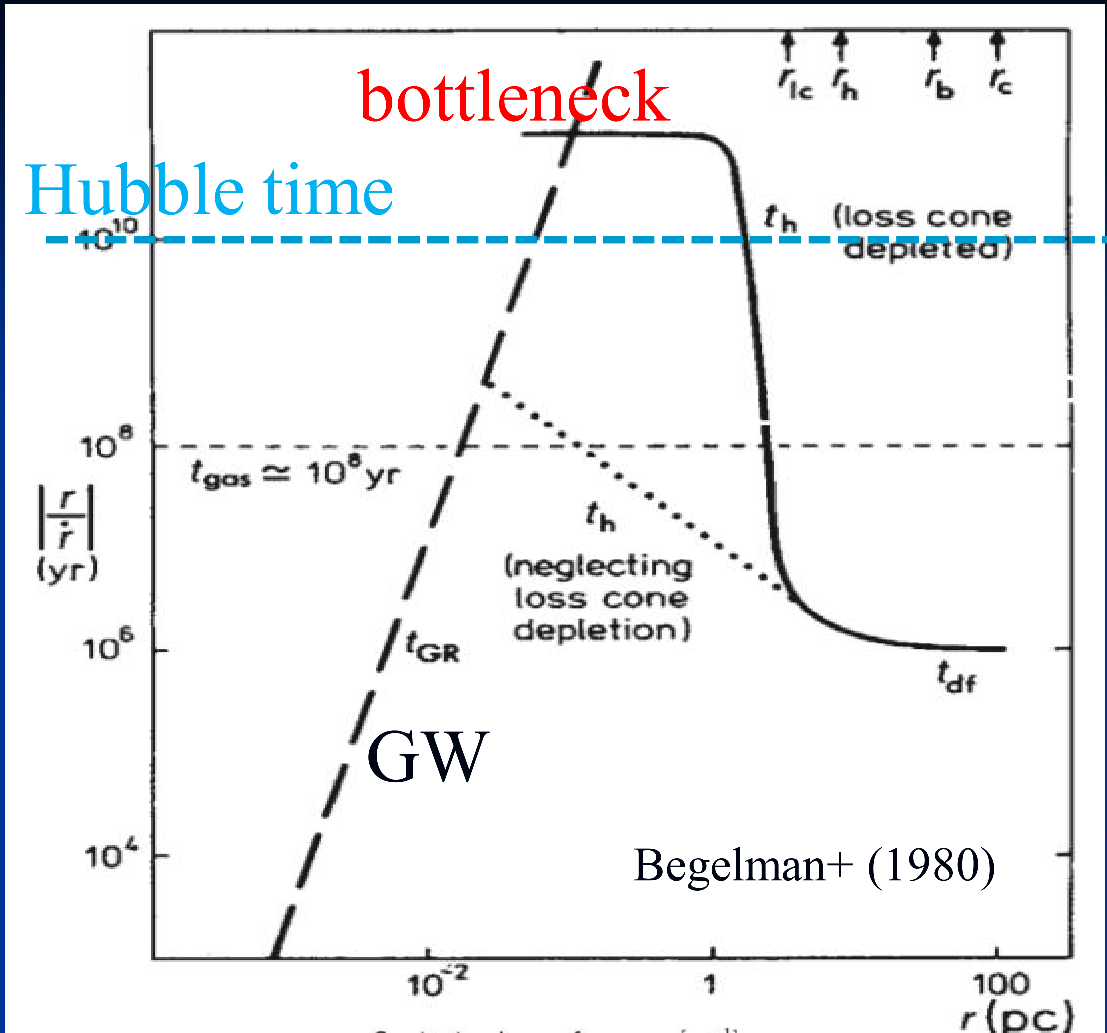
SPark, DBak,JLee,IPark JCAP 2022 (Pyultralight+ Mulguisin)

# Dynamical friction

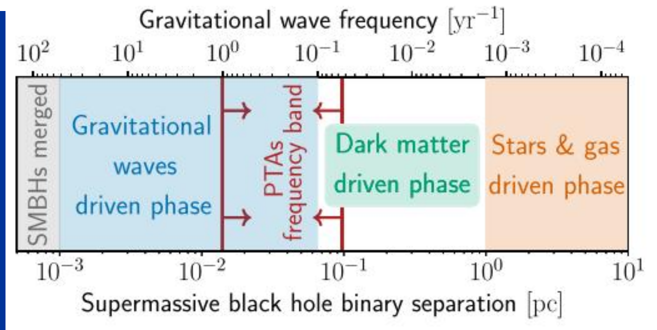



Dynamical friction by ULDM
Wang+ 2110.03428

# Final pc problem



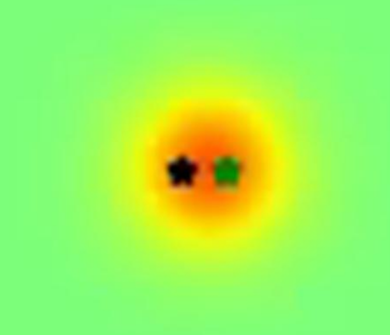


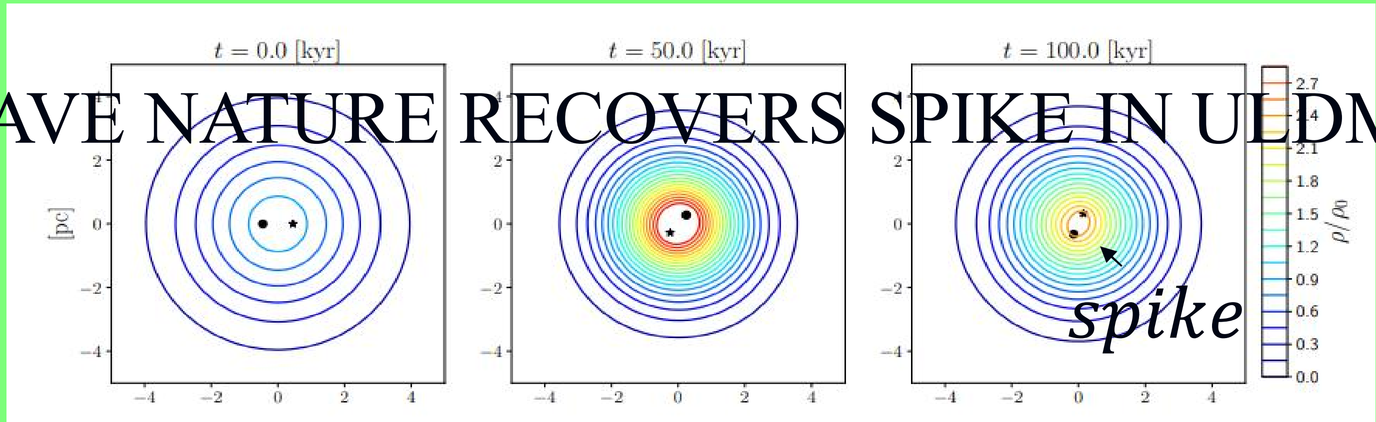



Alonso-Álvarez+

# BH binary (Pyultralight)

may solve final pc problem (Koo+ 2311.03412 PLB )

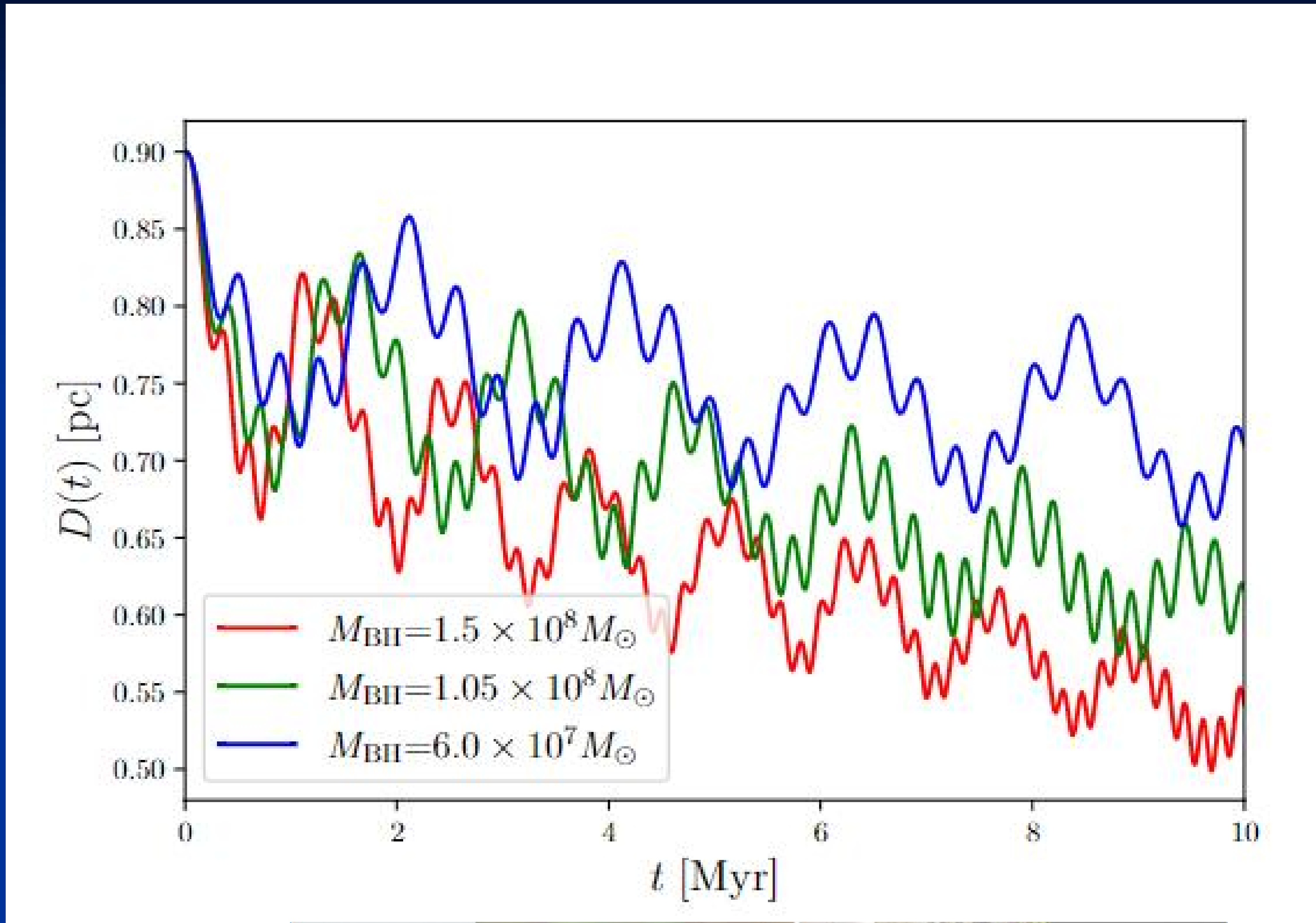





WAVE NATURE RECOVERS SPIKE IN ULDM

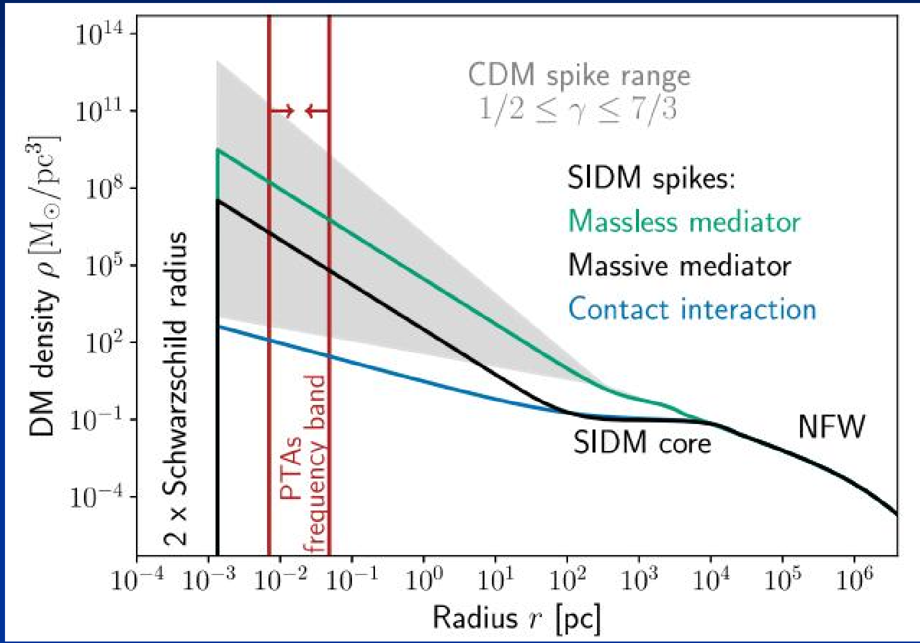
# FDM solution to final pc

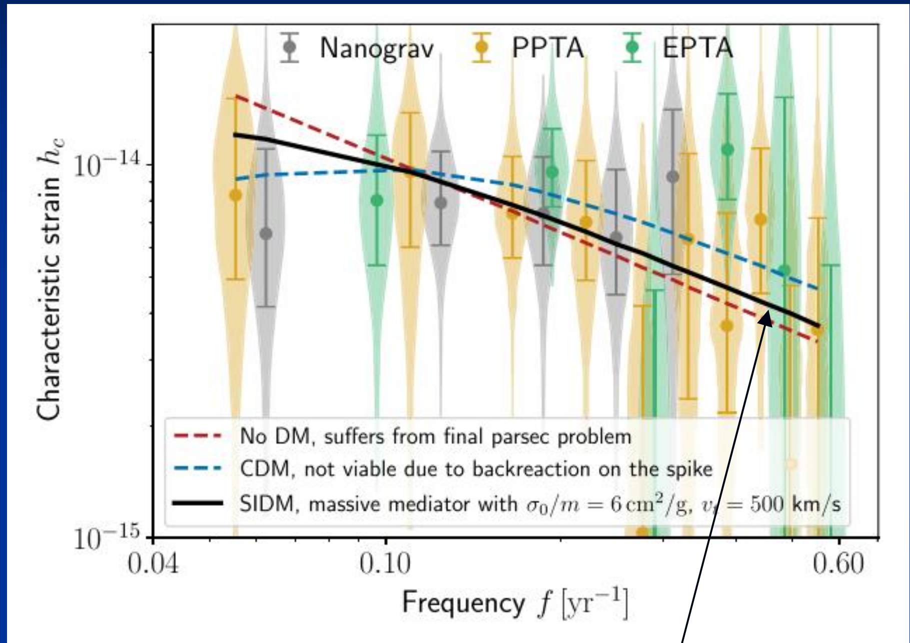
Koo+ 2311.03412 PLB



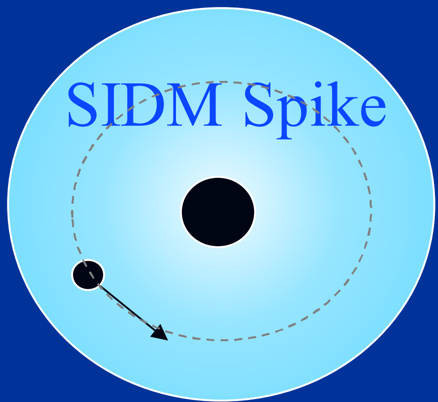

# SIDM(particle) solution to final pc

Alonso-Álvarez+ PRL



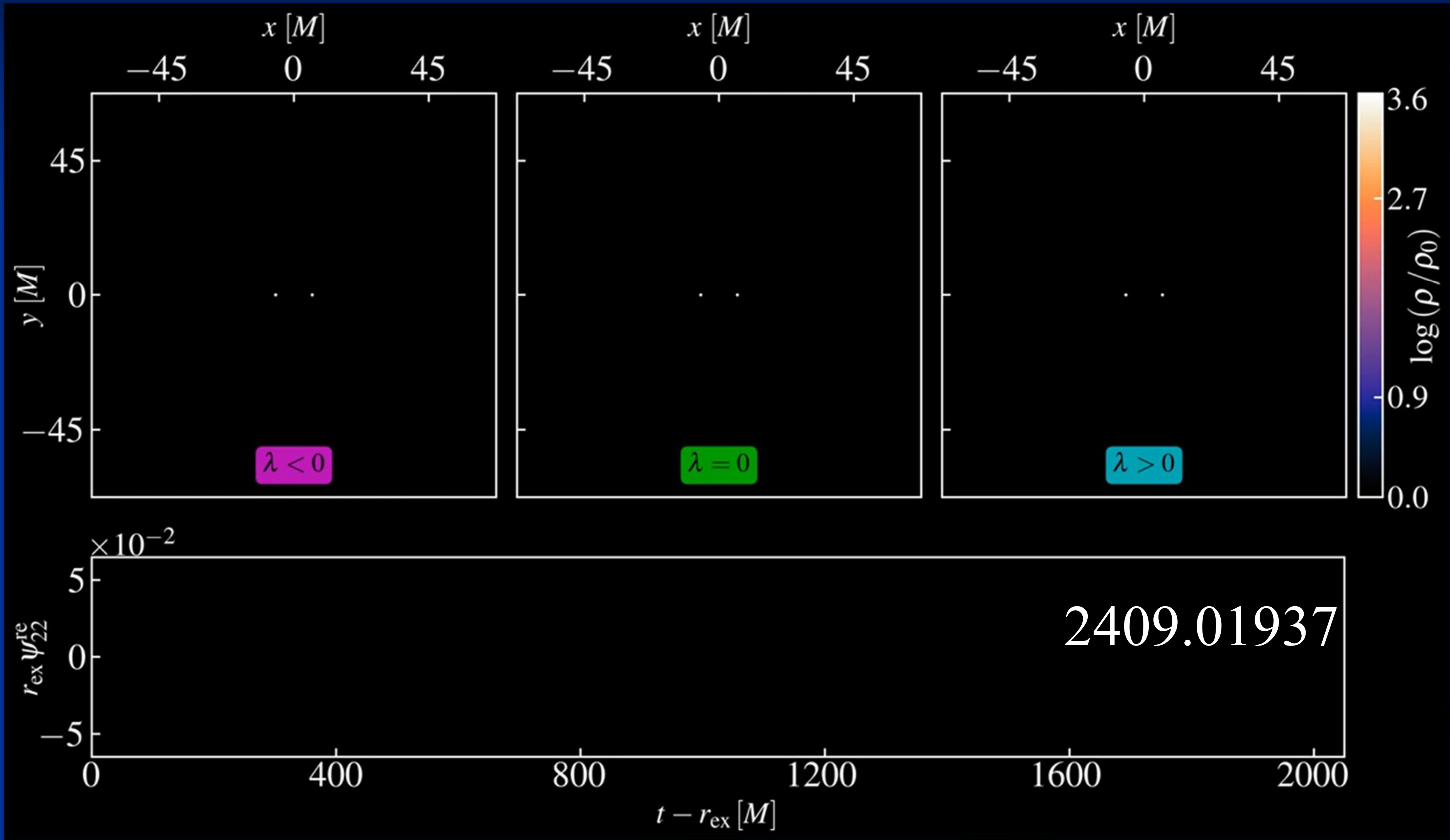




Thermal equilibrium recovers spike



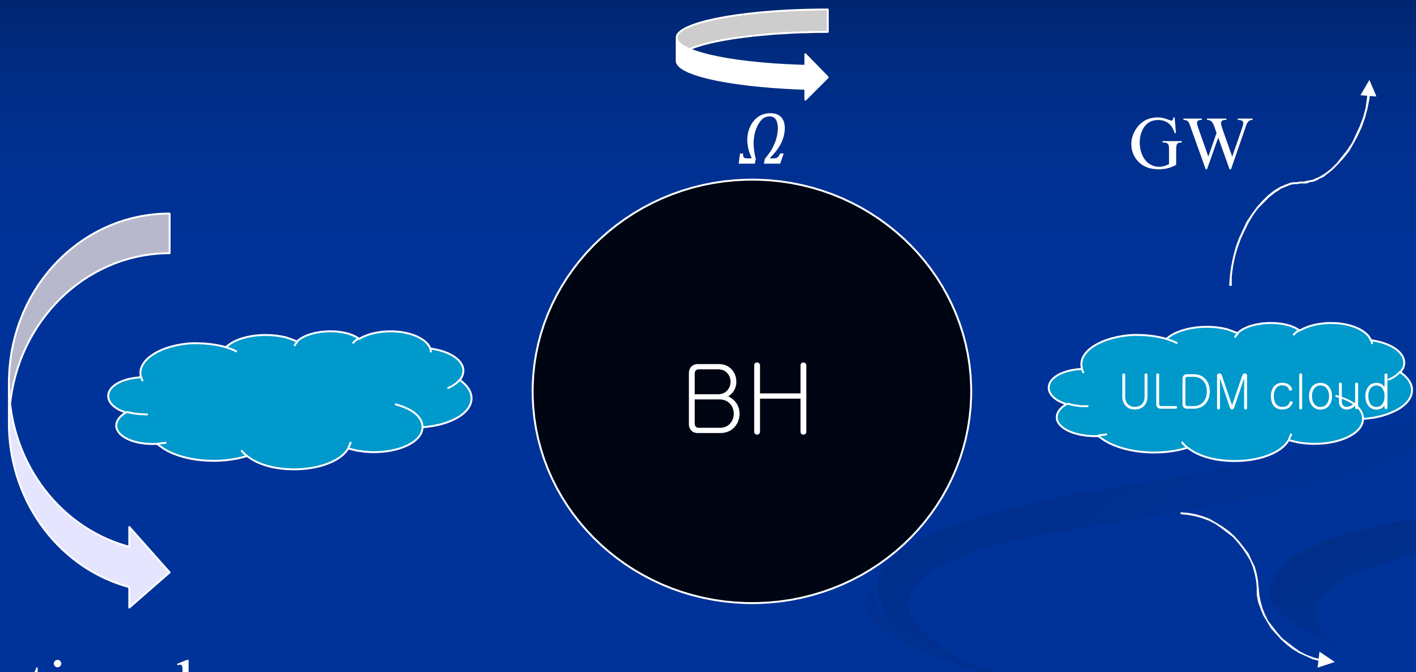

SIDM reduces GW due to DF

# Self-interacting scalar dark matter around binary black holes (Aurrekoetxea+ : GRChombo)




GW + BH + GR + DM

# Superradiance (Penrose process)




Gravitational atom

$$\Phi(t, r, \theta, \phi) = e^{-i\omega t} e^{im\phi} S(\theta) R(r)$$

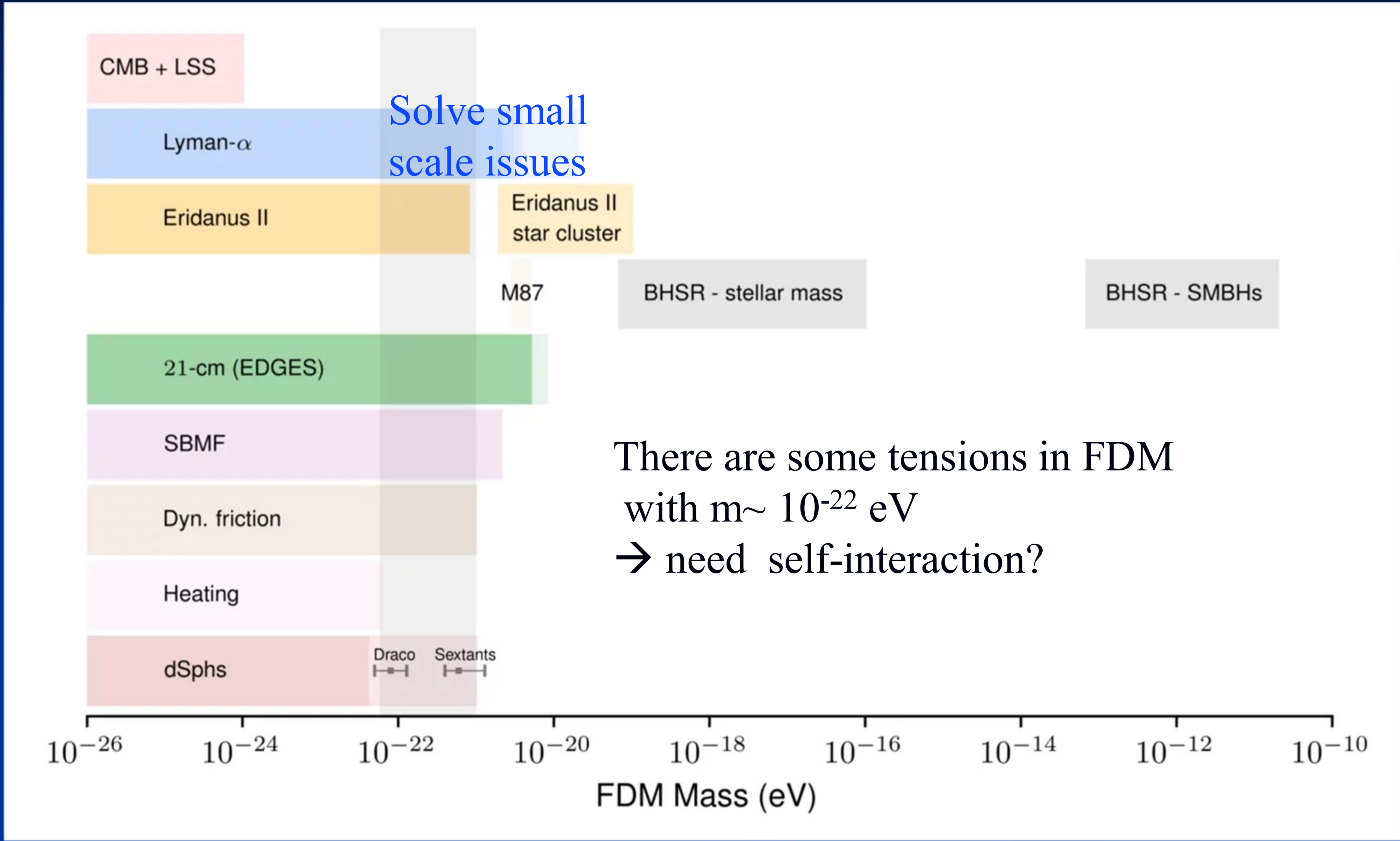
$$R(r) \propto e^{-i(\omega - m\Omega\ \ ) r_*}$$

m: mode

- outgoing mode if $\omega_{nlm} < m\Omega$ → BH spin changes
- can change spectrum from accretion disk & GW patterns from BHs

Zhang & Yang 2018

# Constraints on FDM mass




There are some tensions in FDM with m~ $10^{-22}$ eV

→ need self-interaction?

may be m > $10^{-21}$ eV?

MPA

# Why we study self-interacting ULDM?

We can

- allow wider mass range

→avoid some tensions of FDM
- detect ULDM! (coupling to SM induces self.int)
- calculate abundance (i.e., F)
- find the particle physics model and BSM

Thermal equilibrium also require interactions.

# Self-Interacting ULDM

Lee and Koh (PRD 53, hep-ph/9507385)

Galactic DM is described by coherent scalar field

Action

$$S = \int \sqrt{-g} d^4x \left[ \frac{-R}{16\pi G} - \frac{g^{\mu\nu}}{2}\phi^*_{;\mu}\phi_{;\nu} - \frac{m^2}{2}|\phi|^2 - \frac{\lambda}{4}|\phi|^4 \right]$$

typical phi4 theory with gravity

Metric

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega \quad \textit{Spherical.}$$

Field

$$\phi(r,t) = (4\pi G)^{-\frac{1}{2}}\sigma(r)e^{-i\omega t}$$

Stationary spherical

Exact ground state

$$\sigma_* = \sqrt{\frac{\gamma_0 \operatorname{Sin}(\sqrt{2}r_*)}{\sqrt{2}r_*}}$$

$$r_* = r\Lambda^{-1/2}$$

$\Lambda \equiv \frac{\lambda m_P^2}{4\pi m^2}$, $\Lambda >> 1$ (Newtonian & TF limit)

*New length scale* $R_{TF} \approx \frac{\sqrt{\Lambda}}{m} = \sqrt{\frac{\pi\hbar^3\lambda}{8cGm^4}}$

*& mass scale* $M_{\max} = \sqrt{\Lambda}\frac{m_P^2}{m}$

Even tiny self-interaction drastically changes the scales!

→ allows wider range for m

# Detection

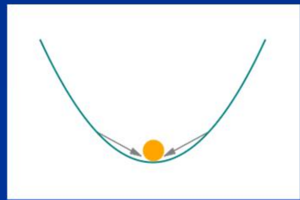
dilatonic coupling

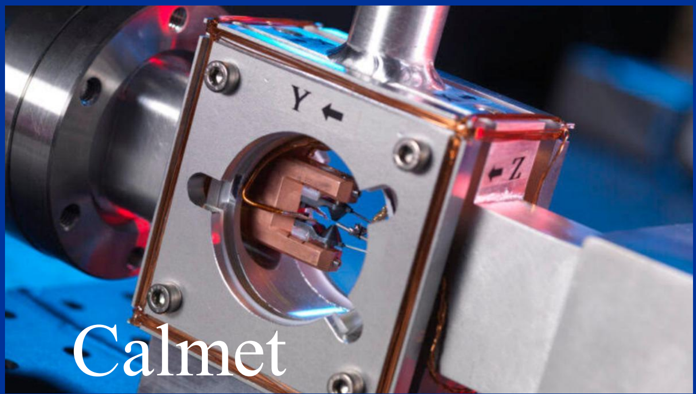
$$\mathcal{L} \supset \varphi \frac{d_e}{4\mu_0} F_{\mu\nu} F^{\mu\nu},$$

Using atomic clocks to detect ULDM by mimickin g time variations of fundamental constants

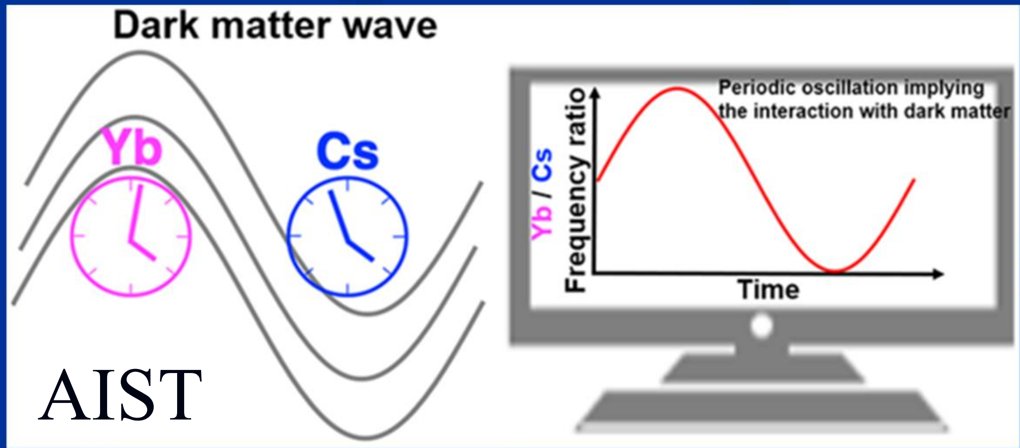
$$\alpha(t) \approx \alpha\ [1 + d_e \varphi_0 \cos(\omega t + \delta)]$$

Oscillation of fine structure constant






Due to nuclear and atomic structure Yb and Cs have different frequency dependency on α (special relativistic effects)



# Typical scales for selfint. ULDM

are functions of $\tilde{m} = \frac{m}{\lambda^{1/4}}$ ~ energy scale of ULDM

1) time

$t_c \simeq (G\bar{\rho})^{-1/2}$ : Hubble time

2) length ( Jeans length)

$$\lambda_J = 2\pi/k_J = \sqrt{\frac{\pi\hbar^3\lambda}{2cGm^4}} = 0.978kpc\left(\frac{\tilde{m}}{10\mathrm{eV}}\right)^{-2}$$ t-indep. → $\tilde{m} \sim 10\mathrm{eV}$

3) mass $$M_J = \frac{4\pi}{3}\left(\frac{\lambda_J}{2}\right)^3\bar{\rho} = \frac{\pi^{5/2}}{\sqrt{288}}\left(\frac{\hbar^3\lambda}{cGm^4}\right)^{3/2}\bar{\rho} = 4.908\times 10^6 M_\odot\left(\frac{\tilde{m}}{10eV}\right)^{-6}\left(\frac{\bar{\rho}}{10^{-2}M_\odot/pc^3}\right)$$

4) velocity $$v_c \equiv x_c/t_c = \frac{2^{7/4}\left(\frac{cG^3m^1}{\hbar^3\lambda}\right)^{1/4}}{\pi^{1/4}}\sqrt{M} = 59.28\,\mathrm{km/s}\left(\frac{M}{10^8M_\odot}\right)^{1/2}\left(\frac{\tilde{m}}{10\mathrm{eV}}\right)$$

5) Angular momentum

$$L_c = M\mathbf{x}_c v_c = \left(\frac{32\pi G\hbar^3\lambda}{cm^4}\right)^{1/4} M^{3/2} = 3.375\times 10^{96}\hbar\left(\frac{M}{10^8 M_\odot}\right)^{3/2}\left(\frac{10\mathrm{eV}}{\tilde{m}}\right)$$

6) acceleration

$$a_c = x_c/t_c^2 = \frac{16cG^2m^4M}{\pi\hbar^3\lambda} = 1.163\times 10^{-10}\ \mathrm{meter/s^2}\left(\frac{\tilde{m}}{10\mathrm{eV}}\right)^4\left(\frac{M}{10^8 M_\odot}\right)$$

cf) MOND scale $a_0 = 1.2\times 10^{-10} meter/s^2$

7) potential $V_c$ =1 gives Max. Galaxy mass M ~ $10^{16} M_\odot$

$$V_c = \frac{GM}{x_c} = \frac{GM\sqrt{\frac{2}{\pi}}}{\sqrt{\frac{\hbar^3\lambda}{cGm^4}}} = 4.888\times 10^{-9}c^2\left(\frac{\tilde{m}}{10\mathrm{eV}}\right)^2\left(\frac{M}{10^8 M_\odot}\right)$$

8) density

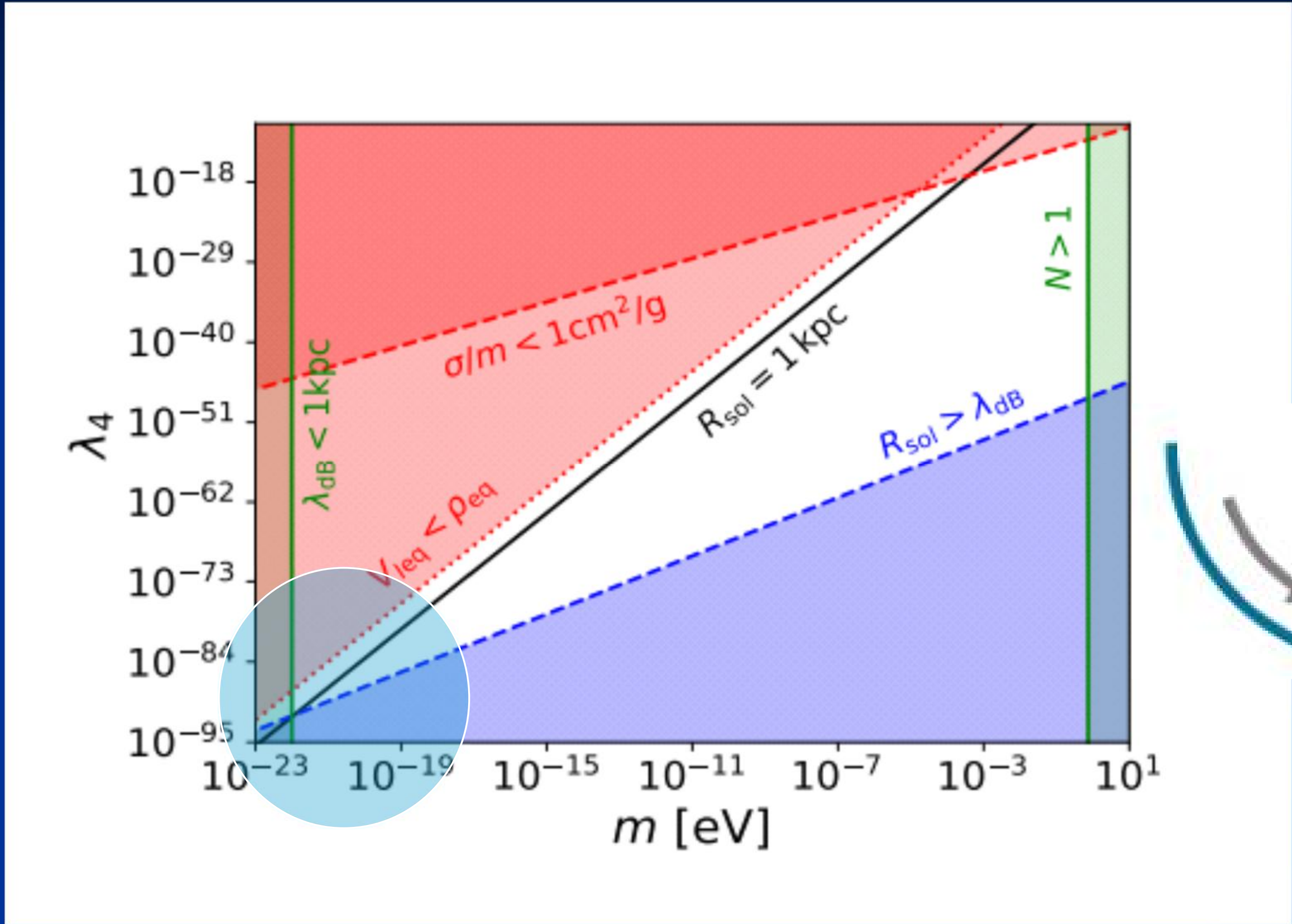
$$\rho_c = \frac{2\sqrt{2}M\left(\frac{cGm^4}{\lambda}\right)^{3/2}}{\hbar^{9/2}\pi^{3/2}} = 0.106 M_\odot/pc^3\left(\frac{\tilde{m}}{10\mathrm{eV}}\right)^6\left(\frac{M}{10^8 M_\odot}\right)$$

9) surface density observed $\Sigma_c = 10^{2.15\pm 0.2} M_\odot \mathrm{pc}^{-2}$

$$\Sigma = \frac{\hbar\pi^{3/2}\sqrt{\frac{\hbar\lambda}{CGm^4}}\rho}{6\sqrt{2}} = 5.124 M_\odot/pc^2\left(\frac{\tilde{m}}{10eV}\right)^{-2}\left(\frac{\bar{\rho}}{10^{-2}M_\odot/pc^3}\right)$$

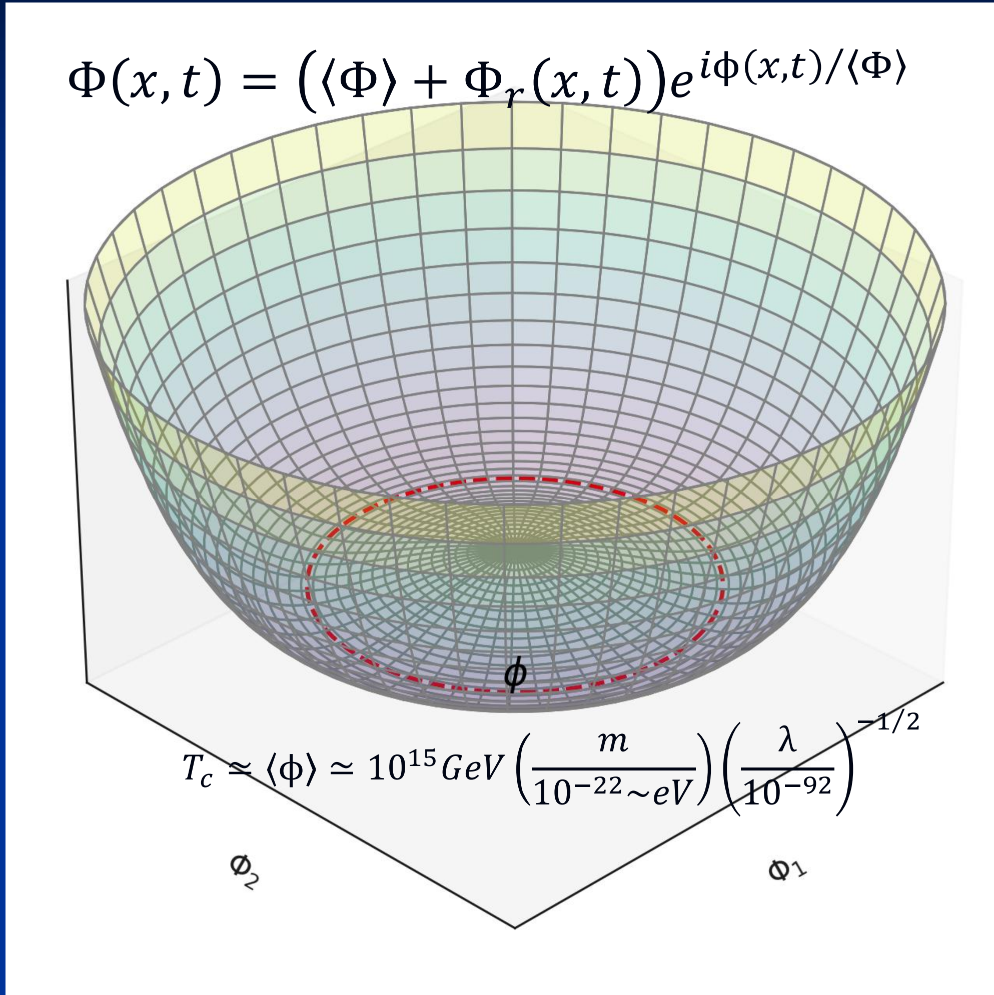
# cosmological constraints

Garcia + 2304.10221




# SSB

**JLee 2410.02842**




Galaxy observation gives GUT scale!

Proposal: ULDM is a pNGB associated with GUT scale SSB

$$T_c \simeq v \sim m/\sqrt{\lambda}$$

JLee 2410.02842

# Neutrino mass

$$\mathcal{L}_{Yukawa} = -y\phi\bar{\nu}^{c}\nu$$ Majonara $\nu$

real

$$m_\nu = 0.1\text{eV}\left(\frac{y}{10^{-25}}\right)\left(\frac{v}{10^{15}\text{GeV}}\right)$$

$$y = \frac{m_\nu\sqrt{\lambda}}{m_\phi} = 10^{-25}\left(\frac{m_\nu}{0.1\text{eV}}\right)\left(\frac{m_\phi}{10^{-22}\text{eV}}\right)^{-1}\left(\frac{\lambda}{10^{-92}}\right)^{1/2}$$

One-loop quantum correction from the Yukawa is $O(y^4) \simeq 10^{-8}\lambda \ll \lambda$

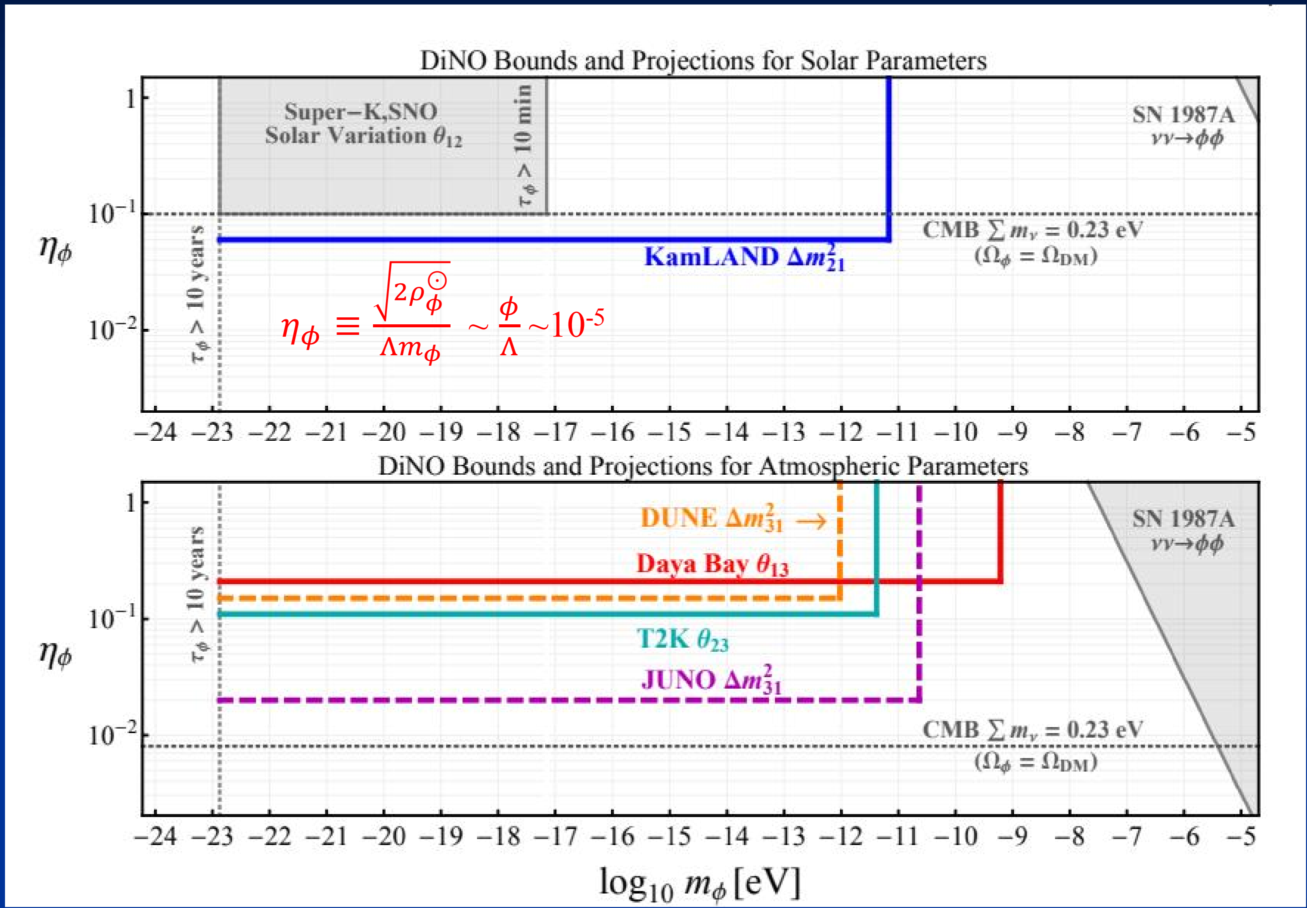
0.06 eV< $\Sigma m_\nu$ <0.071 eV (DESI)

# Hierarchy

**JLee 2410.02842**

| | |
|---|---|
| GUT | $T_c \simeq m/\sqrt{\lambda} = \tilde{m}^2/m \sim 10^{15}$ GeV |
| neutrino | $\tilde{m} \equiv m/\lambda^{1/4} \sim 10$ eV |
| | reverting Type I seesaw |
| EW | $T_{EW} \sim (T_c\tilde{m})^{1/2} \sim 10^3$ GeV |
| PNGB ULDM | m $\sim 10^{-22}$ eV, $\lambda \sim 10^{-92}$ |
| | galaxy observations |

# neutrino oscillation with ULDM




Krnjaic+ PRD

$$\mathcal{L}_{eff} = -m_\nu\left(1 + y_1\frac{\phi}{\Lambda}\right)\nu\nu,\ \ \phi(x,t) \simeq \frac{\sqrt{2\rho_\phi^\odot}}{m_\phi}\cos[m_\phi(t - \vec{v}\cdot\vec{x})] \sim 10^{10}\text{GeV}$$

# Conclusion

- *FDM with m~$10^{-22}$ eV or*
  *Self-interacting ULDM with* $\frac{m}{\lambda^{1/4}}$ $\sim 10eV$ *is consistent with many cosmological observations*

- *Tc is about GUT scale and SSB of ULDM can give neutrino masses and EW scale*

*→Oscillation of ULDM can be detected by neutrino osc. and other experiments (GW, atomic clock)*

*ULDM possibly explain satellite plane, final pc, Hubble tension, S8, and many other mysteries*